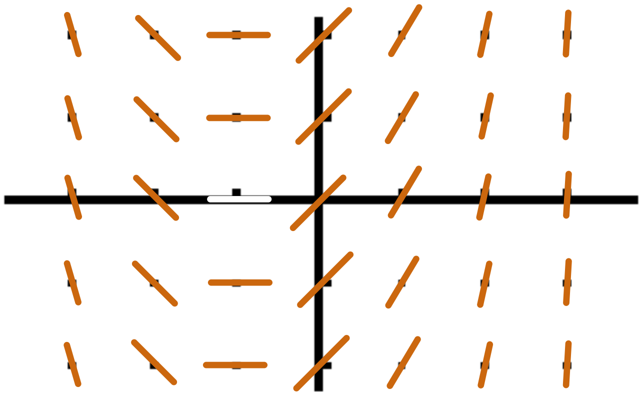
Name KEY Date____________________ Period______

Worksheet 5.2—Slope Fields
Show all work when applicable.

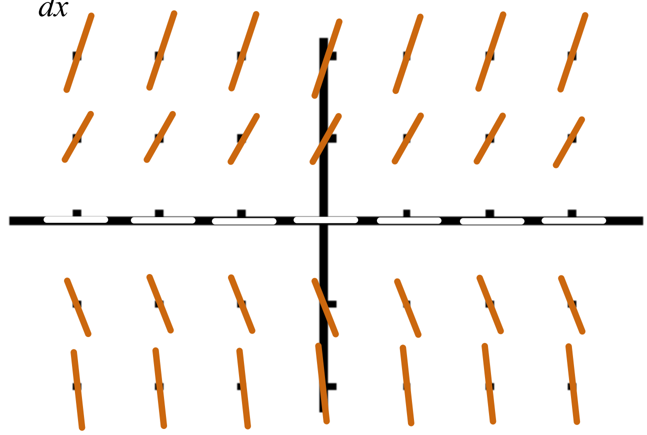
Short Answer and Free Response:

Draw a slope field for each of the following differential equations.

