Practice Exam BC-1

Calculus BC
Section I, Part A
Time — 55 minutes
Number of questions — 28

No calculator is allowed for these questions.

x	$f(x)$	$f'(x)$
0	1	2
$\frac{1}{2}$	2	4
1	3	5
$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$
2	$\frac{3}{2}$	−2

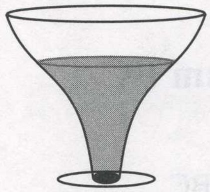
Questions 1 and 2 refer to the table above.

1. If f is a differentiable function on the interval $0 < x < 2$, find the derivative of the inverse function $f^{-1}(x)$ at $x = \frac{1}{2}$.

(A) −4 (B) −2 (C) −1 (D) $-\frac{1}{8}$ (E) $-\frac{1}{16}$

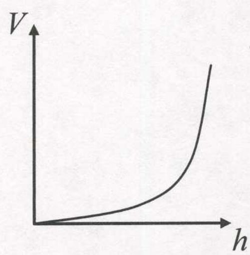
2. Using the table above and the fact that $f'(x)$ is continuous on the interval $0 \le x \le 2$, $\int_0^2 f'(x)\,dx =$

(A) −4 (B) −2 (C) 0 (D) $\frac{1}{2}$ (E) $\frac{3}{2}$

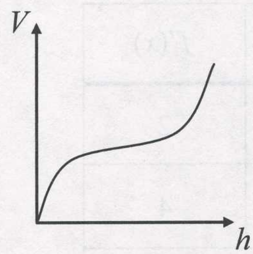


3. The glass above is initially empty, then gradually filled with water. Which of the following graphs best represents the volume V of water versus the height h of the water?

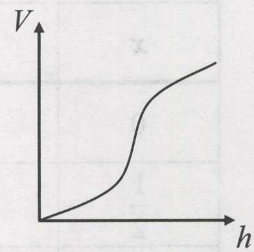
(A)



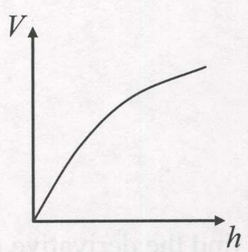
(B)



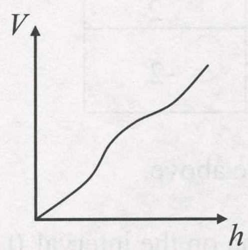
(C)



(D)



(E)



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4. If $f(x)=\sum_{n=0}^{\infty}\frac{(2x+1)^{n+1}}{n!}$, then $f''\left(-\frac{1}{2}\right)=$

(A) 0 (B) 1 (C) 2 (D) 4 (E) 8

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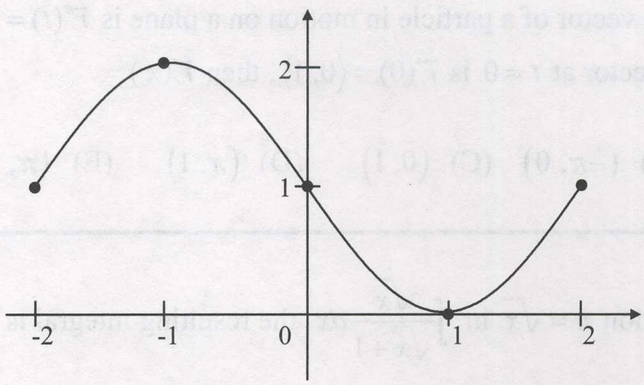
5. If $f(x)$ is a continuous and even function and $\int_0^4 f(x)\,dx=-5$ and $\int_4^6 f(t)\,dt=2$, then the average value of $f(x)$ over the interval from $x=-6$ to $x=4$ is

(A) −0.2 (B) −0.8 (C) 0.2 (D) 1.2 (E) 2



6. Given the graph of $y = f(x)$ shown above, which of the following values is the largest?

(A) $f(0)$ (B) $f'(0)$ (C) $\lim_{h\to 0}\frac{f(h)-1}{h}$

(D) $\frac{f(1)-f(-1)}{2}$ (E) $\frac{f'(1)-f'(-1)}{2}$

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7. $\lim_{h\to 0}\left(\frac{1}{h}\int_1^{1+h} e^{-t^2}\,dt\right) =$

