Name KEY Date Period

Calculus Prerequisites

Worksheet—Day 1

Work the following on <u>notebook paper</u>. All work must be shown. Use your **calculator only on problems 14-17 and problems 30-32.**

Find the equations of the asymptotes (horizontal, vertical, and slant), symmetry, and intercepts, then sketch the graph.

5. $y = \dfrac{2x^2 - 8}{x^2 - 16}$

$y = \dfrac{2(x-2)(x+2)}{(x-4)(x+4)}$

y has VAs @ $x = \pm 4$ and an HA @ $y = 2$

6. $y = \dfrac{x^2 - 2x - 3}{x+2}$

$y = \dfrac{(x-3)(x+1)}{(x+2)}$

$$\begin{array}{r|l} & x-4 \\ x+2 & x^2-2x-3 \\ & -(x^2+2x) \\ & -4x-3 \\ & -(-4x-8) \\ & 5 \end{array}$$

y has a VA @ $x = -2$ & a SA at $y = x - 4$

Solve.

7. $(x+1)(x-3)^3(x+2)^2 \geq 0$

Test values: $x = -1, 3, -2$

x	−3	−2	−1.5	−1	0	3	4
sign	+		+		−		+

$x \in (-\infty, -1] \cup [3, \infty)$

8. $\dfrac{4x-3}{x+1} < 0$

Test values: $x = -1, \frac{3}{4}$

x	−2	−1	0	3/4	1
sign	+		−		+

$x \in (-1, \frac{3}{4})$

Solve. Show all steps. Give decimal answers correct to **<u>three</u>** decimal places.

14. $e^{2x} - e^x - 12 = 0$

$(e^x - 4)(e^x + 3) = 0$

$e^x = 4$ or $e^x = -3$ (No Soln)

$x = \ln 4$

$x \approx 1.386$

15. $\log_3(x+4) - \log_3(x-1) = 2$

$\log_3\left(\dfrac{x+4}{x-1}\right) = 2$

$\dfrac{x+4}{x-1} = 3^2$

$\dfrac{x+4}{x-1} - \dfrac{9}{1}\left(\dfrac{x-1}{x-1}\right) = 0$

$\dfrac{x+4-9x+9}{x-1} = 0$

$\dfrac{13-8x}{x-1} = 0$

when $13 - 8x = 0$

$x = \frac{13}{8} = 1.625$

16. $\log_2(\log_3(\log_5 x)) = 0$

$2^0 = \log_3(\log_5 x)$

$3^1 = \log_5 x$

$x = 5^3$

$x = 125$

17. The number of junior and senior students at NBHS infected with the "Math Bug" t days after exposure is modeled by the function $P(t)=\dfrac{500}{1+3^{3-t}}$.

a. How many students were originally infected?
b. How many of these students were infected after four days?
c. When will 200 of these students be infected?
d. What is the maximum number of students that will be infected?

(a) $P(0)=\dfrac{500}{1+3}$
$=\dfrac{500}{4}$
$=125$ students

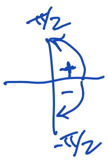
(b) $P(4)=\dfrac{500}{1+3^{-1}}$
$=\dfrac{500}{4/3}$
$=125\cdot 3$
$=375$ students

(c) $P(t)=200$
$\dfrac{500}{1+3^{3-t}}=200$
$\frac{5}{2}=1+3^{3-t}$
$\frac{3}{2}=3^{3-t}$
$\log_3\left(\frac{3}{2}\right)=3-t$
$t=3-\log_3\left(\frac{3}{2}\right)$
$t=2.630$ days

(d) $\lim\limits_{t\to\infty}\dfrac{500}{1+3^{3-t}}=500$ students

Evaluate.

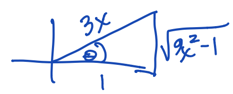
18. $\cos\left(Sin^{-1}\left(-\dfrac{\sqrt{3}}{2}\right)\right)$



$\cos\left(-\frac{\pi}{3}\right)$

$\frac{1}{2}$

19. $\tan\left(Arc\sec\left(\frac{3x}{1}\right)\right)$



$\tan\theta$

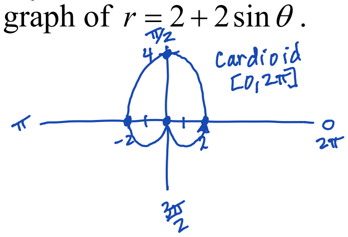
$\dfrac{\sqrt{9x^2-1}}{1}$

$\sqrt{9x^2-1}$

20. Convert $r^2+6r\cos\theta=0$ into rectangular form.

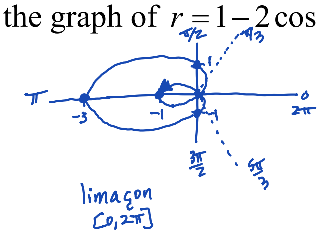
$r^2+6r\cdot\frac{x}{r}=0$
$r^2+6x=0$
$x^2+y^2+6x=0$
$x^2+6x+9+y^2=9$
$(x+3)^2+y^2=9$ circle

21. Sketch the graph of $r=2+2\sin\theta$.



22. Sketch the graph of $r=1-2\cos\theta$.

Polar zero
$1-2\cos\theta=0$
$\cos\theta=\frac{1}{2}$
$\theta=\frac{\pi}{3},\frac{5\pi}{3}$



Find the limit.

