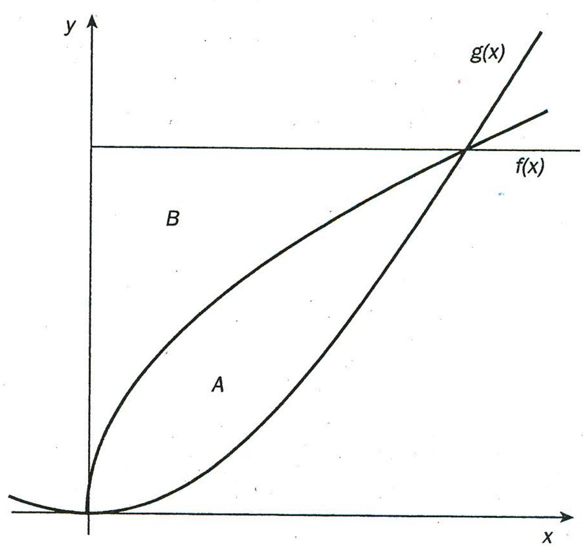
PART A

Time: 30 Minutes
2 Problems

A GRAPHING CALCULATOR IS ALLOWED FOR THIS PORTION OF THE EXAM

1. The rate of change of charge passing into a battery is modeled by the function $C(t) = 10 + 6\sin\left(\frac{t^2}{3}\right)$, for $t \geq 0$. $C(t)$ has units in coulombs per hour and t has units in hours. At $t = 0$ the battery is empty of any charge.
   (a) Is the amount of charge in the battery increasing or decreasing at $t = 4$? Give a reason for your answer.
   (b) Is the rate of change of charge in the battery increasing or decreasing at $t = 4$? Give a reason for your answer.
   (c) What is the average rate of change of the charge between $t = 2$ and $t = 10$?
   (d) The battery can hold a charge of 160 coulombs. How long, in hours, does it take to charge the battery?

2. Let f and g be the functions given by the equations $f(x) = \sqrt{x}$ and $g(x) = 1 - \cos\left(\frac{\pi x}{2}\right)$. Let A be the region in the first quadrant enclosed by the graphs of f and g, and let B be the region in the first quadrant enclosed by the graph of f, the y-axis, and the line $y = 1$, as shown in the figure below.
   (a) Find the area of A.
   (b) Find the volume of the solid generated when A is rotated about the x-axis.
   (c) Find the volume of the solid generated when B is rotated about the y-axis.



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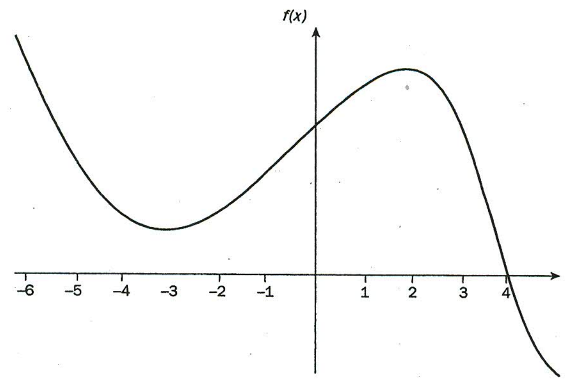
PART B

Time: 60 Minutes
4 Problems

NO CALCULATOR IS ALLOWED ON THIS PORTION OF THE EXAM

Note: If you have extra time, you can go back and work on Part A of Section II, but you cannot use a calculator to complete your work at this time.

3. The graph of the function $f'(x)$ appears below, where $f'(x)$ is the derivative of some function $f(x)$. The domain of $f'(x)$ here is the set of real numbers x satisfying the inequalities $-6 < x < 6$.



(a) Determine all points at which f has a relative extremum (if any exist) and state whether each is a maximum or minimum. Justify your answers.

(b) Determine all points of inflection for f. Justify your answers.

(c) Determine whether the function f is concave upward or concave downward on the interval $(-6, -3)$.

4. Let C be the curve defined by the equation $xy^2 + y^3 = 5$.

(a) Find $\frac{dy}{dx}$ as a function of x and y.

(b) Determine all points (if any exist) where a horizontal tangent occurs.