(A) $-\frac{1}{2e}$ (B) $-\frac{2}{e}$ (C) 0 (D) $\frac{1}{e}$

(E) the limit does not exist

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8. If the differential equation $\frac{dy}{dx} = y - 2y^2$ has a solution curve $y = f(x)$ containing point $\left(0, \frac{1}{4}\right)$, then $\lim_{x\to\infty} f(x) =$

(A) 0 (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) 2

(E) the limit does not exist

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9. If the acceleration vector of a particle in motion on a plane is $\vec{r}''(t) = (t\sin t, \sin t)$ and the velocity vector at $t = 0$ is $\vec{r}'(0) = \langle 0, 1 \rangle$, then $\vec{r}'(\pi) =$

(A) $(-\pi, 2)$ (B) $(-\pi, 0)$ (C) $(0, 1)$ (D) $(\pi, 1)$ (E) $(\pi, 3)$

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10. After the substitution $u = \sqrt{x}$ in $\int \frac{\sqrt{x}}{\sqrt{x}+1}\,dx$, the resulting integral is

(A) $\int (1+u)\,du$ (B) $\int \frac{1}{u+1}\,du$ (C) $\int \frac{u}{u+1}\,du$

(D) $2\int (u+u^2)\,du$ (E) $2\int \frac{u^2}{u+1}\,du$

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11. If $f(x) = \tan^{-1} x$ then

$$\lim_{x\to\sqrt{3}} \frac{f'(x) - f'(\sqrt{3})}{x - \sqrt{3}} =$$

(A) $-\frac{\sqrt{3}}{8}$ (B) $-\frac{1}{4\sqrt{3}}$ (C) $\frac{1}{4}$ (D) $\frac{\pi}{6}$ (E) $\frac{\pi}{3}$

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12. If

$$f(x) = \begin{cases} \frac{|x|-2}{x-2}, & x \neq 2 \\ k, & x = 2 \end{cases},$$

then the value of k for which $f(x)$ is continuous for all real values of x is $k =$

(A) –2 (B) –1 (C) 0 (D) 1 (E) 2

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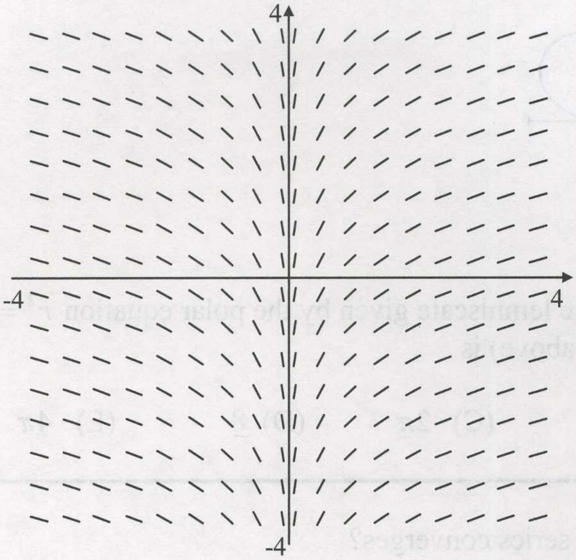
13. The radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(3x+4)^n}{n}$ is

(A) 0 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{4}$ (E) $\frac{4}{3}$



14. The slope field shown above matches which differential equation?

(A) $\frac{dy}{dx} = \frac{1}{x}$ (B) $\frac{dy}{dx} = \frac{1}{x^2}$ (C) $\frac{dy}{dx} = \frac{y}{x}$

(D) $\frac{dy}{dx} = \frac{\ln x}{x}$ (E) $\frac{dy}{dx} = \frac{\sin x}{x}$

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15. The graph of $f(x) = \frac{\sin x}{|x|}$ has