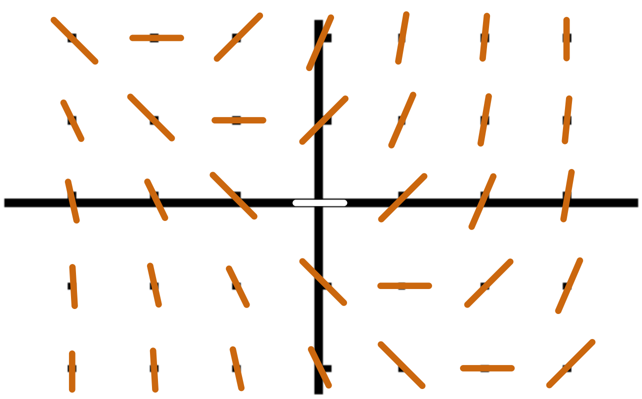
1. $\dfrac{dy}{dx} = x+1$



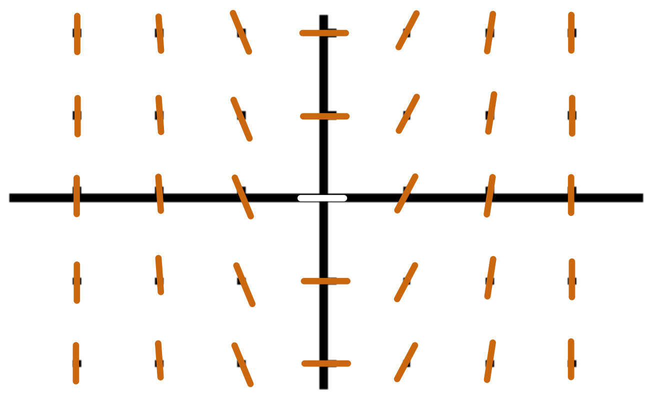
2. $\dfrac{dy}{dx} = 2y$



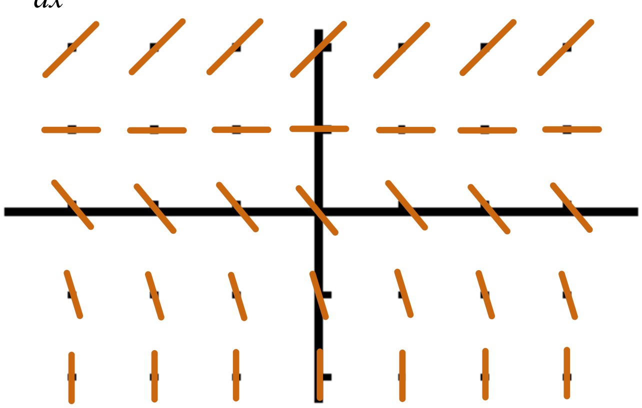
3. $\dfrac{dy}{dx} = x+y$



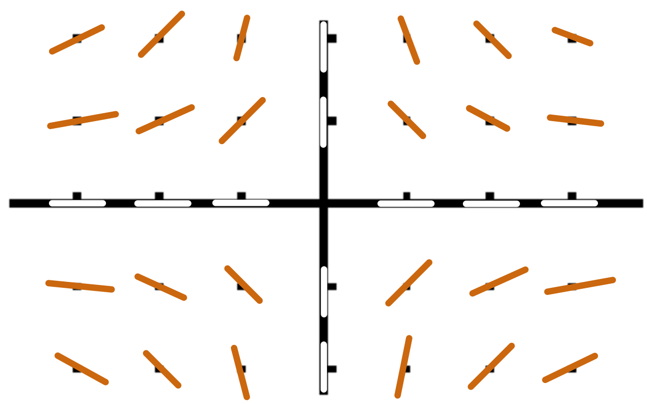
4. $\dfrac{dy}{dx} = 2x$



5. $\dfrac{dy}{dx} = y-1$

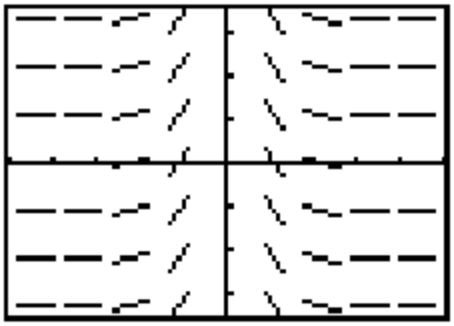


6. $\dfrac{dy}{dx} = -\dfrac{y}{x}$

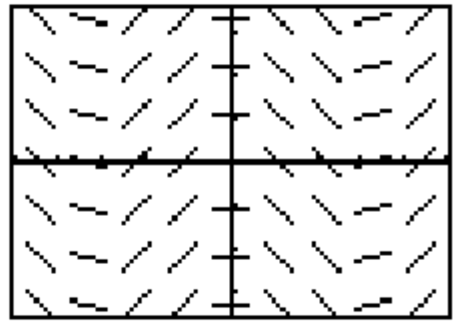


For 7 – 12, match each slope field with the **equation** that the slope field could represent.