(c) Determine the point on C where the tangent line is vertical.

5. Let R be the region enclosed by the graphs of $f(x) = ax(2 - x)$ and $g(x) = ax$ for some positive real number a.

(a) Find the area of the region R.

(b) Find the volume of the solid of revolution generated when R is rotated about the x-axis.

(c) Assume a solid exists with a cross-section area of R and uniform thickness π. Find the value of a for which this solid has the same volume as the solid in (b).

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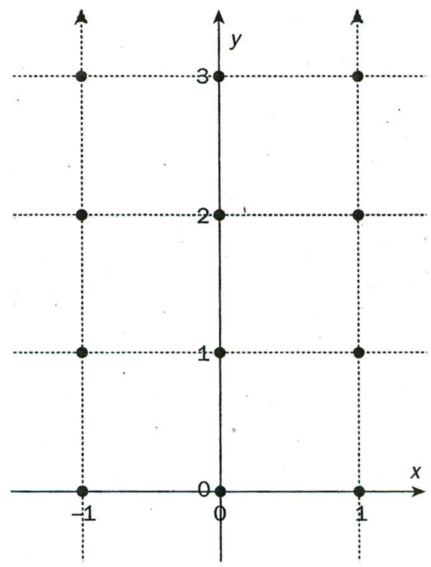
6. Consider the differential equation $\frac{dy}{dx} = x^3(y-1)$. Let $f(x)$ be a solution to the differential equation satisfying $f(2) = 2$.



(a) On the axes provided, sketch a slope field for the given differential equation at the 12 points indicated.

(b) Write an equation for the tangent line to the graph of $f(x)$ at $x = 2$.

(c) Find the solution $y = f(x)$ to the given differential equation with the initial condition $f(2) = 2$.

PRACTICE TEST 1: CALCULUS AB

SECTION I, PART A

Time: 55 Minutes
28 Questions

Note: Part A and Part B of Section I combined count for 50 percent of your exam grade.

No Calculator Allowed

> **Directions:** Solve the following problems, using available space for scratchwork. After examining the form of the choices, decide which one is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

Note: For the actual test, no scrap paper is provided.

In this test:

(1) The domain of a function f is the set of all real numbers x for which $f(x)$ is a real number, unless otherwise specified.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. Given $\lim\limits_{x\to 0} \frac{1-\cos x}{x} = 0$, then $\lim\limits_{x\to 0}\left(\frac{3x^2+5\cos x-5}{2x}\right) =$

(A) 0
(B) $\frac{5}{2}$
(C) 3
(D) 5
(E) Does not exist

2. $f(x) = \frac{3x^2-6x-9}{x^2-x-2}$ will have vertical asymptotes at

(A) $x = 2$
(B) $x = -1$ and 2
(C) $y = 3$
(D) $x = 3$
(E) There are no vertical asymptotes.

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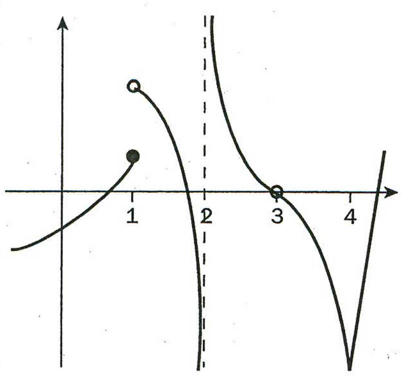
3. Which of the following functions grows the **fastest?**

(A) $a(u) = \left(\frac{1}{2}\right)^u$
(B) $b(u) = u^{100} + u^{99}$
(C) $c(u) = 4^u$
(D) $d(u) = 200e^u$
(E) $e(u) = 3^u + u^3$

4. Which of the following is true about the graph below?



(A) An infinite discontinuity appears to occur at $x = 4$.
(B) The function does not appear to be continuous at $x = 4$.
(C) A jump discontinuity appears to occur at $x = 3$.
(D) A removable discontinuity appears to occur at $x = 2$.
(E) A jump discontinuity appears to occur at $x = 1$.

5. The cost of producing x units of a certain item is $c(x) = 2000 + 8.6x + 0.5x^2$. What is the instantaneous rate of change of c with respect to x when $x = 300$?

