

CALCULUS BC

SECTION I, Part A

Time—55 Minutes

Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test: Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

1. If $7 = xy - e^{xy}$, then $\frac{dy}{dx} =$

(A) $x - e^y$

(B) $y - e^x$

(C) $\frac{ye^{xy} + y}{x - xe^{xy}}$

(D) $\frac{-y}{x}$

(E) $\frac{ye^{xy} + y}{x + xe^{xy}}$

2. The volume of the solid that results when the area between the curve $y = e^x$ and the line $y = 0$, from $x = 1$ to $x = 2$, is revolved around the x -axis is

(A) $2\pi(e^4 - e^2)$

(B) $\frac{\pi}{2}(e^4 - e^2)$

(C) $\frac{\pi}{2}(e^2 - e)$

(D) $2\pi(e^2 - e)$

(E) $2\pi e^2$

GO ON TO THE NEXT PAGE 

3. $\int \frac{x - 18}{(x + 3)(x - 4)} dx =$

(A) $\int \frac{5dx}{(x + 3)(x - 4)}$

(B) $\int \frac{dx}{(x + 3)(x - 4)}$

(C) $\int \frac{3dx}{x + 3} + \int \frac{2dx}{x - 4}$

(D) $\int \frac{15dx}{x + 3} - \int \frac{14dx}{x - 4}$

(E) $\int \frac{3dx}{x + 3} - \int \frac{2dx}{x - 4}$

4. If $y = 5x^2 + 4x$ and $x = \ln t$ then $\frac{dy}{dt} =$

(A) $\frac{10}{t} + 4$

(B) $10t \ln t + 4t$

(C) $\frac{10 \ln t + 4}{t}$

(D) $\frac{5}{t^2} + \frac{4}{t}$

(E) $10 \ln t + \frac{4}{t}$

GO ON TO THE NEXT PAGE 

5. $\int_0^{\frac{\pi}{2}} \sin^5 x \cos x dx =$

(A) $\frac{1}{6}$

(B) $-\frac{1}{6}$

(C) 0

(D) -6

(E) 6

6. The tangent line to the curve $y = x^3 - 4x + 8$ at the point $(2,8)$ has an x -intercept at

(A) $(-1, 0)$

(B) $(1, 0)$

(C) $(0, -8)$

(D) $(0, 8)$

(E) $(8, 0)$

7. The graph in the xy -plane represented by $x = 3\sin(t)$ and $y = 2\cos(t)$ is

(A) a circle

(B) an ellipse

(C) a hyperbola

(D) a parabola

(E) a line

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8. $\int \frac{dx}{\sqrt{4-9x^2}} =$

(A) $\frac{1}{6} \sin^{-1}\left(\frac{3x}{2}\right) + C$

(B) $\frac{1}{2} \sin^{-1}\left(\frac{3x}{2}\right) + C$

(C) $6 \sin^{-1}\left(\frac{3x}{2}\right) + C$

(D) $3 \sin^{-1}\left(\frac{3x}{2}\right) + C$

(E) $\frac{1}{3} \sin^{-1}\left(\frac{3x}{2}\right) + C$

9. $\lim_{x \rightarrow \infty} 4x \sin\left(\frac{1}{x}\right)$ is

(A) 0

(B) 2

(C) 4

(D) 4π

(E) nonexistent

GO ON TO THE NEXT PAGE 

10. The position of a particle moving along the x -axis at time t is given by $x(t) = e^{\cos(2t)}$, $0 \leq t \leq \pi$. For which of the following values of t will $x'(t) = 0$?

I. $t = 0$

II. $t = \frac{\pi}{2}$

III. $t = \pi$

(A) I only

(B) II only

(C) I and III only

(D) I and II only

(E) I, II, and III

11. $\lim_{h \rightarrow 0} \frac{\sec(\pi + h) - \sec(\pi)}{h} =$

(A) -1

(B) 0

(C) $\frac{1}{\sqrt{2}}$

(D) 1

(E) $\sqrt{2}$

12. Use differentials to approximate the change in the volume of a cube when the side is decreased from 8 to 7.99 cm. (in cm^3)

(A) -19.2

(B) -15.36

(C) -1.92

(D) -0.01

(E) -0.0001

GO ON TO THE NEXT PAGE 

13. The radius of convergence of $\sum_{n=1}^{\infty} \frac{a^n}{(x+2)^n}$; $a > 0$ is

- (A) $(a - 2) \leq x \leq (a + 2)$
 - (B) $(a - 2) < x < (a + 2)$
 - (C) $(-a - 2) > x > (a - 2)$
 - (D) $(a - 2) > x > (-a - 2)$
 - (E) $(a - 2) \leq x \leq (-a - 2)$
-