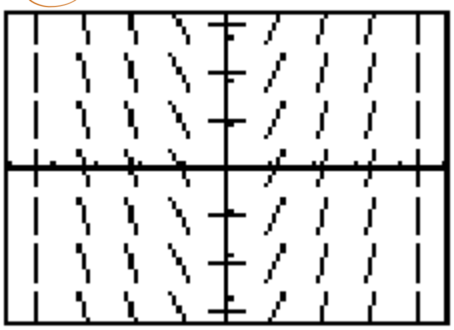
7. E



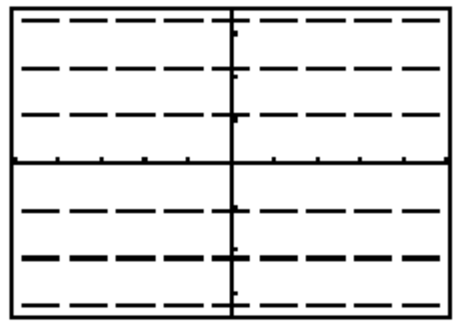
8. G



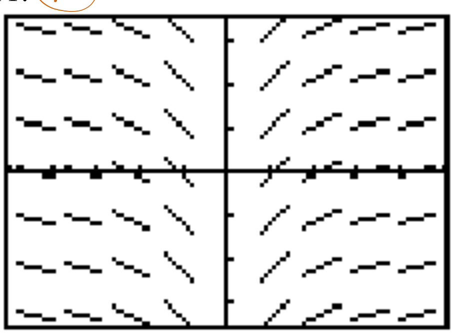
9. C



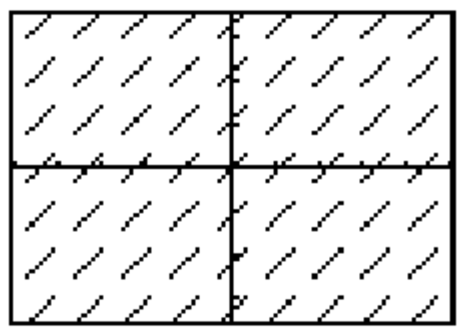
10. A



11. H



12. B



(A) $y = 1$

(B) $y = x$

(C) $y = x^2$

(D) $y = \frac{1}{6}x^3$

(E) $y = \frac{1}{x^2}$

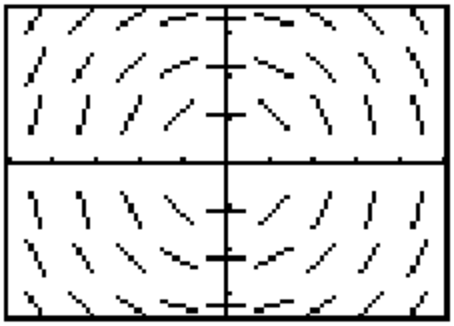
(F) $y = \sin x$

(G) $y = \cos x$

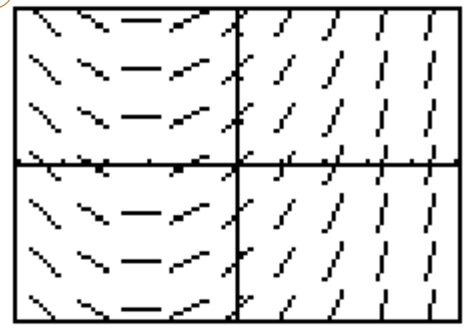
(H) $y = \ln|x|$

For 13 – 16, match the slope fields with their differential equations.

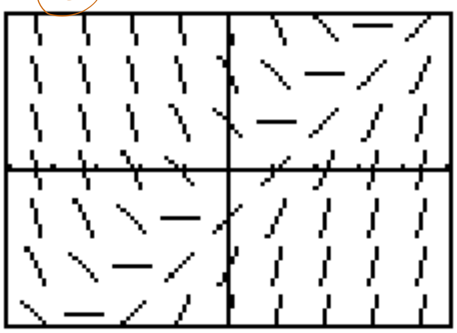
13. D



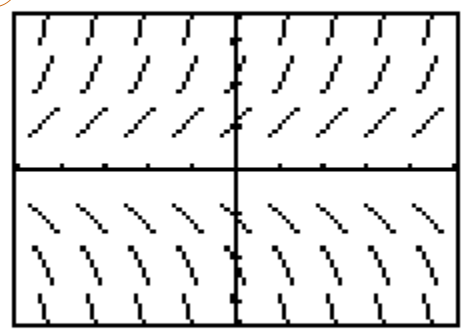
14. A



15. B



16. C



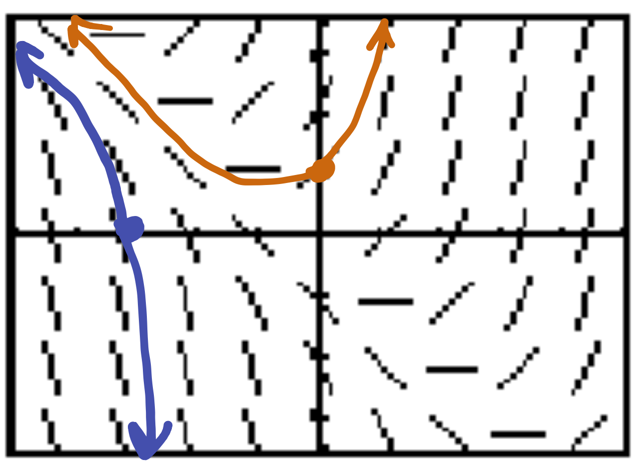
(A) $\frac{dy}{dx} = \frac{1}{2}x + 1$

(B) $\frac{dy}{dx} = x - y$

(C) $\frac{dy}{dx} = y$

(D) $\frac{dy}{dx} = -\frac{x}{y}$

17. The calculator-drawn slope field for the differential equation $\frac{dy}{dx} = x + y$ is shown in the figure below.



(a) Sketch the solution curve through the point $(0,1)$.

