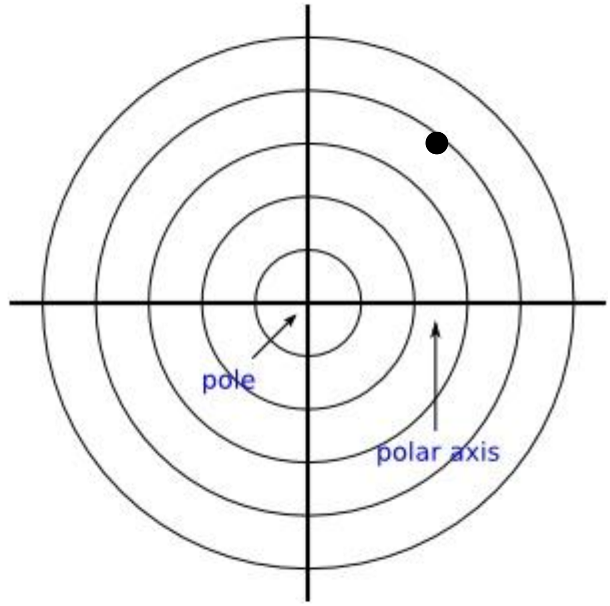
§8.1—Polar Intro & Derivatives



A rectangular coordinate system is only one way to navigate through a Euclidean plane. Such coordinates, (x, y), known as **rectangular coordinates**, are useful for expressing functions of *y* in terms of *x*. Curves that are not functions are often more easily expressing in an alternative coordinate system called **polar coordinates**.

In a polar coordinate system, we still have the traditional *x*- and *y*- axes. The intersection of these axes, the old origin, is called the **pole**. Similar to navigating on the Unit Circle, we can now get to any point in 2-D space by specifying an independent choice of an angle, θ, from the initial ray, **polar axis**, then walking out along that terminal ray a specified amount, *r*, in either direction.

Although the angle is the independent variable, we express the point in the polar plane as (r, θ). The point to the right would have coordinates of $\left(4, \frac{\pi}{4}\right)$.

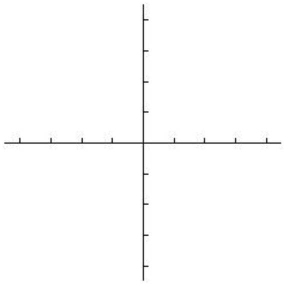
Example 1:
Find several other equivalent polar coordinates for the point shown above, then find the equivalent rectangular coordinate.

Why use polar coordinates? Graphs that aren't functions in rectangular form $f(x)$ can still be functions in polar form $r(\theta)$. Some of these curves can be quite elaborate and are more easily expressed as polar, rather than rectangular equations, as the following calculator exploration will demonstrate.

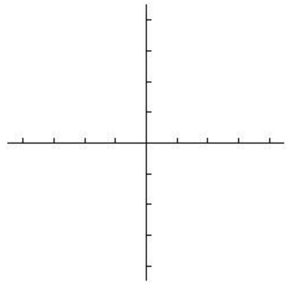
Put your graphing calculator in POLAR mode and RADIAN mode. Graph the following equations on your calculator, sketch the graphs on this sheet, and answer the questions.

Example 2:

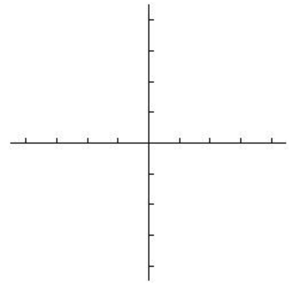
$r = \cos\theta$



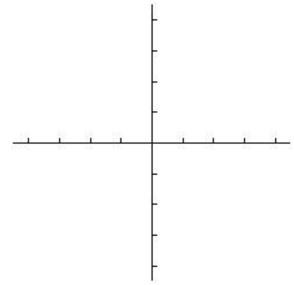
$r = 2\cos\theta$



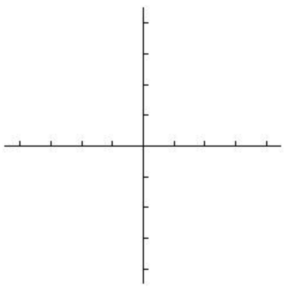
$r = 3\cos\theta$



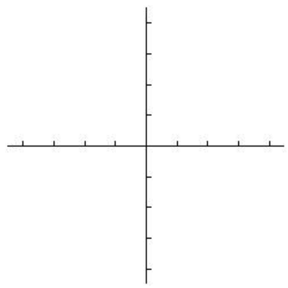
$r = -3\cos\theta$



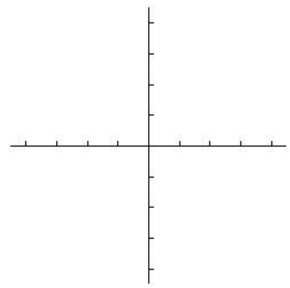
$r = \sin\theta$



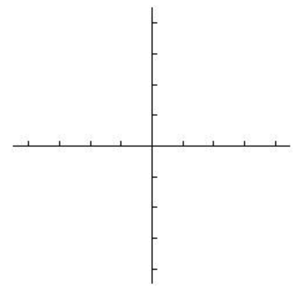
$r = 2\sin\theta$



$r = 3\sin\theta$



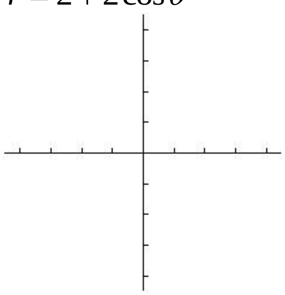
$r = -3\sin\theta$



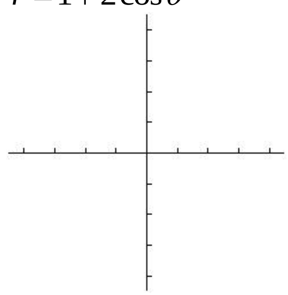
What do you notice about these graphs?

Example 3:

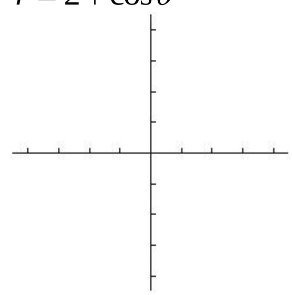
$r = 2 + 2\cos\theta$



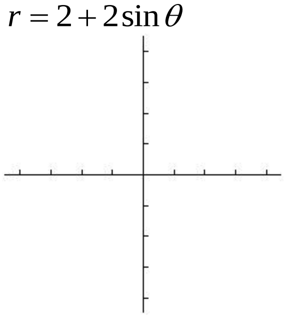
$r = 1 + 2\cos\theta$



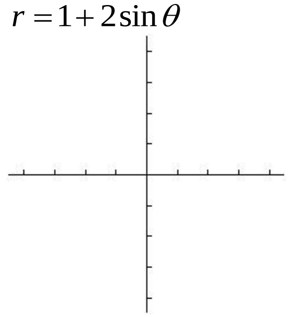
$r = 2 + \cos\theta$



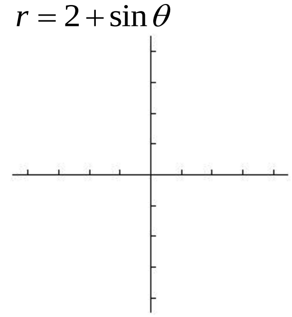
$r = 2 + 2\sin\theta$



$r = 1 + 2\sin\theta$



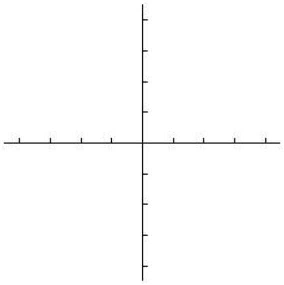
$r = 2 + \sin\theta$



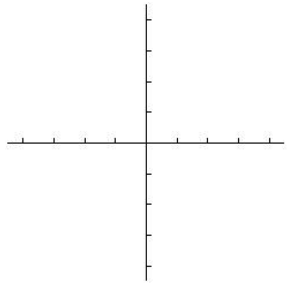
Which graphs go through the pole?
Which ones do not go through the pole?
Which ones have an inner loop?
What causes these things to happen? (Hint: Go to FORMAT and set your calculator to see the Polar Graphing Coordinates when you trace.)

Example 4:

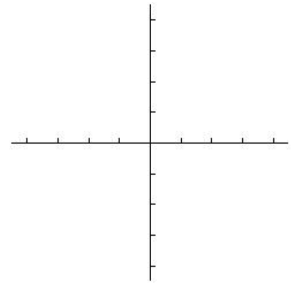
$r = 3\cos 3\theta$



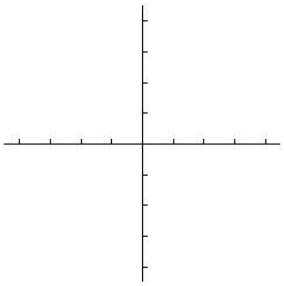
$r = 2\cos 5\theta$



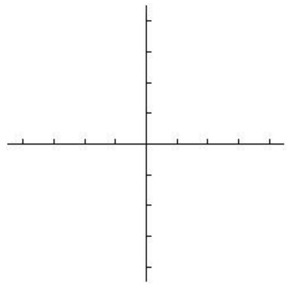
$r = 4\cos 7\theta$



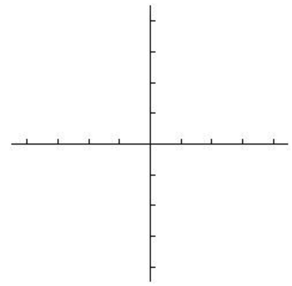
$r = 3\sin 3\theta$



$r = 2\sin 5\theta$



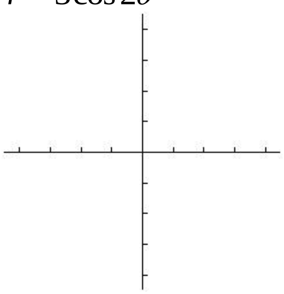
$r = 4\sin 7\theta$



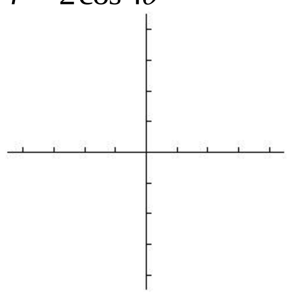
What do you notice about these graphs?

Example 5:

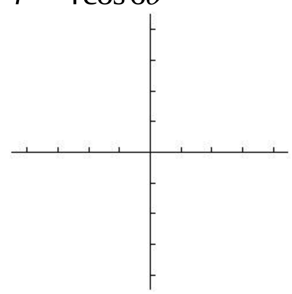
$r = 3\cos 2\theta$



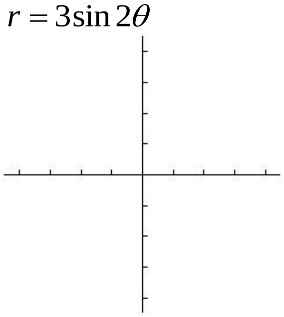
$r = 2\cos 4\theta$



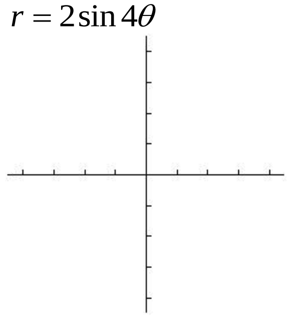
$r = 4\cos 6\theta$



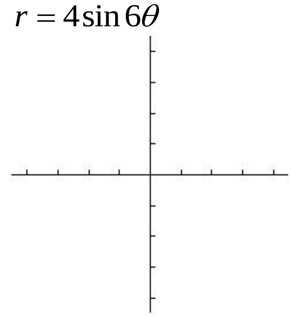
$r = 3\sin 2\theta$



$r = 2\sin 4\theta$



$r = 4\sin 6\theta$



What do you notice about these graphs?

Before we jump into the calculus of polar, we need to develop some proficiency converting back and forth from polar to rectangular coordinates and equations.

Rectangular/Polar Coordinates and Equations

Rectangular coordinates are in the form (x, y), where x is the independent variable.
Polar coordinates are in the form (r, θ), where θ is the independent variable.

For coordinate conversions:

Polar to Rectangular

$x = r\cos\theta$
$y = r\sin\theta$

Rectangular to Polar

$r = \sqrt{x^2 + y^2}$

$\tan\theta = \dfrac{y}{x}$

Example 6:
Convert the following coordinates as specified.

a) Convert $\left(2, \dfrac{5\pi}{6}\right)$ to rectangular coordinates.

b) Convert $(3, -3)$ to polar coordinates.

Example 7:
Convert the following equations from rectangular to polar.

a) $y = 2x + 1$

b) $y = 4$

c) $x^2 + y^2 = 16$

Example 8:
Convert the following equations from polar to rectangular.

a) $r = 2\cos\theta$

b) $\theta = \dfrac{3\pi}{4}$

c) $r = \csc\theta$

Now it's time to B.O.T.C.

Slope of a Polar Equation

To find the slope of a tangent line to a polar graph $r = f(\theta)$, we can use $x = r\cos\theta$ and $y = r\sin\theta$ and the product rule:

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{r\cos\theta + r'\sin\theta}{-r\sin\theta + r'\cos\theta}\text{, provided that } \frac{dx}{d\theta} \neq 0$$

Or . . . . create $y(\theta)$ and $x(\theta)$ first, then find $y'(\theta)$ and $x'(\theta)$.

Example 9:

Find $\frac{dy}{dx}$ and the slope of the graph of the polar curve at the given value of θ.

$$r = 2 + 2\sin\theta \text{ at } \theta = \frac{\pi}{2}$$

Example 10:

Find the **points,** (r,θ) of horizontal and vertical tangency to the graph of $r = 2 - 2\cos\theta$. Beware of $\frac{0}{0}$.