14. $\int_0^1 \sin^{-1}(x) dx =$

- (A) 0
 - (B) $\frac{\pi + 2}{2}$
 - (C) $\frac{\pi - 2}{2}$
 - (D) $\frac{\pi}{2}$
 - (E) $\frac{-\pi}{2}$
-

15. The equation of the line *normal* to $y = \sqrt{\frac{5-x^2}{5+x^2}}$ at $x = 2$ is

- (A) $81x - 60y = 142$
 - (B) $81x + 60y = 182$
 - (C) $20x + 27y = 49$
 - (D) $20x + 27y = 31$
 - (E) $81x - 60y = 182$
-

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16. If c satisfies the conclusion of the Mean Value Theorem for derivatives for $f(x) = 2 \sin x$ on the interval $[0, \pi]$, then c could be

(A) 0

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) π

(E) There is no value of c on $[0, \pi]$

17. The average value of $f(x) = x \ln x$ on the interval $[1, e]$ is

(A) $\frac{e^2 + 1}{4}$

(B) $\frac{e^2 + 1}{4(e + 1)}$

(C) $\frac{e + 1}{4}$

(D) $\frac{e^2 + 1}{4(e - 1)}$

(E) $\frac{3e^2 + 1}{4(e - 1)}$

18. A 17-foot ladder is sliding down a wall at a rate of -5 feet/sec. When the top of the ladder is 8 feet from the ground, how fast is the foot of the ladder sliding away from the wall (in feet/sec)?

(A) $\frac{75}{8}$

(B) $\frac{8}{3}$

(C) $\frac{3}{8}$

(D) -16

(E) $\frac{-75}{3}$

GO ON TO THE NEXT PAGE 

19. If $\frac{dy}{dx} = 3y \cos x$, and $y = 8$ when $x = 0$, then $y =$

(A) $8e^{3\sin x}$

(B) $8e^{3\cos x}$

(C) $8e^{3\sin x} + 3$

(D) $3\frac{y^2}{2}\cos x + 8$

(E) $3\frac{y^2}{2}\sin x + 8$

20. The length of the curve determined by $x = 3t$ and $y = 2t^2$ from $t = 0$ to $t = 9$ is

(A) $\int_0^9 \sqrt{9t^2 + 4t^4} dt$

(B) $\int_0^{162} \sqrt{9 - 16t^2} dt$

(C) $\int_0^{162} \sqrt{9 + 16t^2} dt$

(D) $\int_0^3 \sqrt{9 - 16t^2} dt$

(E) $\int_0^9 \sqrt{9 + 16t^2} dt$

GO ON TO THE NEXT PAGE 

21. If a particle moves in the xy -plane so that at time $t > 0$ its position vector is (e^{t^2}, e^{-t^3}) , then its velocity vector at time $t = 3$ is
- (A) $(\ln 6, \ln(-27))$
 - (B) $(\ln 9, \ln(-27))$
 - (C) (e^9, e^{-27})
 - (D) $(6e^9, -27e^{-27})$
 - (E) $(9e^9, -27e^{-27})$
-

22. The graph of $f(x) = \sqrt{11 + x^2}$ has a point of inflection at
- (A) $(0, \sqrt{11})$
 - (B) $(-\sqrt{11}, 0)$
 - (C) $(0, -\sqrt{11})$
 - (D) $(\sqrt{\frac{11}{2}}, \sqrt{\frac{33}{2}})$
 - (E) There is no point of inflection.
-

GO ON TO THE NEXT PAGE 

23. What is the volume of the solid generated by rotating about the y -axis the region enclosed by $y = \sin x$ and the x -axis, from $x = 0$ to $x = \pi$?
- (A) π^2 (B) $2\pi^2$ (C) $4\pi^2$ (D) 2 (E) 4
-

24. $\int_{\frac{2}{\pi}}^{\infty} \frac{\sin\left(\frac{1}{t}\right)}{t^2} dt =$

- (A) 1 (B) 0 (C) -1 (D) 2 (E) Undefined
-

25. A rectangle is to be inscribed between the parabola $y = 4 - x^2$ and the x -axis, with its base on the x -axis. A value of x that maximizes the area of the rectangle is