(b) Sketch the solution curve through the point $(-3,0)$.

(c) Approximate $y(-3.1)$ using the equation of the tangent line to $y = f(x)$ at the point $(-3,0)$.

$\left.\frac{dy}{dx}\right|_{(-3,0)} = -3+0=-3$

$L(x) = 0-3(x+3)$

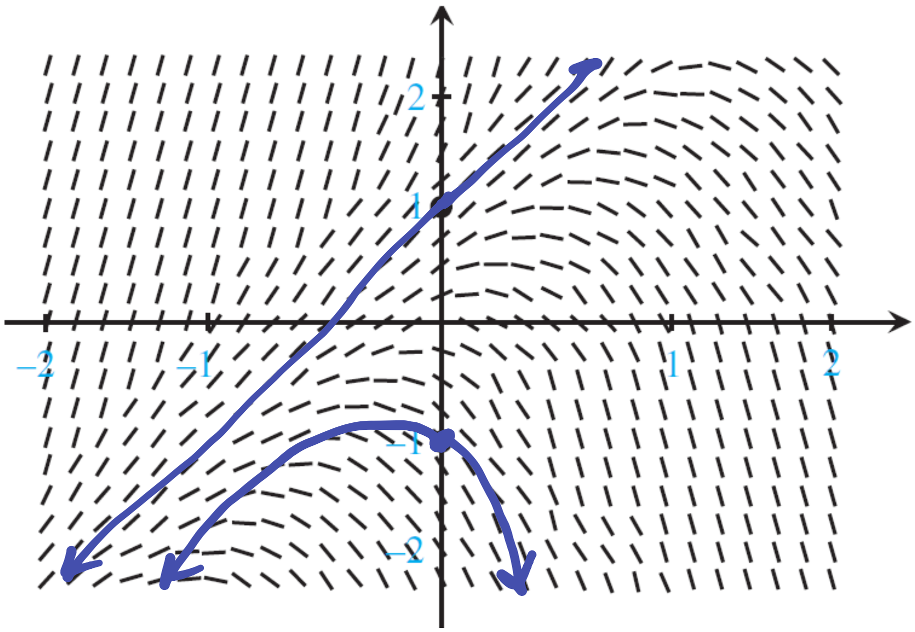
$y(-3.1) \approx L(-3.1) = -3(-3.1+3)$

$= -3(-.1)$

$= 0.3$

18. Consider the differential equation $\frac{dy}{dx} = 2y - 4x$.

(a) The slope field for the differential equation is shown below. Sketch the solution curve that passes through the point $(0,1)$ and sketch the solution curve that goes through the point $(0,-1)$.



(b) There is a value of b for which $y = 2x + b$ is a solution to the differential equation. Find this value of b. Justify your answer.

if a solution is linear, it has no concavity, so $\frac{d^2y}{dx^2} = 0$

$\frac{dy}{dx} = 2y - 4x$

$\frac{d}{dt}: \frac{d^2y}{dx^2} = 2\frac{dy}{dx} - 4 = 0$

$2(2y-4x) - 4 = 0$

$4y - 8x - 4 = 0$

$4y = 8x + 4$

$y = 2x + 1$

so, $b = 1$

(c) Let g be the function that satisfies the given differential equation with the initial condition $g(0) = 0$. It appears from the slope field that g has a local maximum at the point $(0,0)$. Using the differential equation, prove analytically that this is so.

