Name_____ Date_____ Period_____

Worksheet 2.6—The Chain Rule

Short Answer

Show all work, including rewriting the original problem in a more useful way. No calculator unless otherwise stated.

1. Find the derivative of the following functions with respect to the independent variable. (You do not need to simplify your final answers here.)

(a) $y=(2x-7)^3$ (b) $y=\dfrac{1}{t^2+3t-1}$ (c) $y=\left(\dfrac{1}{t-3}\right)^2$

(d) $y=\csc^3\left(\dfrac{3x}{2}\right)$ (e) $y=3\sec^2(\pi t-1)$ (f) $y=\sin\sqrt[3]{x}+\sqrt[3]{\sin x}$

(g) $y=x^2\tan\dfrac{1}{x}$ (h) $r=\sec 2\theta\tan 2\theta$ (i) $f(x)=\sqrt[3]{\csc^5 7}$

2. Find the equation of the tangent line (in Taylor Form) for each of the following at the indicated point.

(a) $s(t)=\sqrt{t^2+2t+8}$ at $x=2$ (b) $f(t)=\dfrac{3t+2}{t-1}$ at $(0,-2)$

3. Determine the point(s) in the interval $(0,2\pi)$ at which the graph of $f(x)=2\cos x+\sin 2x$ has a horizontal tangent.

4. Find the second derivative of each of the following functions. Remember to simplify early and often.

(a) $f(x)=2\left(x^2-1\right)^3$ (b) $f(x)=\sin\left(x^2\right)$

5. If $h(x)=\tan(2x)$, evaluate $h''(x)$ at $\left(\frac{\pi}{6},\sqrt{3}\right)$. Simplify early and often.

6. If $g(5)=-3$, $g'(5)=6$, $h(5)=3$, and $h'(5)=-2$, find $f'(5)$ (if possible) for each of the following. If it is not possible, state what additional information is required.

(a) $f(x)=\frac{g(x)}{h(x)}$ (b) $f(x)=g(h(x))$ (c) $f(x)=g(x)h(x)$

(d) $f(x)=[g(x)]^3$ (e) $f(x)=g(x+h(x))$ (f) $f(x)=(g(x)+h(x))^{-2}$

7. Find the derivative of $f(x)=\sin^2 x+\cos^2 x$ two different ways,

(a) By using the chain rule on the given expression.

(b) By using an identity first, then differentiating.

(c) What's the moral of THIS story? (Hint: It is NOT *"Flattery is a dangerous weapon in the hands of the enemy."*)

8. Using calculus and trig Identities, prove that if $f(x)=\tan^2 x$ and $g(x)=\sec^2 x$, then $f'(x)=g'(x)$.

9. Using the chain rule,

(a) Prove that the derivative of an odd function is an even function. That is if $f(-x)=-f(x)$, then $f'(-x)=f'(x)$.

(b) What type of function do you think the derivative of an even function is? Justify in a manner similar to part (a).

10. As demonstrated on the last example in the notes,

(a) Using the chain rule, prove that if $|g(x)| = \sqrt{g^2(x)}$ then $\frac{d}{dx}\left[|g(x)|\right] = \frac{g(x)}{|g(x)|} \cdot g'(x),\ g(x) \neq 0$.

(b) Use the result from part (a) to find $\frac{d}{dx}\left[|x^2 - 4|\right]$.

11. What is the largest value possible for the slope of the curve of $y = \sin\left(\frac{x}{2}\right)$? Justify.

12. Find the equation of the <u>normal</u> line to the curve $y = 2\tan\left(\frac{\pi x}{4}\right)$ at $x = 1$.

13. After the chain rule is applied to find the derivative of a function $F(x)$, the function $F'(x) = f(x) = 4(\cos(3x))^3 \cdot (-\sin(3x)) \cdot 3$ is obtained. Give a possible function for $F(x)$. Check your work by taking the derivative of your guess using the chain rule.

Multiple Choice

_____ 14. If $f(x)=\frac{1}{\sqrt{x^2+3}}$, find $f'(x)$.

(A) $f'(x)=-\frac{x}{\sqrt{\left(x^2+3\right)^3}}$

(B) $f'(x)=\frac{x}{\sqrt{x^2+3}}$

(C) $f'(x)=-\frac{x}{\left(x^2+3\right)\sqrt{2x}}$

(D) $f'(x)=-\frac{1}{2\sqrt{\left(x^2+3\right)^3}}$

(E) $f'(x)=-\frac{x^2+3x}{x^2+3}$

_____15. If $g(x)=(1-x)^3(4x+1)$, then $g'(x)=$

(A) $-12(1-x)^2$

(B) $(1-x)^2(1+8x)$

(C) $(1-x)^2(1-16x)$

(D) $3(1-x)^2(4x+1)$

(E) $(1-x)^2(16x+7)$

_____ 16. $\frac{d}{dx}\left[\left(\frac{x^2-3}{5x^2-9}\right)^5\right]=$

(A) $\frac{10x(x^2-3)^4(10x^2-17)}{(5x^2-9)^6}$

(B) $\frac{-10x(x^2-3)^4(5x^2-16)}{(5x^2-9)^5}$

(C) $\frac{-240x(x^2-3)^4}{(5x^2-9)^6}$

(D) $\frac{60x(x^2-3)^4}{(5x^2-9)^6}$

(E) $\frac{100x(x^2-3)^4}{(5x^2-9)^6}$

_____ 17. A derivative of a function $f(x)$ is obtained using the chain rule. The result is $f'(x)=3\sec^3 x\tan x$. Which of the following could be $f(x)$?

I. $f(x)=-\pi+\frac{3}{4}\sec^4 x$

II. $f(x)=8+\sec^3 x$

III. $f(x)=\sec x+\sec x\tan^2 x$

(A) I only (B) II only (C) III only (D) II and III only (E) I, II, and III