23. $\lim\limits_{x\to 3}\dfrac{x-3}{x^3-27}$ $\frac{0}{0}$

$$\begin{array}{r|rrrr} 3 & 1 & 0 & 0 & -27 \\ & \downarrow & 3 & 9 & 27 \\ \hline & 1 & 3 & 9 & \boxed{0} \end{array}$$

$\lim\limits_{x\to 3}\dfrac{\cancel{(x-3)}}{\cancel{(x-3)}(x^2+3x+9)}$

$\dfrac{1}{9+9+9}$

$\dfrac{1}{27}$

24. $\lim\limits_{y\to 0}\dfrac{\sqrt{2-y}-\sqrt{2}}{y}\cdot\dfrac{\sqrt{2-y}+\sqrt{2}}{\sqrt{2-y}+\sqrt{2}}$ $\frac{0}{0}$

$\lim\limits_{y\to 0}\dfrac{2-y-2}{y(\sqrt{2-y}+\sqrt{2})}$

$\lim\limits_{y\to 0}\dfrac{-\cancel{y}}{\cancel{y}(\sqrt{2-y}+\sqrt{2})}$

$\dfrac{-1}{2\sqrt{2}}$

25. $\lim\limits_{x\to -\infty}\dfrac{3x^2-4x^3}{5x^3+2x}$

$-\dfrac{4}{5}$

Use the power rule to find the derivative when needed.

26. Find the slope of the line tangent to $f(x)=x^3-3x^2+4x$ at the point $(1,2)$.

$f'(x)=3x^2-6x+4$

$f'(1)=3-6+4$

$=-3+4$

$=1$

27. Find an equation of a line that is tangent to $g(x)=5-x^2$ and is perpendicular to the line $x+6y-7=0$

$6y=-x+7$

$y=-\frac{1}{6}x+\frac{7}{6}$

Slope $=-\frac{1}{6}$

perp slop $=6=g'(x)$

$g'(x)=-2x$

So, $-2x=6$

$x=-3$

$g(-3)=5-9$

$=-4$

pt: $(-3,-4)$, $m=6$

eq: $y=-4+6(x+3)$ (Taylor Form)

Use the limit definition of the derivative to find the derivative of each function.

28. $f(x)=\dfrac{2}{x+1}$ $\frac{0}{0}$

$f'(x)=\lim\limits_{h\to 0}\dfrac{\frac{2}{x+h+1}-\frac{2}{x+1}}{h}\cdot\dfrac{(x+h+1)(x+1)}{(x+h+1)(x+1)}$

$=\lim\limits_{h\to 0}\dfrac{2(x+1)-2(x+h+1)}{h(x+h+1)(x+1)}$

$=\lim\limits_{h\to 0}\dfrac{2x+2-2x-2h-2}{h(x+h+1)(x+1)}$

$=\lim\limits_{h\to 0}\dfrac{-2\cancel{h}}{\cancel{h}(x+h+1)(x+1)}$

$f'(x)=\dfrac{-2}{(x+1)^2}$

29. $f(x)=-2\sqrt{x+1}$

$f'(x)=\lim\limits_{h\to 0}\dfrac{-2\sqrt{x+h+1}+2\sqrt{x+1}}{h}$ $\frac{0}{0}$

$=\lim\limits_{h\to 0}\dfrac{-2(\sqrt{x+h+1}-\sqrt{x+1})}{h}\cdot\dfrac{\sqrt{x+h+1}+\sqrt{x+1}}{\sqrt{x+h+1}+\sqrt{x+1}}$

$=\lim\limits_{h\to 0}\dfrac{-2(x+h+1-x-1)}{h(\sqrt{x+h+1}+\sqrt{x+1})}$

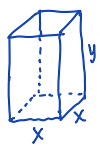
$=\lim\limits_{h\to 0}\dfrac{-2\cancel{h}}{\cancel{h}(\sqrt{x+h+1}+\sqrt{x+1})}$

$=\dfrac{-2}{2\sqrt{x+1}}$

$f'(x)=\dfrac{-1}{\sqrt{x+1}}$

Write a function, and use your graphing calculator to solve. Give decimal answers correct to **three** decimal places.