$\frac{dy}{dx} = 2y - 4x$

$\left.\frac{dy}{dx}\right|_{(0,0)} = 0 - 0 = 0$

so, $(0,0)$ is a critical value of $y = g(x)$.

from part (b): $\frac{d^2y}{dx^2} = 4y - 8x - 4$

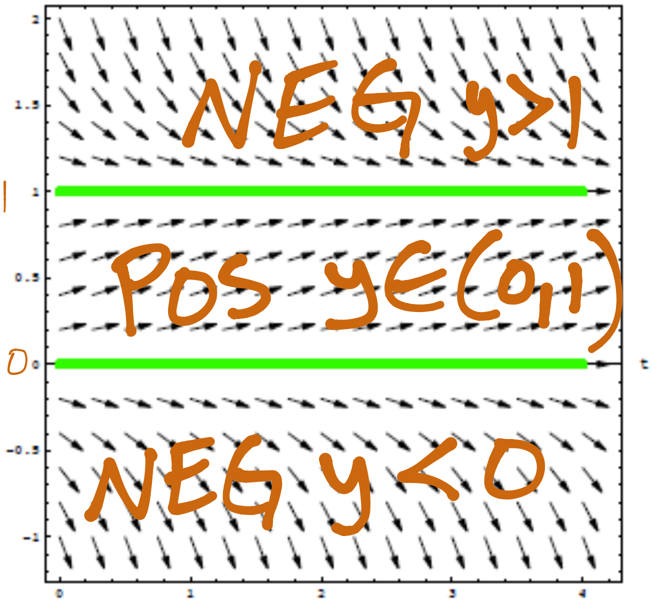
$\left.\frac{d^2y}{dx^2}\right|_{(0,0)} = 0 - 0 - 4 = -4 < 0$

so, $y = g(x)$ is concave down at a critical value, so $y = g(x)$ has a local max at $(0,0)$ by the 2nd Derivative Test.

Multiple Choice:

B

19. Given the following slope field (with **equilibrium solutions**, meaning the slopes are zero and a the existence of a horizontal asymptote on the solution graph, at $y = 0$ and $y = 1$), find the matching differential equation.



(A) $\frac{dy}{dx} = y(y-1)$ (B) $\frac{dy}{dx} = y(1-y)$

(C) $\frac{dy}{dx} = \frac{1}{y(1-y)}$ (D) $\frac{dy}{dx} = 1 - e^{y(1-y)}$

(E) $\frac{dy}{dx} = \frac{y}{y-1}$

C

20. A slope field for the differential equation $\frac{dy}{dx} = 42 - y$ will show

(A) a line with slope -1 and y-intercept of 42 (B) a vertical asymptote at $x = 42$

(C) a horizontal asymptote at $y = 42$ (D) a family of parabolas opening downward

(E) a family of parabolas opening to the left

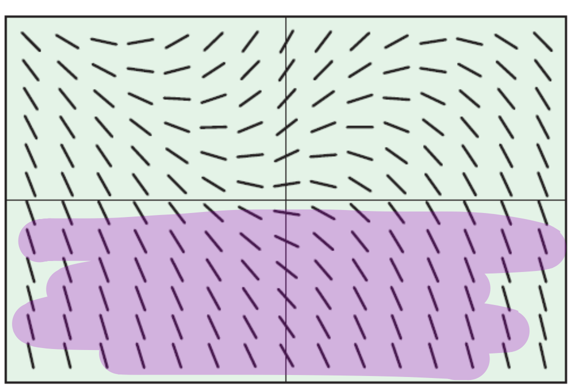
E

21. For which of the following differential equations will a slope field show nothing but negative slopes in the fourth quadrant? $x>0, y<0 \rightarrow \frac{dy}{dx}<0$

(A) $\frac{dy}{dx} = -\frac{x}{y}$ (B) $\frac{dy}{dx} = xy + 5$ (C) $\frac{dy}{dx} = xy^2 - 2$ (D) $\frac{dy}{dx} = \frac{x^3}{y^2}$ (E) $\frac{dy}{dx} = \frac{y}{x^2} - 3$

