Name KEY Date________________________ Period______

Worksheet 7.2—Parametric & Vector Accumulation
Show all work. No calculator except unless specifically stated.

Short Answer/Free Response

1. If $x = e^{2t}$ and $y = \sin(3t)$, find $\frac{dy}{dx}$ in terms of t.

7.1

$x' = 2e^{2t}$ $y' = 3\cos(3t)$

$\frac{dy}{dx} = \frac{y'(t)}{x'(t)} = \frac{3\cos(3t)}{2e^{2t}}$

2. Write an integral expression to represent the length of the path described by the parametric equations $x = \cos^3 t$ and $y = \sin^2 t$ for $0 \le t \le \frac{\pi}{2}$.

7.1

$x'(t) = 3\cos^2 t(-\sin t)$ $y'(t) = 2\sin t\cos t$

$L = \int_0^{\frac{\pi}{2}} \sqrt{(-3\sin t\cos^2 t)^2 + (2\sin t\cos t)^2}\, dt$

reword notes to this

3. For what value(s) of t does the curve given by the parametric equations $x = t^3 - t^2 - 1$ and $y = t^4 + 2t^2 - 8t$ have a vertical tangent?

7.1

$x'(t) = 3t^2 - 2t$

$y' = 4t^3 + 4t - 8$

$\frac{dy}{dx} = \frac{4t^3 + 4t - 8}{3t^2 - 2t}$

Vertical tangent

$\frac{dy}{dx} = \frac{\neq 0}{0}$

$3t^2 - 2t = 0$

$t(3t - 2) = 0$

$t = 0$ $3t - 2 = 0$

$t = \frac{2}{3}$

* $\left.\frac{dy}{dt}\right|_{t=0}$ & $\left.\frac{dy}{dt}\right|_{t=2/3}$ $\neq 0$

So, curve has vertical tangents @ $t = 0$, $t = \frac{2}{3}$

4. Find the equation of the tangent line to the curve given by the parametric equations $x(t) = 3t^2 - 4t + 2$ and $y(t) = t^3 - 4t$ at the point on the curve where $t = 1$.

7.1

$x'(t) = 6t - 4$

$y'(t) = 3t^2 - 4$

pt: $(x(1), y(1))$
$= (3-4+2, 1-4)$
$= (1, -3)$

$m: \left.\frac{dy}{dx}\right|_{t=1} = \frac{3(1^2)-4}{6(1)-4}$
$= \frac{-1}{2}$

So, $L(x) = -3 - \frac{1}{2}(x-1)$

7.1

5. If $x(t) = e^t + 1$ and $y = 2e^{2t}$ are the equations of the path of a particle moving in the $xy-$plane, write an equation for the path of the particle in terms of x and y.

eliminate the parameter

$x = e^t + 1$, $y = 2e^{2t}$
$x - 1 = e^t$ So, $y = 2e^{2\ln(x-1)}$
$t = \ln(x-1)$ $y = 2e^{\ln(x-1)^2}$
$y = 2(x-1)^2$

6. (Calculator) A particle moves in the $xy-$plane so that its position at any time t is given by $x = \cos(5t)$ and $y = t^3$. What is the speed of the particle when $t = 2$?

7.1

$x'(t) = -5\sin 5t$, $y'(t) = 3t^2$

$\vec{v}(2) = \langle -5\sin 10, 12 \rangle$

Speed $= \sqrt{(-5\sin 10)^2 + (12)^2}$

$= 12.304$

7.1

7. (Calculator) The position of a particle at time $t \ge 0$ is given by the parametric equations

$X_1 = x(t) = \frac{(t-2)^3}{3} + 4$ and $y_1 = y(t) = t^2 - 4t + 4$.

put into calculator

(a) Find the magnitude of the velocity vector at $t = 1$.

$$\vec{v}(1) = \langle x'(1), y'(1) \rangle = \underbrace{\langle 1, -2 \rangle}_{\text{from calculator}} = \langle A, B \rangle$$

$\boxed{\text{MATH}}$ $\boxed{8}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{TRACE}}$

store as A & B when ugly decimals

$$\|\vec{v}(1)\| = \sqrt{A^2 + B^2} = \sqrt{1 + 2^2} = \sqrt{5} \approx 2.236$$

(b) Find the total distance traveled by the particle from $t = 0$ to $t = 1$.

$$x'(t) = (t-2)^2$$
$$y'(t) = 2t - 4$$

$$D = \mathcal{L} = \int_0^1 \sqrt{\left((t-2)^2\right)^2 + (2t-4)^2}\, dt$$
$$= 3.815 \text{ or } 3.816$$

(c) When is the particle at rest? What is its position at that time?

When BOTH $x'(t)$ & $y'(t) = 0$

$$x'(t) = 0 \qquad y'(0) = 0$$
$$(t-2)^2 = 0 \qquad 2t - 4 = 0$$
$$t = 2 \qquad t = 2$$

So, particle is at rest when $t = 2$.

$$(x(2), y(2)) = (0 + 4, 4 - 8 + 4) = (4, 0)$$

So, at $t = 2$, particle is at $(4, 0)$

7.1

8. (Calculator) An object moving along a curve in the $xy-$plane has position $\left(x(t), y(t)\right)$ at time $t \geq 0$ with $\dfrac{dx}{dt} = 1 + \tan\left(t^2\right)$ and $\dfrac{dy}{dt} = 3e^{\sqrt{t}}$. Find the acceleration vector and the speed of the object when $t = 5$.