(A) no horizontal asymptotes and no vertical asymptotes
(B) one horizontal asymptote and no vertical asymptotes
(C) one horizontal asymptote and one vertical asymptote
(D) one horizontal asymptote and two vertical asymptotes
(E) two horizontal asymptotes and one vertical asymptote

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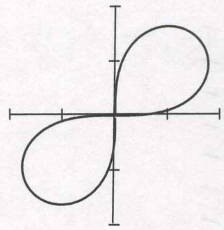
16. If $f(x) = 4^{3x}$ then $f'(x) =$

(A) $3x\left(4^{3x-1}\right)$ (B) $4^{3x}(\ln 4)$ (C) $3\left(4^{3x}\right)(\ln 4)$

(D) $\frac{4^{3x}}{\ln 4}$ (E) $\frac{4^{3x}}{x \ln 4}$



17. The area enclosed by the lemniscate given by the polar equation $r^2 = 4\sin 2\theta$ (whose graph is shown above) is

(A) 2 (B) 4 (C) 2π (D) 8 (E) 4π

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18. Which of the following series converges?

(A) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n+1}$ (B) $\sum_{n=2}^{\infty} \frac{1}{\ln n}$ (C) $\sum_{n=1}^{\infty} \sqrt{\frac{n}{n+1}}$

(D) $\sum_{n=1}^{\infty} \frac{1}{n \ln n}$ (E) $\sum_{n=1}^{\infty} \frac{\ln n}{n^2}$

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19. If $f(x) = e^x$ then $\frac{d}{dx}\left[f(f(x))\right] =$

(A) e^{x^2} (B) e^{e^x} (C) e^{x^e} (D) e^{2e^x} (E) e^{x+e^x}

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20. If $\frac{dy}{dx} = 1 - \frac{x}{y}$ and $y(1) = 1$, then when Euler's method with a step size of 0.5 is used to approximate $y(2)$, the approximation is

(A) 0 (B) 0.375 (C) 0.5 (D) 0.75 (E) 1.5

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21. For $x > 0$, $\frac{d}{dx}\int_x^{2x} \ln t \, dt =$

(A) $-\frac{1}{2x}$ (B) $\ln 2$ (C) $\ln 4$ (D) $\ln(2x)$ (E) $\ln(4x)$

22. Suppose the first three terms of the Maclaurin series for e^x are used to approximate $\frac{1}{\sqrt{e}}$. If a is the approximate value of $\frac{1}{\sqrt{e}}$ obtained and $b = \left|\frac{1}{\sqrt{e}} - a\right|$, then

(A) $a = \frac{5}{8}$ and $\frac{1}{24} \le b < \frac{1}{8}$

(B) $a = \frac{5}{8}$ and $\frac{1}{48} \le b < \frac{1}{24}$

(C) $a = \frac{5}{8}$ and $b < \frac{1}{48}$

(D) $a = \frac{3}{4}$ and $\frac{1}{24} \le b < \frac{1}{8}$

(E) $a = \frac{3}{4}$ and $b < \frac{1}{48}$

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23. If the region underneath $y = \frac{10}{x^2}$ and above the x-axis for $x \ge 1$ is divided into two regions with equal areas by the line $x = a$, then $a =$

(A) 1 (B) 2 (C) 5 (D) 10 (E) 100

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24. A series expansion for $f(x) = \frac{x}{1+x^2}$ is

(A) $1 - x^2 + x^4 - x^6 + \ldots$

(B) $x + x^3 + x^5 + x^7 + \ldots$

(C) $x - x^3 + x^5 - x^7 + \ldots$

(D) $x^2 - x^3 + x^4 - x^5 + \ldots$

(E) $x^2 - \frac{x^4}{3} + \frac{x^6}{5} - \frac{x^8}{7} + \ldots$

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25. $\int_0^2 x\sqrt{4-x^2}\,dx =$

