

Name KEY Date \_\_\_\_\_ Period \_\_\_\_\_

**Worksheet 8.1—Polar Intro & Derivatives**

Show all work. No calculator except unless specifically stated.

**Short Answer**

Convert the following equations to polar form.

1.  $y = 4$

$r \sin \theta = 4$

$r = \frac{4}{\sin \theta}$

$r = 4 \csc \theta$

2.  $3x - 5y + 2 = 0$

$3(r \cos \theta) - 5(r \sin \theta) + 2 = 0$

$r(3 \cos \theta - 5 \sin \theta) = -2$

$r = \frac{-2}{3 \cos \theta - 5 \sin \theta}$   
OR

$r = \frac{2}{5 \sin \theta - 3 \cos \theta}$

3.  $x^2 + y^2 = 25$

$r^2 = 25$

$r = -5$  or  $r = 5$

Convert the following equations to rectangular form.

4.  $r = 3 \sec \theta$

$r = 3 \left( \frac{r}{x} \right)$

$1 = \frac{3}{x}$

$x = 3$

5.  $r = 2 \sin \theta$

$r = 2 \left( \frac{y}{r} \right)$

$r^2 = 2y$

$x^2 + y^2 = 2y$

6.  $\theta = \frac{5\pi}{6}$

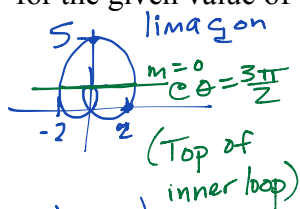
$\tan \theta = \tan \frac{5\pi}{6}$

$\frac{y}{x} = -\frac{\sqrt{3}}{3}$

$y = -\frac{\sqrt{3}}{3}x$

For the following, find  $\frac{dy}{dx}$  for the given value of  $\theta$ .

7.  $r = 2 + 3 \sin \theta, \theta = \frac{3\pi}{2}$



$x(\theta) = r \cos \theta$

$x(\theta) = (2 + 3 \sin \theta) \cos \theta$

$x'(\theta) = (3 \cos \theta)(\cos \theta) + (2 + 3 \sin \theta)(-\sin \theta)$   
 $= 3 \cos^2 \theta - 2 \sin \theta - 3 \sin^2 \theta$

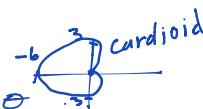
$y(\theta) = r \sin \theta$

$y(\theta) = (2 + 3 \sin \theta) \sin \theta$

$y'(\theta) = 3 \cos \theta \sin \theta + (2 + 3 \sin \theta) \cos \theta$

$\frac{dy}{dx} \Big|_{\theta = \frac{3\pi}{2}} = \frac{y'(\frac{3\pi}{2})}{x'(\frac{3\pi}{2})} = \frac{0 + (2 - 3)(0)}{0 + 2 - 3} = 0$

8.  $r = 3(1 - \cos \theta), \theta = \frac{\pi}{2}$



$x(\theta) = (3 - 3 \cos \theta) \cos \theta$

$x'(\theta) = (3 \sin \theta) \cos \theta + (3 - 3 \cos \theta)(-\sin \theta)$

$x'(\frac{\pi}{2}) = 0 + (3)(-1) = -3$

$y(\theta) = (3 - 3 \cos \theta) \sin \theta$

$y'(\theta) = (3 \sin \theta) \sin \theta + (3 - 3 \cos \theta) \cos \theta$

$y'(\frac{\pi}{2}) = 3 + (3)(0) = 3$

So,  $\frac{dy}{dx} \Big|_{\theta = \frac{\pi}{2}} = \frac{y'(\frac{\pi}{2})}{x'(\frac{\pi}{2})} = \frac{3}{-3} = -1$

9.  $r = 4 \sin \theta, \theta = \frac{\pi}{3}$

$$\begin{aligned} x &= (4 \sin \theta) \cos \theta & y &= (4 \sin \theta) \sin \theta \\ x' &= 4 \cos^2 \theta - 4 \sin^2 \theta & y &= 4 (\sin \theta)^2 \\ x' &= 4 \cos 2\theta & y' &= 8 \sin \theta \cos \theta \\ x'(\frac{\pi}{3}) &= 4 \cos \frac{2\pi}{3} & y' &= 4 \sin 2\theta \\ &= 4(-\frac{1}{2}) & y'(\frac{\pi}{3}) &= 4 \sin \frac{2\pi}{3} \\ &= -2 & &= 4(\frac{\sqrt{3}}{2}) \\ & & &= 2\sqrt{3} \end{aligned}$$

$$\text{So, } \frac{dy}{dx} \Big|_{\theta = \frac{\pi}{3}} = \frac{2\sqrt{3}}{-2} = -\sqrt{3}$$

