Name_______________________________________ Date_______________________ Period_____

Worksheet 2.7—Implicit Differentiation

Show all work. No calculator unless otherwise stated.

Short Answer

1. Find $\frac{dy}{dx}$. Show all necessary rewrites and show $\frac{d}{dx}[\text{Left Side}] = \frac{d}{dx}[\text{Right Side}]$.

(a) $x^3 - 3x^2 y + 4xy^2 = 12$

(b) $\sqrt{xy} = x + 3y$

(c) $4\sin 2y\cos x = 2$

(d) $\left(y^2 + 2\sec y\right)^2 = 4\left(x+1\right)^2$

(e) $x = y\sec\left(\frac{5}{y}\right)$

2. Find $\frac{dy}{dx}$ at the indicated point, then find the equation of the indicated line at the point.

(a) $y^2 = \frac{x^2 - 4}{x^2 + 4}$ at $(2,0)$, tangent line

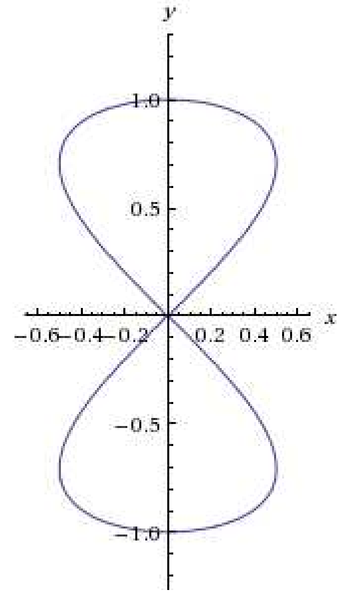
(b) $(x+y)^3 = x^3 + y^3$ at $(-1,1)$, normal line

3. Find $\frac{d^2y}{dx^2}$ in terms of x and y.

(a) $1 - xy = x - y$

(b) $\sqrt[3]{x^2} + \sqrt[3]{y^2} = 1$

4. The graph of the equation $y^4 = y^2 - x^2$ is shown at right.



Graph generated by WolframAlpha

(a) Verify that $\frac{dy}{dx} = \frac{2x}{2y - 4y^3}$. Show the work that leads to your answer.

(b) Find the point(s) (x, y) at which the graph has a horizontal tangent line. Show the work that leads to your answer.

(c) Find the *y*-value(s) which the graph has a vertical tangent line. Show the work that leads to your answer.

(d) While the graph exists at $(0,0)$, the slope does not. Using $\frac{dy}{dx}$, explain why this is so.

5. Consider the curve given by $6+x^3y=xy^2$

(a) Show that $\dfrac{dy}{dx}=\dfrac{y^2-3x^2y}{x^3-2xy}$

(b) Find every point(s) on the graph of the curve that has an x-coordinate of 1, then write an equation for the tangent line at every/each of these point(s).

(c) The graph of the curve has vertical tangent lines. Find the x-coordinate of each of these vertical tangent lines. Show the work that leads to your answer.

6. Find the two points where the curve $x^2+xy+y^2=7$ crosses the x-axis, and show that the tangents to the curve at these points are parallel. What is the common slope of these tangents?

7. Find the equations of the **normal** lines to the curve $xy + 2x - y = 0$ that are parallel to the line $2x + y = 0$

8. The slope of the tangent line is -1 at the point $(0,1)$ on the graph of $x^3 - 6xy - ky^3 = a$, where k and a are constants. The values of the constants a and k are what?

Multiple Choice

_____ 9. Find y' when $xy + 5x + 2x^2 = 4$.

(A) $y' = \dfrac{5 + 2x - y}{x}$

(B) $y' = -\dfrac{y + 5 + 4x}{x}$

(C) $y' = -(y + 5 + 4x)$

(D) $y' = -\dfrac{y + 5 + 2x}{x}$

(E) $y' = \dfrac{y + 5 + 4x}{x}$

_____ 10. Find $\frac{dy}{dx}$ when $\frac{3}{\sqrt{x}}+\frac{2}{\sqrt{y}}=4$

(A) $\frac{dy}{dx}=-\frac{3}{2}\left(\frac{y}{x}\right)^{3/2}$

(B) $\frac{dy}{dx}=\frac{3}{2}(xy)^{1/2}$

(C) $\frac{dy}{dx}=-\frac{2}{3}\left(\frac{x}{y}\right)^{3/2}$

(D) $\frac{dy}{dx}=\frac{2}{3}\left(\frac{x}{y}\right)^{3/2}$

(E) $\frac{dy}{dx}=\frac{3}{2}\left(\frac{y}{x}\right)^{3/2}$

_____ 11. Find the equation of the tangent line to the graph of $y^2-xy-12=0$ at the point $(1,4)$.

(A) $3y=2x+10$

(B) $3y+2x=10$

(C) $y=4x$

(D) $7y=4x+24$

(E) $7y+4x=24$

_____ 12. The points *P* and *Q* on the graph of $y^2-xy+8=0$ have the same *x*-coordinate, $x=6$. The point of intersection of the tangents to the graph at *P* and *Q* is

(A) $\left(\frac{8}{3},\frac{16}{3}\right)$

(B) $\left(\frac{16}{3},\frac{8}{3}\right)$

(C) $\left(\frac{16}{3},\frac{16}{3}\right)$

(D) $\left(\frac{8}{3},\frac{8}{3}\right)$

(E) $\left(\frac{8}{3},\frac{2}{3}\right)$

_____ 13. Determine $\frac{d^2y}{dx^2}$ when $4x^2+3y^2=4$

(A) $\frac{d^2y}{dx^2}=\frac{16}{9y^2}$

(B) $\frac{d^2y}{dx^2}=-\frac{16}{9y^2}$

(C) $\frac{d^2y}{dx^2}=-\frac{4}{9y^3}$

(D) $\frac{d^2y}{dx^2}=-\frac{16}{9y^3}$

(E) $\frac{d^2y}{dx^2}=\frac{16}{9y^3}$

_____ 14. When an object is placed at a distance p from a convex lens having focal length of 5 cm, the image will be at a distance q cm from the lens, with $\frac{1}{5}=\frac{1}{p}+\frac{1}{q}$. Find the rate of change of p with respect to q. (Hint: Based on your answer choices, you should try to explicitly solve for either p or q before differentiating).

(A) $\frac{dp}{dq}=\frac{25}{q-5}$

(B) $\frac{dp}{dq}=\frac{25}{(q-5)^2}$

(C) $\frac{dp}{dq}=-\frac{25}{q-5}$

(D) $\frac{dp}{dq}=-\frac{5}{(q-5)^2}$

(E) $\frac{dp}{dq}=-\frac{25}{(q-5)^2}$