(A) 313.6
(B) 308.6
(C) 300.0
(D) 297.2
(E) 200.0

6. A tank holds 10,000 liters of gasoline. At the bottom of the tank, a lever can be turned to allow the gasoline to be dispensed. The tank can be emptied in exactly 40 minutes. Below is a table which gives the volume v of gasoline (in liters) which remain in the tank after t minutes of draining have taken place.

t(minutes)	0	5	10	15	20	25	30	35	40
v(liters)	4700	4100	3200	2400	2000	1400	800	500	0

During which of the following 10-minute intervals is the average rate of gasoline draining from the tank the least?

(A) $t = 0$ to $t = 10$ minutes
(B) $t = 10$ to $t = 20$ minutes
(C) $t = 15$ to $t = 25$ minutes
(D) $t = 25$ to $t = 35$ minutes
(E) $t = 30$ to $t = 40$ minutes

7. Which of the following gives the derivative of the function $f(x) = x^2$ at the point (2, 4)?

(A) $\lim\limits_{h\to 0} \frac{(x+2)^2 - x^2}{4}$
(B) $\lim\limits_{h\to \infty} \frac{(2+h)^2 - 2^2}{h}$
(C) $\frac{(2+h)^2 - 2^2}{h}$
(D) $\lim\limits_{h\to 0} \frac{(2+h)^2 - 2^2}{h}$
(E) $\lim\limits_{h\to 0} \frac{(4+h)^2 - 4^2}{h}$

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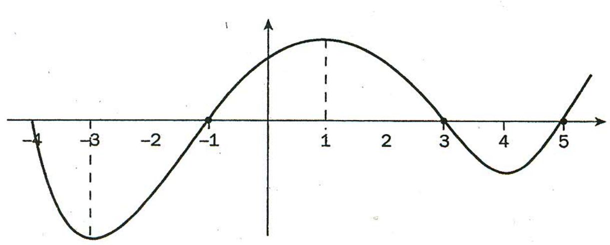
8. Let $f(x) = 5x\sec x + x^3\cos x + 17\pi$.
Determine $\frac{d}{dx}f(x)$.
(A) $5\sec x\tan x + 3x^2\cos x + 17\pi$
(B) $5\sec^2 x - x^3\sin x$
(C) $5\sec x\tan x - 3x^2\sin x$
(D) $5\sec x + 5x\sec x\tan x + 3x^2\cos x - x^3\sin x$
(E) $5\sec x + 5x\sec x\tan x - 3x^2\cos x + x^3\sin x + 17\pi$

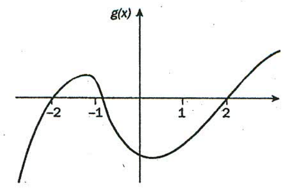
9. Find the derivative of $g(x) = 5\sin^2(6x) + 5\cos^2(6x)$ with respect to x.
(A) $30\cos^2(6x) - 30\sin^2(6x)$
(B) $5\cos^2(6x) - 5\sin^2(6x)$
(C) $120\sin(6x)\cos(6x)$
(D) 30
(E) 0

10. The graph of the function $y = h(x)$ appears below.

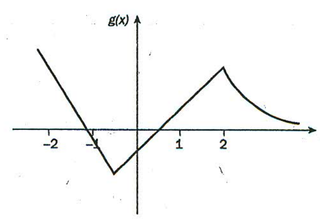


Determine the zeros of the derivative function h′(x).
(A) $x = -4, -1, 3, 5$
(B) $x = -2, -1, 2.5$
(C) $x = 1, 4$
(D) $x = -3, 1, 4$
(E) $x = -2, 2.5$

11. Below is the graph of the function $y = g(x)$.



Which graph below is the graph of $y = g'(x)$?

