Name_____ Date_____ Period_____

Worksheet 3.3—Increasing, Decreasing, and 1st Derivative Test

Show all work. No calculator unless otherwise stated.

Multiple Choice

_____ 1. Determine the increasing and decreasing open intervals of the function $f(x)=(x-3)^{4/5}(x+1)^{1/5}$ over its domain. Tip: factor out least powers from the derivative to put it into full-fledged-factored-form!

(A) Inc: $\left(-1,-\frac{1}{5}\right)$, Dec: $\left(-\frac{1}{5},\infty\right)$

(B) Inc: $\left(-1,-\frac{1}{5}\right)\cup(3,\infty)$, Dec: $\left(-\frac{1}{5},3\right)$

(C) Inc: $(-\infty,-1)\cup(3,\infty)$, Dec: $(-1,3)$

(D) Inc: $\left(-\infty,-\frac{1}{5}\right)\cup(3,\infty)$, Dec: $\left(-\frac{1}{5},3\right)$

(E) Inc: $\left(-\frac{1}{5},3\right)\cup(3,\infty)$, Dec: $\left(-1,\frac{1}{5}\right)\cup(3,\infty)$

_____ 2. Let *f* be the function defined by $f(x)=x-\cos 2x,\ -\pi\le x\le\pi$. Determine all open interval(s) on which *f* is decreasing.

(A) $\left(-\frac{5\pi}{12},-\frac{\pi}{12}\right),\left(\frac{7\pi}{12},\frac{11\pi}{12}\right)$

(B) $\left(-\frac{5\pi}{12},-\frac{\pi}{6}\right),\left(\frac{\pi}{6},\frac{11\pi}{12}\right)$

(C) $\left(-\frac{5\pi}{12},-\frac{\pi}{8}\right),\left(\frac{3\pi}{8},\frac{11\pi}{12}\right)$

(D) $\left(-\frac{\pi}{6},-\frac{\pi}{12}\right),\left(\frac{\pi}{6},\frac{11\pi}{12}\right)$

(E) $\left(-\pi,-\frac{5\pi}{12}\right),\left(\frac{7\pi}{12},\pi\right)$

3. Let $f(x) = x\left(4 + x^2 - \frac{x^4}{5}\right)$.

_____ (i) Which of the following is $f'(x)$?

(A) $f'(x) = (1+x^2)(5-x^2)$

(B) $f'(x) = (1+x^2)(4-x^2)$

(C) $f'(x) = (1-x^2)(5+x^2)$

(D) $f'(x) = (1-x^2)(4+x^2)$

(E) $f'(x) = (1-x^2)(4-x^2)$

_____ (ii) Find the open interval(s) on which f is increasing.

(A) $(-\infty,-2)\cup(2,\infty)$

(B) $(-\infty,-\sqrt{5})\cup(\sqrt{5},\infty)$

(C) $(-2,2)$

(D) $(-\infty,-1)\cup(1,\infty)$

(E) $(-1,1)$

_____ 4. The derivative of a function f is given for all x by $f'(x) = (2x^2+4x-16)(1+g^2(x))$ where g is some unspecified function. At which value(s) of x will f have a local maximum?

Note: $g^2(x) = (g(x))^2$

(A) $x = -4$

(B) $x = 4$

(C) $x = -2$

(D) $x = 2$

(E) $x = -4,\ 2$

_____ 5. Which of the following statements about the absolute maximum and absolute minimum values of $f(x)=\dfrac{x^3-4x^2-6x-1}{x+1}$ on the interval $[0,\infty)$ are correct? (Hint: Think of what type of discontinuity does $f(x)$ have??? $\dfrac{0}{0}$ or $\dfrac{\neq 0}{0}$)

(A) Max $=13$, No Min

(B) No Max, Min $=-\dfrac{29}{4}$

(C) Max $=13$, Min $=-\dfrac{29}{4}$

(D) Max $=5$, No Min

(E) No Max, Min $=-1$

_____ 6. (**Calculator Permitted**) The first derivative of the function f is defined by $f'(x)=\cos\left(x^3-x\right)$ for $0\le x\le 2$. On what intervals is f increasing?

(A) $0\le x\le 1.445$ only

(B) $1.445\le x\le 1.875$

(C) $1.691\le x\le 2$

(D) $0\le x\le 1$ and $1.691\le x\le 2$

(E) $0\le x\le 1.445$ and $1.875\le x\le 2$

Short Answer

7. For each of the following, find the critical values (on the indicated intervals, if indicated.) Remember, a critical value MUST be in the domain of the function, though it may not be in the domain of the derivative.

(a) $f(x)=x^2(3-x)$ (b) $f(x)=\dfrac{\sin x}{1+\cos^2 x},\ [0,2\pi]$ (c) $f(x)=\dfrac{x^2}{x^2-9}$

8. Determine the local extrema of each of the following functions (on the indicated interval, if indicated). Be sure to say which type each is. **Justify** (this means write a sentence.)

(a) $f(x)=\cos^2(2x),\ [0,2\pi]$ (b) $f(x)=x+\dfrac{1}{x}$

(c) $f(x)=\sin^2 x+\sin x,\ [0,2\pi]$ (d) $f(x)=\dfrac{x^2-3x-4}{x-2}$

9. Assume that f is differentiable for all x. The signs of f' are as follows.

$$f'(x) > 0 \text{ on } (-\infty,-4)\cup(6,\infty) \text{ and } f'(x) < 0 \text{ on } (-4,6)$$

Let $g(x)$ be a transformation of $f(x)$. Supply the appropriate inequality $(>,<,\geq,\leq)$ for the indicated value of c in the given blank.

Function	**Sign of $g'(c)$**
(a) $g(x) = f(x) + 5$	$g'(0)$_____ 0
(b) $g(x) = 3f(x) - 3$	$g'(-5)$_____ 0
(c) $g(x) = -f(x)$	$g'(-6)$_____ 0
(d) $g(x) = -f(x)$	$g'(0)$_____ 0
(e) $g(x) = f(x-10)$	$g'(0)$_____ 0
(h) $g(x) = f(x-10)$	$g'(8)$_____ 0
(i) $g(x) = f(-x)$	$g'(8)$_____ 0
(j) $g(x) = f(-x)$	$g'(-8)$_____ 0