- (A) 0 (B) $\frac{2}{\sqrt{3}}$ (C) $\frac{2}{3}$ (D) $\frac{4}{3}$ (E) $\frac{\sqrt{3}}{2}$
-

GO ON TO THE NEXT PAGE 

26. $\int \frac{dx}{\sqrt{9-x^2}} =$

- (A) $\sin^{-1} 3x + C$
- (B) $\ln|x + \sqrt{9-x^2}| + C$
- (C) $\frac{1}{3} \sin^{-1} x + C$
- (D) $\sin^{-1} \frac{x}{3} + C$
- (E) $\frac{1}{3} \ln|x + \sqrt{9-x^2}| + C$

27. Find $\lim_{x \rightarrow \infty} x^{\frac{1}{x}}$

- (A) 0
 - (B) 1
 - (C) ∞
 - (D) -1
 - (E) $-\infty$
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GO ON TO THE NEXT PAGE 

28. What is the sum of the Maclaurin series $\pi - \frac{\pi^3}{3!} + \frac{\pi^5}{5!} - \frac{\pi^7}{7!} + \dots + (-1)^n \frac{\pi^{2n+1}}{(2n+1)!} + \dots$?

(A) 1

(B) 0

(C) -1

(D) e

(E) There is no sum.

STOP

END OF PART A SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS BC

SECTION I, Part B

Time—50 Minutes

Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

In this test:

1. The **exact** numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.
2. Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

29. The first three non-zero terms in the Taylor series about $x = 0$ for $f(x) = \cos x$

(A) $x + \frac{x^3}{3!} + \frac{x^5}{5!}$

(B) $x - \frac{x^3}{3!} + \frac{x^5}{5!}$

(C) $1 - \frac{x^2}{2!} + \frac{x^4}{4!}$

(D) $1 - \frac{x^2}{2!} - \frac{x^4}{4!}$

(E) $1 + \frac{x^2}{2!} + \frac{x^4}{4!}$

GO ON TO THE NEXT PAGE 

30. $\int \cos^3 x \, dx =$

(A) $\frac{\cos^4 x}{4} + C$

(B) $\frac{\sin^4 x}{4} + C$

(C) $\sin x - \frac{\sin^3 x}{3} + C$

(D) $\sin x + \frac{\sin^3 x}{3} + C$

(E) $\sin^3 x + C$

31. If $f(x) = (3x)^{(3x)}$ then $f'(x) =$

(A) $(3x)^{(3x)}(3\ln(3x) + 3)$

(B) $(3x)^{(3x)}(3\ln(3x) + 3x)$

(C) $(9x)^{(3x)}(\ln(3x) + 1)$

(D) $(3x)^{(3x-1)}(3x)$

(E) $(3x)^{(3x-1)}(9x)$

GO ON TO THE NEXT PAGE 

32. To what limit does the sequence $S_n = \frac{3+n}{3^n}$ converge as n approaches infinity?

(A) 1

(B) $\frac{1}{3}$

(C) 0

(D) ∞

(E) 3

33. $\int \frac{18x-17}{(2x-3)(x+1)} dx =$

(A) $8\ln|2x-3| + 7\ln|x+1| + C$

(B) $2\ln|2x-3| + 7\ln|x+1| + C$

(C) $4\ln|2x-3| + 7\ln|x+1| + C$

(D) $7\ln|2x-3| + 2\ln|x+1| + C$

(E) $\frac{7}{2}\ln|2x-3| + 4\ln|x+1| + C$

34. A particle moves along a path described by $x = \cos^3 t$ and $y = \sin^3 t$. The distance that the particle travels along the path from $t = 0$ to $t = \frac{\pi}{2}$ is

(A) 0.75

(B) 1.50

(C) 0

(D) -3.50

(E) -0.75

GO ON TO THE NEXT PAGE 

35. The sale price of an item is $800 - 35x$ dollars and the total manufacturing cost is $2x^3 - 140x^2 + 2,600x + 10,000$ dollars, where x is the number of items. What number of items should be manufactured in order to optimize the manufacturer's total profit?

(A) 35

(B) 25

(C) 10

(D) 15

(E) 20

36. The area enclosed by the polar equation $r = 4 + \cos \theta$, for $0 \leq \theta \leq 2\pi$, is

(A) 0

(B) $\frac{9\pi}{2}$

(C) 18π

(D) $\frac{33\pi}{2}$

(E) $\frac{33\pi}{4}$

37. Use the trapezoid rule with $n = 4$ to approximate the area between the curve $y = x^3 - x^2$ and the x -axis from $x = 3$ to $x = 4$.