(A)


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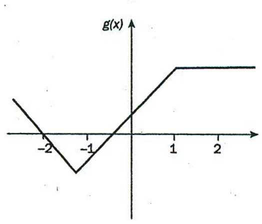

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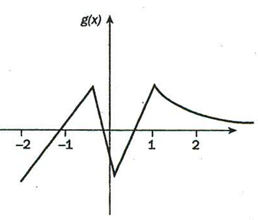

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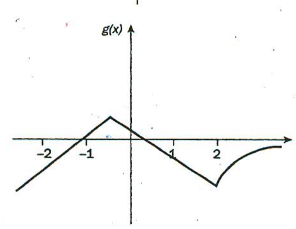
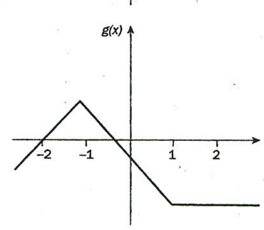


(E)


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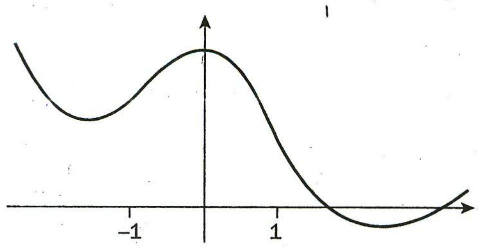
12. The graph of $y=f(x)$ is given below.



Which of the following is true?

(A) The graph is concave down (for all values of x).

(B) The graph is concave up (for all values of x).

(C) The graph is concave up for $x>1$ and concave down for $-1<x<1$.

(D) The graph is concave up for $-1<x<1$ and concave down for $x<-1$.

(E) Nothing can be said about the concavity of the graph above without knowing the rule for the function.

13. A curve is generated by the equation $x^2+4y^2=16$. Determine the number of points on this curve whose corresponding tangent lines are horizontal.

(A) 0

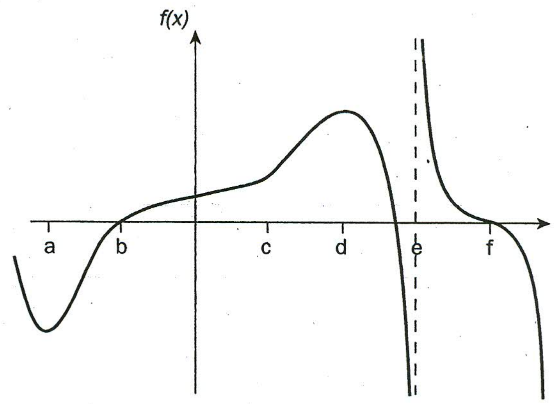
(B) 1

(C) 2

(D) 3

(E) 4

14. Determine all the points on the graph below where the first derivative of the function is 0.



(A) a, b, e

(B) b, c, f

(C) a, d, e

(D) a, b, d, e

(E) a, d

15. A 13-foot ladder is leaning against a 20-foot vertical wall when it begins to slide down the wall. During this sliding process, the bottom of the ladder is sliding away from the bottom of the wall at a rate of $\frac{1}{2}$ foot per second. Determine the rate at which the top of the ladder is sliding down the vertical wall when the tip of the ladder is exactly 5 feet above the ground.

(A) $-\frac{6}{5}$ feet per second

(B) $\frac{5}{6}$ feet per second

(C) $-\frac{12}{13}$ feet per second

(D) -2 feet per second

(E) Not enough information is given to solve this problem.

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16. Find the solution to the differential equation $\frac{dy}{dx} = \frac{\sin x}{e^y}$, where $y\left(\frac{\pi}{4}\right) = 0$.