A

22. Which of the following differential equations would produce the slope field shown below?



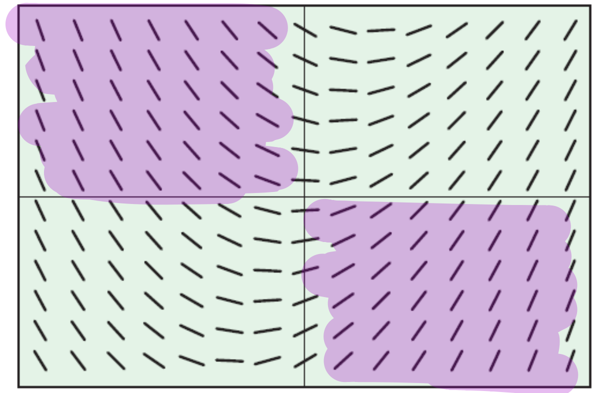
[–3, 3] by [–1.98, 1.98]

neg slopes when y<0

(A) $\frac{dy}{dx} = y - |x|$ (B) $\frac{dy}{dx} = |y| - x$ (C) $\frac{dy}{dx} = |y - x|$ (D) $\frac{dy}{dx} = |y + x|$ (E) $\frac{dy}{dx} = |y| - |x|$

23. Which of the following differential equations would produce the slope field shown below?

neg slopes when $x<0, y>0$



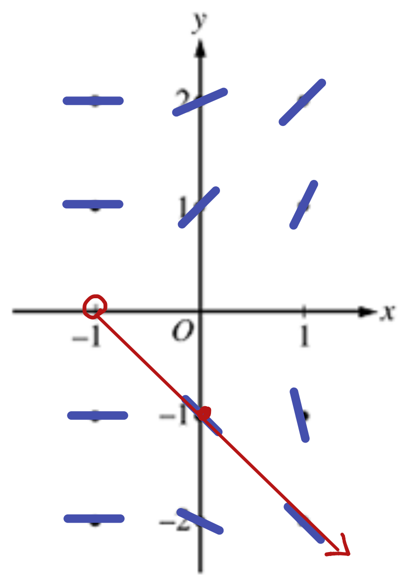
pos slopes when $x>0, y<0$

(A) $\frac{dy}{dx} = y - 3x$ (B) $\frac{dy}{dx} = y - \frac{x}{3}$ (C) $\frac{dy}{dx} = y + \frac{x}{3}$ (D) $\frac{dy}{dx} = x + \frac{y}{3}$ (E) $\frac{dy}{dx} = x - \frac{y}{3}$

24. AP 2010B-5 (No Calculator)

Consider the differential equation $\frac{dy}{dx} = \frac{x+1}{y}$.

(a) On the axes provided at right, sketch a slope field for the given differential equation at the twelve points indicated, and for $-1 < x < 1$, sketch the solution curve that passes through the point $(0,-1)$.



(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane for which $y \neq 0$. Describe all points in the xy-plane, $y \neq 0$, for which $\frac{dy}{dx} = -1$.

$\frac{x+1}{y} = -1$
$x+1 = -y$
$y = -x-1$

So, $\frac{dy}{dx} = -1$ for all points along the line $y = -x-1$ for $y \neq 0$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = -2$.

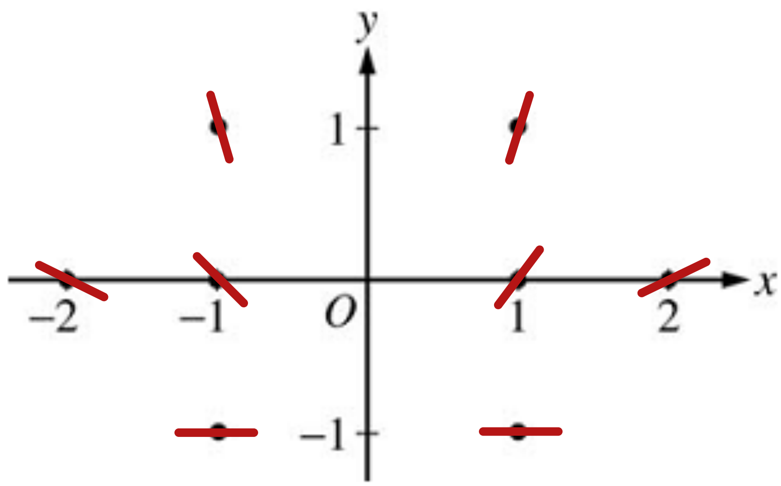
$\frac{dy}{dx} = \frac{x+1}{y}$
$y\,dy = (x+1)\,dx$
$\int y\,dy = \int (x+1)\,dx$
$\frac{1}{2}y^2 = \frac{1}{2}x^2 + x + C$
$y^2 = x^2 + 2x + C$
$y = \pm\sqrt{x^2+2x+C}$