10.  $r = 2 \sin(3\theta), \theta = \frac{\pi}{4}$

$$\begin{aligned} x &= 2 \sin(3\theta) \cos \theta & y &= 2 \sin(3\theta) \sin \theta \\ x' &= 6 \cos(3\theta) \cos \theta - 2 \sin(3\theta) \sin \theta & y' &= 6 \cos(3\theta) \sin \theta + 2 \sin(3\theta) \cos \theta \\ x'(\frac{\pi}{4}) &= 6 \cos(\frac{3\pi}{4}) \cos \frac{\pi}{4} - 2 \sin(\frac{3\pi}{4}) \sin \frac{\pi}{4} & y'(\frac{\pi}{4}) &= 6 \cos \frac{3\pi}{4} \sin \frac{\pi}{4} + 2 \sin \frac{3\pi}{4} \cos \frac{\pi}{4} \\ &= 6(-\frac{\sqrt{2}}{2})(\frac{\sqrt{2}}{2}) - 2(\frac{\sqrt{2}}{2})(\frac{\sqrt{2}}{2}) & &= 6(-\frac{\sqrt{2}}{2})(\frac{\sqrt{2}}{2}) + 2(\frac{\sqrt{2}}{2})(\frac{\sqrt{2}}{2}) \\ &= -3 - 1 & &= -3 + 1 \\ &= -4 & &= -2 \end{aligned}$$

$$\text{So, } \frac{dy}{dx} \Big|_{\theta = \frac{\pi}{4}} = \frac{-2}{-4} = \frac{1}{2}$$

11. Find the point of horizontal and vertical tangency for  $r = 1 + \sin \theta$ . Give your answers in polar form  $(r, \theta)$ .

$$\begin{aligned} x &= (1 + \sin \theta) \cos \theta & y &= (1 + \sin \theta) \sin \theta \\ x' &= \cos^2 \theta - (1 + \sin \theta) \sin \theta & y' &= \cos \theta \sin \theta + (1 + \sin \theta) \cos \theta \\ x' &= \cos^2 \theta - \sin \theta - \sin^2 \theta & y' &= \cos \theta \sin \theta + \cos \theta + \cos \theta \sin \theta \\ x' &= \underbrace{(\cos^2 \theta - \sin^2 \theta)}_{\downarrow \text{PID}} - \sin \theta - \sin^2 \theta & y' &= 2 \cos \theta \sin \theta + \cos \theta \\ x' &= -2 \sin^2 \theta - \sin \theta + 1 \end{aligned}$$

Horz

$$\begin{aligned} \frac{dy}{dx} &= 0 \\ \frac{dy}{dx} &\neq 0 \\ y'(\theta) &= 0 \\ 2 \cos \theta \sin \theta + \cos \theta &= 0 \\ \cos \theta (2 \sin \theta + 1) &= 0 \\ \cos \theta = 0 & \sin \theta = -\frac{1}{2} \\ \theta = \frac{\pi}{2}, \frac{3\pi}{2}, \theta = \frac{7\pi}{6}, \frac{11\pi}{6} \end{aligned}$$

Vert

$$\begin{aligned} \frac{dy}{dx} &= \frac{0}{0} \\ x'(\theta) &= 0 \\ -2 \sin^2 \theta - \sin \theta + 1 &= 0 \\ 2 \sin^2 \theta + \sin \theta - 1 &= 0 \\ (2 \sin \theta - 1)(\sin \theta + 1) &= 0 \\ \sin \theta = \frac{1}{2} & \sin \theta = -1 \\ \theta = \frac{\pi}{6}, \frac{5\pi}{6} & \theta = \frac{3\pi}{2} \end{aligned}$$