(A) $y = \ln\left(\cos x - \frac{\sqrt{2}}{2}\right)$

(B) $y = \ln\left(-\cos x + \frac{\sqrt{2}}{2} + 1\right)$

(C) $y = -\frac{\cos x}{e^y}$

(D) $y = \ln(-\cos x)$

(E) $y = \ln(-\cos x) + \frac{\sqrt{2}}{2}$

17. $\int_0^6 (2 - |4 - x|)dx =$

(A) 24

(B) 12

(C) 6

(D) 2

(E) −4

18. $\int (\sin(2x) + 5\tan^2 x \csc^2 x)dx =$

(A) $-\frac{1}{2}\cos(2x) + 5\tan x + C$

(B) $\cos(2x) - 5\cot x + C$

(C) $-\cos(2x) + 5\sec x \tan x + C$

(D) $-\frac{1}{2}\cos(2x) - 5\cot x + C$

(E) $\frac{1}{2}\sin(2x) + 5\tan x + C$

19. $\int \frac{(7 + x^{\frac{2}{3}})^5}{x^{\frac{1}{3}}} dx =$

(A) $\frac{1}{6}(7 + x^{\frac{2}{3}})^6 + C$

(B) $\frac{1}{6}(7x^{\frac{2}{3}} + x^{\frac{5}{3}})^6 + C$

(C) $\frac{1}{4}(7 + x^{\frac{2}{3}})^6 + C$

(D) $\frac{1}{4}(7 + x^{\frac{1}{3}})^6 + C$

(E) $\frac{1}{6}(7x^{-\frac{1}{3}} + x^{\frac{2}{3}})^6 + C$

20. $\int \sin^4 x \cos x \, dx =$

(A) $\frac{1}{10}\sin^5 x \cos^2 x + C$

(B) $\frac{1}{5}\sin^5 x + C$

(C) $\frac{1}{5}\sin^5 x \cos^2 x + C$

(D) $4\sin^3 x \cos^2 x - \sin^5 x + C$

(E) $\frac{1}{5}\sin^5 x \cos x + \sin^5 x + C$

21. $\frac{d}{dt}\int_2^{t^4} e^{x^2} dx =$

(A) $e^{t^8} - e^4$

(B) $4t^3 e^{t^8} - e^4$

(C) e^{t^8}

(D) $4t^3 e^{t^8}$

(E) Cannot be determined because $\int e^{x^2}$ cannot be determined

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22. Let $g(x) = (\arccos(x^2))^5$. Then $g'(x) =$

(A) $-10\frac{(\arccos(x^2))^4}{\sqrt{1-x^2}}$

(B) $-10\frac{x(\arccos(x^2))^4}{\sqrt{1-x^4}}$

(C) $-10\frac{x(\arcsin(x^2))^4}{\sqrt{1-x^2}}$

(D) $10\frac{x(\arccos(x^2))^4}{\sqrt{1-x^2}}$

(E) $10\frac{(\arccos(x^2))^4}{\sqrt{1-x^4}}$

23. $\frac{d}{dx}(\ln(3x)5^{2x}) =$

(A) $\frac{5^{2x}}{x} + 2\ln(5)\ln(3x)5^{2x}$

(B) $\frac{5^{2x}}{3x} - 2x\ln(3x)5^{2x}$

(C) $\frac{5^{2x}}{x} - \ln(5)\ln(3x)5^{2x}$

(D) $\frac{5^{2x}}{3x} + 2\ln(3x)5^{2x}$

(E) $\frac{5^{2x}}{x} + \ln(5)\ln(3x)5^{2x}$

24. Let $u(t) = e^{3t^2} - e^{-\frac{1}{t}}$. Then $u'(t) =$

(A) $e^{t^3} + \frac{e^{-\frac{1}{t}}}{t^2}$

(B) $6te^{3t^2} + e^{-\frac{1}{t}}$

(C) $e^{3t^2} - e^{-\frac{1}{t}}$

(D) $6te^{3t^2} - \frac{e^{-\frac{1}{t}}}{t^2}$

(E) $e^{t^3} + e^{-\frac{1}{t}}$

25. $\int \frac{e^{3x}}{e^{6x}+1}dx =$

(A) $e^{3x^2} + x + C$

(B) $\ln(e^{6x}+1) + C$

(C) $-\frac{1}{3}e^{-3x} - \frac{1}{6}e^{-6x} + C$

(D) $\frac{1}{6}\arctan(e^{6x}) + C$

(E) $\frac{1}{3}\arctan(e^{3x}) + C$

26. Find the volume of the solid of revolution determined by rotating the area bounded by the graphs of $f(x) = x^2$ and $g(x) = 3x$ about the x-axis.