(A) 1 (B) $\frac{4}{3}$ (C) $\frac{8}{3}$ (D) 2 (E) $\frac{16}{3}$

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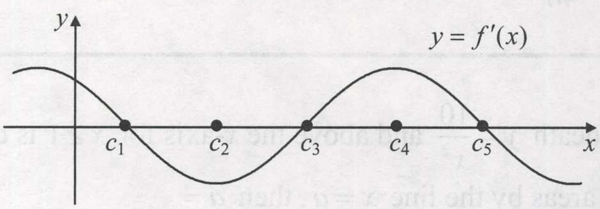
26. The integral expression $\int_1^2 \sqrt{1+\frac{4}{x^2}}\,dx$ could represent the arc length from $x=1$ to $x=2$ for the function $f(x)=$

(A) $-\frac{4}{x}$ (B) $\frac{2}{x}$ (C) $\ln\left(\frac{2}{x}\right)$ (D) $\ln\left(x^2\right)$ (E) $\ln\left(x^4\right)$



27. The graph of $f'(x)$, the derivative of *f*, is shown above. $f(x)$ would increase most rapidly at which of the following domain values?

(A) c_1 (B) c_2 (C) c_3 (D) c_4 (E) c_5

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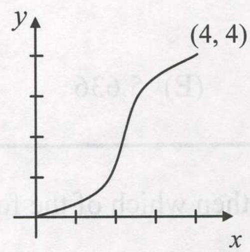
28. What is the slope $\frac{dy}{dx}$ of the polar curve $r=\frac{3}{\theta}$ at $\theta=\frac{\pi}{2}$?

(A) $\frac{-4}{\pi}$ (B) $\frac{-2}{\pi}$ (C) 0 (D) $\frac{2}{\pi}$ (E) $\frac{\pi}{2}$

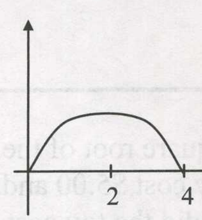
Calculus BC
Section I, Part B
Time — 50 minutes
Number of questions — 17

A graphing calculator is required for some questions.



29. For the graph of $y = f(x)$ defined on $0 \le x \le 4$, as shown above, a graph of $F(x) = \int_x^0 f(t)\,dt$ is best represented by:

(A)



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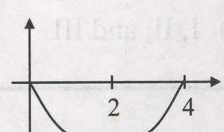

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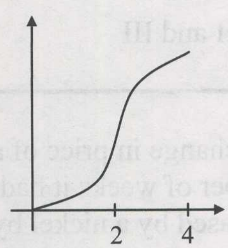

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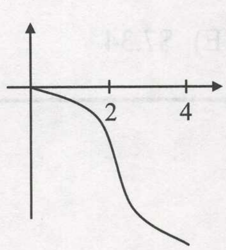
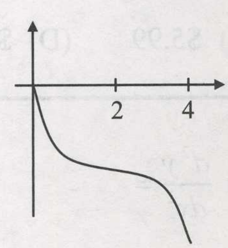


(E)



30. If $f(x)=\int_0^x \cos\left(t^2\right)dt$, then a linear approximation for $f(2)$ using the y value on the tangent line to $f(x)$ at $x=\sqrt{\pi}$ is

(A) −0.891 (B) −0.228 (C) 0.435 (D) 0.663 (E) 1.805

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31. If the region bounded by $y=\sin^{-1}x$, $y=\frac{\pi}{2}$, and $x=0$ is rotated about the y-axis, the volume of the solid formed is

(A) 2.467 (B) 2.605 (C) 2.694 (D) 4.609 (E) 5.636

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32. If $f(x)$ is differentiable over the positive real numbers, then which of the following statements must be true?

I. $f(x)$ is continuous over the positive real numbers
II. $f(-x)$ is differentiable over the negative real numbers
III. $f(|x|)$ is differentiable over all real numbers

(A) I only (B) II only (C) I and II
(D) II and III (E) I, II, and III

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33. The change in price of a popular toy was proportional to the square root of the number of weeks it had been on the market. The toy originally cost \$5.00 and increased by a nickel by the end of the first week. How much did the toy cost (to the nearest penny) at the end of the first 12 weeks?