(A) 35.266

(B) 27.766

(C) 63.031

(D) 31.516

(E) 25.125

GO ON TO THE NEXT PAGE 

38. If $f(x) = \sum_{k=0}^{\infty} (\cos^2 x)^k$, then $f\left(\frac{\pi}{4}\right)$ is

- (A) -2 (B) -1 (C) 0 (D) 1 (E) 2

39. The volume of the solid that results when the area between the graph of $y = x^2 + 2$ and the graph of $y = 10 - x^2$ from $x = 0$ to $x = 2$ is rotated around the x -axis is

- (A) $2\pi \int_0^2 y(\sqrt{y-2}) dy + 2\pi \int_0^2 y(\sqrt{10-y}) dy$
(B) $2\pi \int_2^6 y(\sqrt{y-2}) dy + 2\pi \int_6^{10} y(\sqrt{10-y}) dy$
(C) $2\pi \int_2^6 y(\sqrt{y-2}) dy - 2\pi \int_6^{10} y(\sqrt{10-y}) dy$
(D) $2\pi \int_0^2 y(\sqrt{y-2}) dy - 2\pi \int_0^2 y(\sqrt{10-y}) dy$
(E) $2\pi \int_0^2 y(\sqrt{10-y}) dy - 2\pi \int_0^2 y(\sqrt{y-2}) dy$

40. $\int_0^4 \frac{dx}{\sqrt{9+x^2}} =$

- (A) $\ln 3$ (B) $\ln 4$ (C) $-\ln 2$ (D) $-\ln 4$ (E) Undefined

GO ON TO THE NEXT PAGE 

41. The rate that an object cools is directly proportional to the difference between its temperature (in Kelvins) at that time and the surrounding temperature (in Kelvins). If an object is initially at 35K, and the surrounding temperature remains constant at 10K, it takes 5 minutes for the object to cool to 25K. How long will it take for the object to cool to 20K?
- (A) 6.66 min. (B) 7.50 min. (C) 7.52 min. (D) 8.97 min. (E) 10.00 min.
-

42. $\int e^x \cos x \, dx =$

- (A) $\frac{e^x}{2}(\sin x + \cos x) + C$
(B) $\frac{e^x}{2}(\sin x - \cos x) + C$
(C) $\frac{e^x}{2}(\cos x - \sin x) + C$
(D) $2e^x(\sin x + \cos x) + C$
(E) $e^x(\sin x + \cos x) + C$
-

43. Two particles leave the origin at the same time and move along the y -axis with their respective positions determined by the functions $y_1 = \cos 2t$ and $y_2 = 4 \sin t$ for $0 < t < 6$. For how many values of t do the particles have the same acceleration?
- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4
-

GO ON TO THE NEXT PAGE 

44. The minimum value of the function $y = x^3 - 7x + 11$, $x \geq 0$, is approximately

- (A) 18.128 (B) 9.283 (C) 6.698 (D) 5.513 (E) 3.872
-

45. Use Euler's Method with $h = 0.2$ to estimate $y(1)$, if $y' = y$ and $y(0) = 1$.

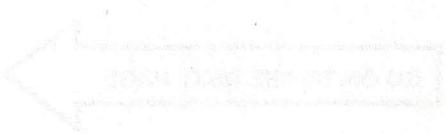
- (A) 1.200 (B) 2.075 (C) 2.488 (D) 4.838 (E) 9.677
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STOP

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS SECTION.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.



GENERAL INSTRUCTIONS

You may wish to look over the problems before starting to work on them, since it is not expected that everyone will be able to complete all parts of all problems. All problems are given equal weight, but the parts of a particular problem are not necessarily given equal weight.

A GRAPHING CALCULATOR IS REQUIRED FOR SOME PROBLEMS OR PARTS OF PROBLEMS ON THIS SECTION OF THE EXAMINATION.

- You should write all work for each part of each problem in the space provided for that part in the booklet. Be sure to write clearly and legibly. If you make an error, you may save time by crossing it out rather than trying to erase it. Erased or crossed-out work will not be graded.
- Show all your work. You will be graded on the correctness and completeness of your methods as well as your answers. Correct answers without supporting work may not receive credit.
- Justifications require that you give mathematical (noncalculator) reasons and that you clearly identify functions, graphs, tables, or other objects you use.
- You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.
- Your work must be expressed in standard mathematical notation rather than calculator syntax.

For example, $\int_1^5 x^2 dx$ may not be written as $\text{fnInt}(X^2, X, 1, 5)$.

