Name KEY Date Period

Worksheet 10.2—Partial Fraction Decomposition
Show all work on a separate sheet of paper. No Calculator

Free Response & Short Answer

1. Evaluate $\int \frac{dx}{2x^3+x^2-x}$

$\int \frac{1}{x(2x^2+x-1)}dx$

$\int \frac{1}{x(2x-1)(x+1)}dx$

$\int\left[\frac{-1}{x}+\frac{4/3}{2x-1}+\frac{1/3}{x+1}\right]dx$

$-\ln|x|+\frac{4}{3}\left(\frac{1}{2}\right)\ln|2x-1|+\frac{1}{3}\ln|x+1|+C$

$\frac{1}{x(2x-1)(x+1)}=\frac{A}{x}+\frac{B}{2x-1}+\frac{C}{x+1}$

$1=A(2x-1)(x+1)+B(x)(x+1)+C(x)(2x-1)$

Let $x=-1$: $1=A(0)+B(-1)(0)+C(-1)(-3)$

$1=3C$

$\boxed{C=\frac{1}{3}}$

Let $x=0$: $1=A(-1)(1)+B(0)+C(0)$

$1=-A$

$\boxed{A=-1}$

Let $x=\frac{1}{2}$: $1=A(0)+B\left(\frac{1}{2}\right)\left(\frac{1}{2}+1\right)+C(0)$

$1=B\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)$

$1=\frac{3}{4}B$

$\boxed{B=\frac{4}{3}}$

2. Evaluate $\int \frac{x^2+12x-5}{(x+1)(x^2-6x-7)}dx$

$\int \frac{x^2+12x-5}{(x+1)(x-7)(x+1)}dx$

$\int \frac{x^2+12x-5}{(x+1)^2(x-7)}dx$

$\int\left[\frac{-1}{(x+1)}+\frac{2}{(x+1)^2}+\frac{2}{x-7}\right]dx$

$-\ln|x+1|-\frac{2}{x+1}+2\ln|x-7|+C$

$\frac{x^2+12x-5}{(x+1)^2(x-7)}=\frac{A}{x+1}+\frac{B}{(x+1)^2}+\frac{C}{x-7}$

$x^2+12x-5=A(x+1)(x-7)+B(x-7)+C(x+1)^2$

Let $x=-1$: $1-12-5=B(-8)$

$-16=-8B$

$\boxed{B=2}$

Let $x=7$: $49+84-5=C(8)^2$

$128=64C$

$\boxed{C=2}$

Let $x=1$: $1+12-5=A(2)(-6)+2(-6)+2(2^2)$

$8=-12A-12+8$

$12=-12A$

$\boxed{A=-1}$

3. Evaluate $\int \frac{8x^2-3x-4}{(4x-1)(x^2+1)}dx$

$\frac{8x^2-3x-4}{(4x-1)(x^2+1)}=\frac{A}{4x-1}+\frac{Bx+C}{x^2+1}$

$8x^2-3x-4=A(x^2+1)+(Bx+C)(4x-1)$

$x=\frac{1}{4}$: $8\left(\frac{1}{16}\right)-\frac{3}{4}-4=A\left(\frac{1}{16}+1\right)+0$

$\frac{1}{2}-\frac{3}{4}-4=\frac{17}{16}A$

$A=\frac{16}{17}\left(-\frac{1}{4}-4\right)$

$A=\frac{16}{17}\left(-\frac{17}{4}\right)$

$\boxed{A=-4}$

$x=1$: $8-3-4=-4(2)+(B+C)(3)$

$1=-8+3(B+C)$

$3(B+C)=9$

$\boxed{B+C=3}$

$x=-1$: $8+3-4=-4(2)+(C-B)(-5)$

$7=-8-5(C-B)$

$15=-5(C-B)$

$\boxed{C-B=-3}\rightarrow C=B-3$

So, $B+B-3=3$

$2B=6$

$\boxed{B=3}$

$C=3-3$

$\boxed{C=0}$

$\int\left[\frac{-4}{4x-1}+\frac{3x+0}{x^2+1}\right]dx$

$\int\left[-4\left(\frac{1}{4x-1}\right)+3\left(\frac{x}{x^2+1}\right)\right]dx$

$(-4)\left(\frac{1}{4}\right)\ln|4x-1|+3\left(\frac{1}{2}\right)\ln|x^2+1|+C$

$-\ln|4x-1|+\frac{3}{2}\ln(x^2+1)+C$

4. Evaluate $\int \frac{3x^3-5x^2-11x+9}{x^2-2x-3}dx$

Long Division 1st since deg num ≥ deg denom

$$\begin{array}{r} 3x+1 \\ x^2-2x-3\enclose{longdiv}{3x^3-5x^2-11x+9} \\ -(3x^3-6x^2-9x) \\ x^2-2x+9 \\ -(x^2-2x-3) \\ 12 \end{array}$$

$\int\left(3x+1+\frac{12}{(x-3)(x+1)}\right)dx$

$\int\left(3x+1+\frac{3}{x-3}+\frac{-3}{x+1}\right)dx$

Heaviside Cover-up Method (Non-repeating linear factors)

$\frac{3}{2}x^2+x+3\ln|x-3|-3\ln|x+1|+C$

$\frac{3}{2}x^2+x+3\ln\left|\frac{x-3}{x+1}\right|+C$