(A) \$5.60 (B) \$5.69 (C) \$5.99 (D) \$7.08 (E) \$7.34

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34. If $x=[f(t)]^2$ and $y=f(t)$, then $\frac{d^2y}{dx^2}=$

(A) $\frac{1}{2f(t)}$ (B) $\frac{f'(t)}{2f(t)}$ (C) $-\frac{1}{4f^2(t)}$

(D) $-\frac{f'(t)}{2f^2(t)}$ (E) $-\frac{1}{4f^3(t)}$

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35. An equation of a tangent line to the parametric curve $x = e^t$, $y = 2^t$ at $t = 0$ is:

(A) $y-1=\frac{1}{2}(x-1)$ (B) $y-1=(\ln 2)(x-1)$ (C) $y=(\ln 2)x$

(D) $y=\frac{1}{\ln 2}(x-1)$ (E) $y-1=\frac{e}{2}(x-1)$

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36. Recall that the diagonal of a cube is $\sqrt{3}$ times its side. The diagonal of a cube is expanding at the rate of 0.5 cm per second. How fast is the volume of the cube changing, in cm^3/sec, when the diagonal is 3 cm?

(A) 1.5 (B) 2.598 (C) 5.196 (D) 9 (E) 10.392

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37. If a particle moves on the curve $x = \sin t$, $y = \sin 2t$, then at time $t = 3$ the speed of the particle is

(A) 0.930 (B) 0.965 (C) 1.379 (D) 1.645 (E) 2.161

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38. The base of a solid is a region bounded by $y = \ln x$, $y = 0$, and $x = e$. Cross sections perpendicular to the base and the y-axis are squares. An integral expression for the volume of this solid is

(A) $\int_0^1 (e-e^y)^2\,dy$ (B) $\int_0^e (e-e^y)^2\,dy$ (C) $\int_1^e (e-\ln y)^2\,dy$

(D) $\int_1^e (\ln x)^2\,dx$ (E) $\int_1^e (e-\ln x)^2\,dx$

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39. The minimum distance from point (5, 6) to the curve $y = x^2 + 1$ is

(A) 2.358 (B) 2.501 (C) 2.701 (D) 2.913 (E) 3.015

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40. If $x+y=\tan^{-1}(xy)$, then $\frac{dy}{dx}=$

(A) $\frac{1+x^2y^2}{x}$ (B) $\frac{y}{1+xy-x}$ (C) $\frac{x-1-x^2y^2}{1+x^2y^2}$

(D) $\frac{1+x^2+y^2}{x-x^2y^2-1}$ (E) $\frac{y-1-x^2y^2}{1+x^2y^2-x}$

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41. $\sum_{k=1}^{2n}\sqrt{\frac{1+\frac{k}{n}}{n^2}}$ is a Riemann sum for which of the following integrals?

(A) $\int_0^1 \sqrt{1+x}\,dx$ (B) $\int_0^2 \sqrt{1+x}\,dx$ (C) $\frac{1}{2}\int_0^1 \sqrt{1+x}\,dx$

(D) $\int_0^1 \sqrt{1+2x}\,dx$ (E) $\int_0^2 \sqrt{1+2x}\,dx$

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42. Particle A's velocity function is $v(t)=4-t^2$ m/sec over $0\le t\le 4$ seconds. For the same time interval, particle B's velocity function is $v(t)=t^3-4t$ m/sec. The difference in the total meters traveled by the two particles over $0\le t\le 4$ is

(A) 24 (B) $26\frac{2}{3}$ (C) $34\frac{2}{3}$ (D) $37\frac{1}{3}$ (E) $45\frac{1}{3}$

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43. $\sum_{n=3}^{\infty}\frac{e^{\frac{n}{2}}}{\pi^n}$ is

(A) 0.304 (B) 0.525 (C) 4.808 (D) 9.624 (E) divergent

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44. A line from the point (4, 1) perpendicular to a tangent line to the graph of $f(x)=x^2$ intersects the graph of $y=f(x)$ at $x=$

(A) 1.392 (B) 1.647 (C) 1.939 (D) 4.472 (E) 7.873