\*  $\frac{3\pi}{2}$  gives  $\frac{dy}{dx} = \frac{0}{0}$   
 So throw out (Not a vert or horz tangent, but a cusp)

Tables Not Shown

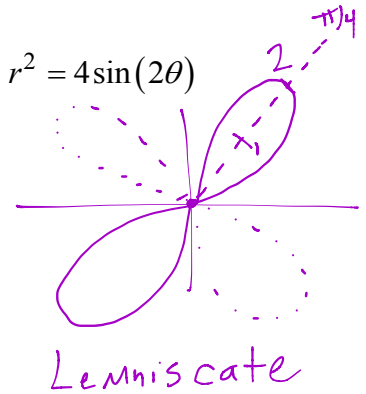
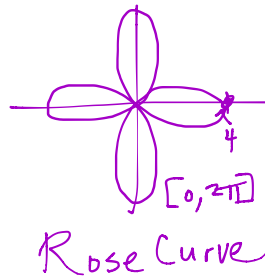
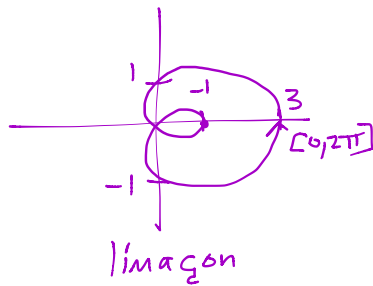
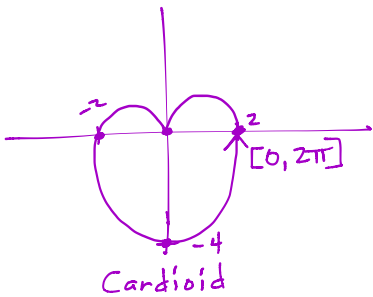
Make a table (of values, not one at which to eat) and sketch the graph.

12.  $r = 2 - 2\sin\theta$

13.  $r = 1 + 2\cos\theta$

14.  $r = 4\cos(2\theta)$

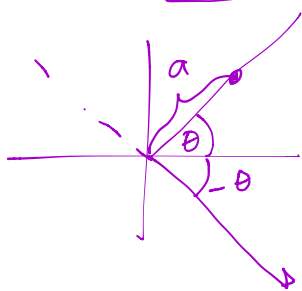
15.  $r^2 = 4\sin(2\theta)$



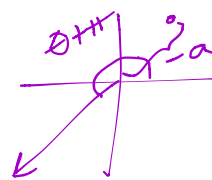
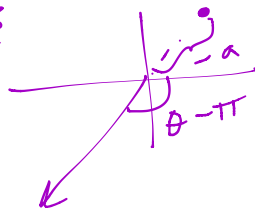
**Multiple Choice**

B 16. If  $a \neq 0$  and  $\theta \neq 0$ , all of the following must represent the same point in polar coordinates *except* which ordered pair?

- (A)  $(a, \theta)$     (B)  $(-a, -\theta)$     (C)  $(-a, \theta - \pi)$     (D)  $(-a, \theta + \pi)$     (E)  $(a, \theta - 2\pi)$



~~A & B~~  
not same



D

17. Which of the following gives the slope of the polar curve  $r = f(\theta)$  graphed in the  $xy$ -plane?

- (A)  $\frac{dr}{d\theta}$     (B)  $\frac{dy}{d\theta}$     (C)  $\frac{dx}{d\theta}$     (D)  $\frac{dy/d\theta}{dx/d\theta}$     (E)  $\frac{dy}{dx} \cdot \frac{dr}{d\theta}$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

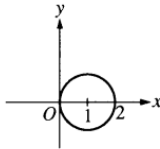
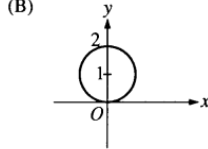
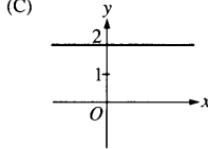
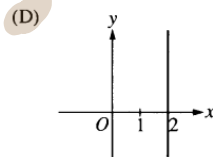
D

18. Which of the following represents the graph of the polar curve  $r = 2\sec\theta$ ?

$$r = 2\left(\frac{r}{x}\right)$$

$$1 = \frac{2}{x}$$

$$x = 2$$

- (A) 
- (B) 
- (C) 
- (D) 
- (E) 