at $(0,-2)$:
$-2 = -\sqrt{0+0+C}$
$4 = C$
So, $y = -\sqrt{x^2+2x+4}$

25. AP 2006-5 (No Calculator)

Consider the differential equation $\frac{dy}{dx} = \frac{1+y}{x}$, where $x \neq 0$.

(a) On the axes provided, sketch a slope field for the given differential equation at the eight points indicated.



(b) Find the particular solution $y = f(x)$ to the differential equation with the initial condition $f(-1) = 1$ and state its domain.

$\frac{dy}{dx} = \frac{1+y}{x}$

$\frac{1}{1+y}\,dy = \frac{1}{x}\,dx$

$\int \frac{1}{1+y}\,dy = \int \frac{1}{x}\,dx$

$\ln|1+y| = \ln|x| + C$

$|1+y| = e^{\ln|x|+C}$

$1+y = Ce^{\ln|x|}$

$1+y = C|x|$

$y = C|x| - 1$

$y = C|x| - 1$

@ $(-1,1)$: $1 = C|-1| - 1$

$1 = C - 1$

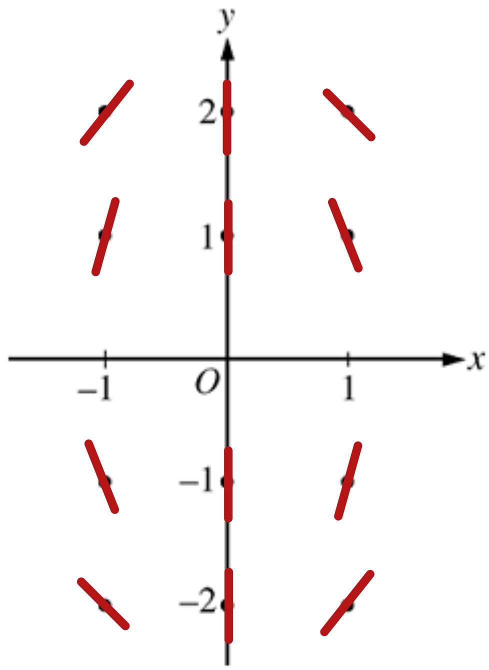
$C = 2$

So, $y = 2|x| - 1$

26. AP 2005-6 (No Calculator)

Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated.



(b) Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$. Write an equation for the line tangent to the graph of f at $(1,-1)$ and use it to approximate $f(1.1)$.

$\frac{dy}{dx} = \frac{-2x}{y}$

$\left.\frac{dy}{dx}\right|_{(1,-1)} = \frac{-2}{-1} = 2$

$\mathcal{L}(x) = -1 + 2(x-1)$

$f(1.1) \approx \mathcal{L}(1.1) = -1 + 2(1.1-1)$

$= -1 + 2(0.1)$

$= -1 + 0.2$

$= -0.8$

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(1) = -1$.

$\frac{dy}{dx} = \frac{-2x}{y}$

$y\,dy = -2x\,dx$

$\int y\,dy = \int -2x\,dx$

$\frac{1}{2}y^2 = -x^2 + c$

$y^2 = -2x + c$

$y = \pm\sqrt{-2x+c}$

$y = \pm\sqrt{-2x+c}$

@ $(1,-1)$: $-1 = -\sqrt{-2+c}$

$1 = -2 + c$

$c = 3$

So, $y = -\sqrt{-2x+3}$

*for what it's worth

$f(1.1) = -\sqrt{-2.2+3}$

$= -\sqrt{.8}$

$= -.894$

& since $\mathcal{L}(1.1) > f(1.1)$,

$f(x)$ is concave up

for $x \in (1, 1.1)$