$\vec{v}(t) = \langle 1 + \tan(t^2), 3e^{\sqrt{t}} \rangle$

speed $= \sqrt{(x'(5))^2 + (y'(5))^2}$

$= \sqrt{(1+\tan 25)^2 + (3e^{\sqrt{5}})^2}$

$= 28.082$ or 28.083

$\vec{a}(5) = \vec{v}\,'(5) = \langle x''(5), y''(5) \rangle$

$= \underbrace{\langle 10.178, 6.276 \rangle}$

$\boxed{\text{MATH}}$ $\boxed{8}$ using $\frac{dx}{dt}$ & $\frac{dy}{dt}$.

7.1

9. (Calculator) A particle moves in the $xy-$plane so that the position of the particle is given by $x(t) = t + \cos t$ and $y(t) = 3t + 2\sin t$, $0 \leq t \leq \pi$. Find the velocity vector when the particle's vertical position is $y = 5$.

$y(t) = 5$

$3t + 2\sin t = 5$

$3t + 2\sin t - 5 = 0$

$t = 1.079... = A$ (Store as A)

$\vec{v}(A) = \langle x'(A), y'(A) \rangle$

$= \langle 0.118, 3.944 \rangle$

10. (Calculator) An object moving along a curve in the xy-plane has position $(x(t), y(t))$ at time t with $\frac{dx}{dt} = 2\sin(t^3)$ and $\frac{dy}{dt} = \cos(t^2)$ for $0 \le t \le 4$. At time $t = 1$, the object is at the position $(3,4)$.

(a) Write an equation for the line tangent to the curve at $(3,4)$.

$$\left.\frac{dy}{dx}\right|_{t=1} = \frac{y'(1)}{x'(1)} = \frac{\cos 1}{2\sin 1} = 0.321$$

So, $\mathcal{L} = 4 + 0.321(x-3)$

(b) Find the speed of the object at time $t = 2$.

$$\text{Speed} = \sqrt{(x'(2))^2 + (y'(2))^2}$$
$$= \sqrt{(2\sin 8)^2 + (\cos 4)^2}$$
$$= 2.083$$

(c) Find the total distance traveled by the object over the time interval $0 \le t \le 1$.

$$\text{Dist} = \mathcal{L} = \int_0^1 \sqrt{(x'(t))^2 + (y'(t))^2}\,dt$$
$$= 1.126$$

(d) Find the position of the object at time $t = 2$.

$$x(2) = x(1) + \int_1^2 x'(t)\,dt \qquad y(2) = y(1) + \int_1^2 y'(t)\,dt$$
$$= 3 + \int_1^2 2\sin(t^3)\,dt \qquad y(2) = 4 + \int_1^2 \cos(t^2)\,dt$$
$$= 3.436 \qquad = 3.556$$

So, position at $t = 2$ is $(x(2), y(2)) = (3.436, 3.556)$

Multiple Choice:

D 11. (Calculator) An object moving along a curve in the $xy-$plane has position $\left(x(t), y(t)\right)$ with $\frac{dx}{dt} = \cos\left(t^2\right)$ and $\frac{dy}{dt} = \sin\left(t^3\right)$. At time $t = 0$, the object is at position $(4, 7)$. Where is the particle when $t = 2$?

(A) $\langle -0.564, 0.989 \rangle$ (B) $\langle 0.461, 0.452 \rangle$ (C) $\langle 3.346, 7.989 \rangle$

(D) $\langle 4.461, 7.452 \rangle$ (E) $\langle 5.962, 8.962 \rangle$

$$X(2) = \left\langle x(0) + \int_0^2 x'(t)\,dt,\ y(0) + \int_0^2 y'(t)\,dt \right\rangle$$

$$= \left\langle 4 + \int_0^2 \cos(t^2)\,dt,\ 7 + \int_0^2 \sin(t^3)\,dt \right\rangle$$

$$= \langle 4.461, 7.452 \rangle$$

or

$$(4.461, 7.452)$$

B 12. (Calculator) The path of a particle moving in the plane is defined parametrically as a function of time t by $x = \sin 2t$ and $y = \cos 5t$. What is the speed of the particle at $t = 2$?

(A) 1.130 (B) 3.018 (C) $\langle -1.307, 2.720 \rangle$ (D) $\langle 0.757, 0.839 \rangle$ (E) $\langle 1.307, 2.720 \rangle$

$$\text{Speed} = \sqrt{(x'(2))^2 + (y'(2))^2}$$

$$= \sqrt{(2\cos(2\cdot 2))^2 + (-5\sin(5\cdot 2))^2}$$

$$= 3.0179\ldots$$

C

13. For what values of t does the curve given by the parametric equations $x = t^3 - t^2 - 1$ and $y = t^4 + 2t^2 - 8t$ have a vertical tangent?

7.1

(A) 0 only (B) 1 only (C) 0 and 2/3 only (D) 0, 2/3, and 1 (E) No value

Vert tangent

$$\frac{dy}{dx} = \frac{\neq 0}{0}$$

$$\frac{dx}{dt} = 3t^2 - 2t = 0$$

$$t(3t-2) = 0$$

$$t = 0,\ t = \frac{2}{3}$$

D

14. The distance traveled by a particle from $t = 0$ to $t = 4$ whose position is given by the vector $\vec{s}(t) = \langle t^2, t \rangle$ is given by $\vec{v} = \vec{s}\,' = \langle 2t, 1 \rangle = \langle x'(t), y'(t) \rangle$

(A) $\int_0^4 \sqrt{4t+1}\,dt$ (B) $2\int_0^4 \sqrt{t^2+1}\,dt$ (C) $\int_0^4 \sqrt{2t^2+1}\,dt$ (D) $\int_0^4 \sqrt{4t^2+1}\,dt$ (E) $2\pi\int_0^4 \sqrt{4t^2+1}\,dt$

$$D = L = \int_0^4 \sqrt{(2t)^2 + (1)^2}\,dt$$

$$= \int_0^4 \sqrt{4t^2 + 1}\,dt$$