30. A container with a square base, vertical sides, and an open top is to be made from 1000 ft^2 of material (assume no waste.) Find the dimensions of the container with greatest volume.



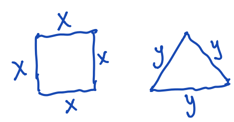
Constraint eq.
$A = x^2 + 4xy = 1000$
$4xy = 1000 - x^2$
$y = \dfrac{1000 - x^2}{4x}$

Primary eq.
$V = x^2 y$
So, $V = x^2\left(\dfrac{1000 - x^2}{4x}\right), x \neq 0$
$V = \frac{1}{4}x\left(1000 - x^2\right)$
$V = 250x - \frac{1}{4}x^3$
$V' = 250 - \frac{3}{4}x^2 = 0$
$x^2 = \frac{4(250)}{3}$

$x = \sqrt{\frac{1000}{3}} = 18.257$ ft

$y = \dfrac{1000 - \frac{1000}{3}}{4\sqrt{\frac{1000}{3}}} = 9.128$ ft

31. A piece of wire 10 m long is cut into two pieces. One piece is bent into a square, and the other is bent into an equilateral triangle. How should the wire be cut so that the total area enclosed is
   a. A maximum?



Constraint eq
$4x + 3y = 10$
$4x = 10 - 3y$
$x = \dfrac{10 - 3y}{4}$

Primary eq.
$A = x^2 + \frac{\sqrt{3}}{4}y^2$
$A = \left(\frac{10 - 3y}{4}\right)^2 + \frac{\sqrt{3}}{4}y^2, \ y \in \left[0, \frac{10}{3}\right]$
$A = \frac{1}{16}(10 - 3y)^2 + \frac{\sqrt{3}}{4}y^2$
$A' = \frac{1}{8}(10 - 3y)(-3) + \frac{\sqrt{3}}{2}y = 0$
$-30 + 9y + 4\sqrt{3}y = 0$
$y(9 + 4\sqrt{3}) = 30$
$y = \frac{30}{9 + 4\sqrt{3}}$ m

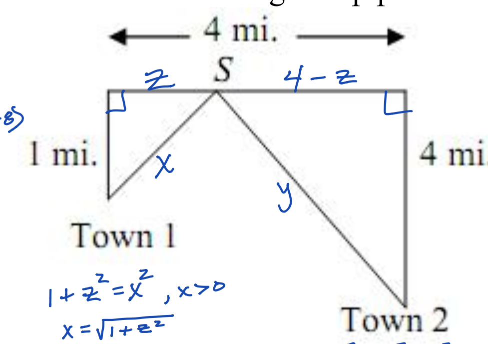
$A'' = -\frac{3}{8}(-3) + \frac{\sqrt{3}}{2}$
$A'' = \frac{9}{8} + \frac{\sqrt{3}}{2} > 0$ for all y
So, A is concave up for all y,
So, $y = \dfrac{30}{9 + 4\sqrt{3}}$ minimizes area absolutely.

$A(0) = 6.25 \text{ m}^2$
$A\left(\frac{30}{9 + 4\sqrt{3}}\right) = 2.718 \text{ m}^2$
$A\left(\frac{10}{3}\right) = 4.811 \text{ m}^2$

So, let $y = 0$ m (use all wire for square only) to maximize area, and let $y = \dfrac{30}{9 + 4\sqrt{3}}$ m to minimize area.

   b. A minimum?

32. On the same side of a straight river are two towns, and the townspeople want to build a pumping station at point *S*. Find the distance from *S* to Town 1 that will minimize the total length of pipe.



pipe = P = x + y
$P = \sqrt{1 + z^2} + \sqrt{z^2 - 8z + 32}$
$P' = \frac{1}{2}(1 + z^2)^{-1/2}(2z) + \frac{1}{2}(z^2 - 8z + 32)^{-1/2}(2z - 8)$
$P' = \dfrac{z}{\sqrt{1 + z^2}} + \dfrac{z - 4}{\sqrt{z^2 - 8z + 32}} = 0$
$z = 0.8$ miles (from calculator)

since $P' < 0$, for all $z < 0.8$ and $P' > 0$, for all $z > 0.8$,
$z = 0.8$ miles minimizes P.

$1 + z^2 = x^2, \ x > 0$
$x = \sqrt{1 + z^2}$

$(4 - z)^2 + 4^2 = y^2, \ y > 0$
$y = \sqrt{16 - 8z + z^2 + 16}$
$y = \sqrt{z^2 - 8z + 32}$

So, $x = \sqrt{1 + (0.8)^2}$ miles $= \sqrt{\frac{41}{25}}$
$= 1.280$ miles $= \frac{\sqrt{41}}{5}$ miles