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

PART A

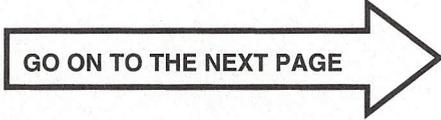
Time—45 minutes

Number of problems—3

A graphing calculator is required for some problems or parts of problems.

During the timed portion for Part A, you may work only on the problems in Part A.

On Part A, you are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your problem, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results.

GO ON TO THE NEXT PAGE 

1. Two particles travel in the xy -plane. For time $t \geq 0$, the position of particle A is given by $x = t + 1$ and $y = (t + 1)^2 - 2t - 2$, and the position of particle B is given by $x = 4t - 2$ and $y = -2t + 2$.
- (a) Find the velocity vector for each particle at time $t = 2$.
 - (b) Set up an integral expression for the distance traveled by particle A from time $t = 1$ to $t = 3$. Do not evaluate the integral.
 - (c) At what time do the two particles collide? Justify your answer.
 - (d) Sketch the path of both particles from time $t = 0$ to $t = 4$. Indicate the direction of each particle along its path.
-

2. Let f be the function given by $f(x) = 2x^4 - 4x^2 + 1$.
- (a) Find an equation of the line tangent to the graph at $(-2, 17)$. Verify your answer.
 - (b) Find the x and y -coordinates of the relative maxima and relative minima.
 - (c) Find the x -coordinates of the points of inflection. Verify your answer.
-

3. Water is draining at the rate of $48\pi \text{ ft}^3/\text{sec}$ from the vertex at the bottom of a conical tank whose diameter at its base is 40 feet and whose height is 60 feet.
- (a) Find an expression for the volume of water (in ft^3) in the tank in terms of its radius at the surface of the water.
 - (b) At what rate (in ft/sec) is the radius of the water in the tank shrinking when the radius is 16 feet?
 - (c) How fast (in ft/sec) is the height of the water in the tank dropping at the instant that the radius is 16 feet?

PART B
Time—45 minutes
Number of problems—3

No calculator is allowed for these problems.

During the timed portion for Part B, you may continue to work on the problems in Part A without the use of any calculator.

4. Let f be the function given by $f(x) = e^{-4x^2}$

- (a) Find the first four non-zero terms and the general term of the power series for $f(x)$ about $x = 0$.
 - (b) Find the interval of convergence of the power series for $f(x)$ about $x = 0$. Show the analysis that leads to your conclusion.
 - (c) Use term-by-term differentiation to show that $f'(x) = -8xe^{-4x^2}$
-

5. Let R be the region enclosed by the graphs of $y = 2 \ln x$ and $y = \frac{x}{2}$, and the lines $x = 2$ and $x = 8$.

- (a) Find the area of R .
 - (b) Set up, but do not integrate, an integral expression, in terms of a single variable, for the volume of the solid generated when R is revolved about the x -axis.
 - (c) Set up, but do not integrate, an integral expression, in terms of a single variable, for the volume of the solid generated when R is revolved about the line $x = -1$.
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GO ON TO THE NEXT PAGE 

6. Let f and g be functions that are differentiable throughout their domains and that have the following properties:

(i) $f(x + y) = f(x)g(y) + g(x)f(y)$

(ii) $\lim_{a \rightarrow 0} f(a) = 0$

(iii) $\lim_{h \rightarrow 0} \frac{g(h) - 1}{h} = 0$

(iv) $f'(0) = 1$

(a) Use L'Hôpital's Rule to show that $\lim_{a \rightarrow 0} \frac{f(a)}{a} = 1$.

(b) Use the definition of the derivative to show that $f'(x) = g(x)$.

(c) Find $\int \frac{g(x)}{f(x)} dx$

END OF EXAMINATION

ANSWER KEY TO SECTION I

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|-------|-------|-------|-------|-------|
| 1. D | 11. B | 21. D | 31. A | 41. D |
| 2. B | 12. C | 22. E | 32. C | 42. A |
| 3. E | 13. C | 23. B | 33. B | 43. D |
| 4. C | 14. C | 24. A | 34. B | 44. E |
| 5. A | 15. A | 25. B | 35. E | 45. C |
| 6. B | 16. C | 26. D | 36. D | |
| 7. B | 17. D | 27. B | 37. D | |
| 8. E | 18. B | 28. B | 38. D | |
| 9. C | 19. A | 29. C | 39. B | |
| 10. E | 20. E | 30. C | 40. A | |