(A) $\frac{162\pi}{5}$

(B) $\frac{27\pi}{2}$

(C) 9π

(D) 28π

(E) $\frac{240\pi}{7}$

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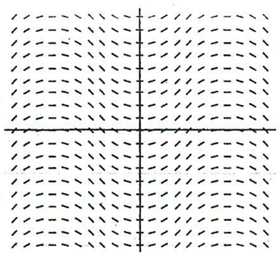
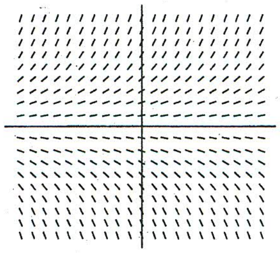
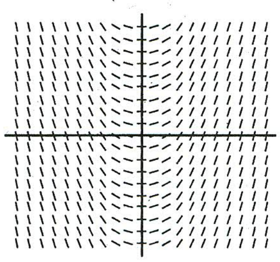
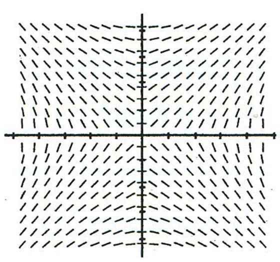
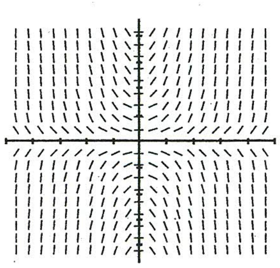
27. Which of the following slope fields describes the differential equation $\frac{dy}{dx} = \frac{x}{y}$?

(A) 

(B) 

(C) 

(D) 

(E) 

28. $\lim\limits_{x \to 5} \frac{x^2 + 2x - 35}{x^2 - 25} =$

(A) 0

(B) $\frac{6}{5}$

(C) 5

(D) 7

(E) The limit does not exist.

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT TURN TO ANY OTHER SECTION IN THE TEST.

STOP

SECTION I, PART B

Time: 50 Minutes
17 Questions

A GRAPHING CALCULATOR MAY BE USED FOR THIS SECTION

Directions: Solve the following problems, using available space for scratchwork. After examining the form of the choices, decide which one is the best of the choices given and fill in the corresponding oval on the answer sheet. No credit will be given for anything written in the test book. Do not spend too much time on any one problem.

Note: For the actual test, no scrap paper is provided.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the numerical value.

(2) The domain of a function f is the set of all real numbers x for which $f(x)$ is a real number, unless otherwise specified.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

29. Let $u(t) = \frac{t^2+4t-21}{t^2-9}$. Assume also that $u(t)$ is continuous for all positive real numbers. Determine $u(t)$.

(A) $\frac{5}{3}$
(B) 3
(C) $-\frac{10}{3}$
(D) 0
(E) This is not possible. The function cannot be continuous at $t = 3$.

30. The cost of producing x units of a certain item is $c(x) = 2{,}000 + 8.6x + 0.5x^2$. What is the average rate of change of c with respect to x when the level of production increases from $x = 300$ to $x = 310$ units?

(A) 313.6
(B) 310
(C) 214.2
(D) 200
(E) 10

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31. Find the values of x for which the function $f(x) = x^6 + 3x^5 - \frac{15}{2}x^4 - 40x^3 - 60x^2 + 8x + 5$ has inflection points. Hint: $x^4 + 2x^3 - 3x^2 - 8x - 4 = (x^2 - 4)(x^2 + 2x + 1)$.

(A) f has no inflection points
(B) $x = -2, 2$
(C) $x = -1, 0, 1$
(D) $x = -2, -1, 2$
(E) $x = 0$

32. Let $a(x) = \sin(\sin x)$. Find, accurate to three decimal places, $a'\left(\frac{\pi}{4}\right) =$

(A) 0.538
(B) −0.999
(C) 0.009
(D) 1.000
(E) 0.866

33. On the interval $(0, 2\pi)$, the function $f(x) = \sin x \cos x$ has critical points at:

(A) $x = \frac{\pi}{4}, \frac{5\pi}{4}$
(B) $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$
(C) $x = \frac{3\pi}{4}, \frac{7\pi}{4}$
(D) $x = \frac{\pi}{2}, \pi, \frac{3\pi}{2}$
(E) There are no critical points for this function on the given interval.

34. Determine the slope of the normal line to the curve $x^3 + xy^2 = 10y$ at the point (2, 1).

(A) 0
(B) 2
(C) $-\frac{7}{3}$
(D) $-\frac{6}{13}$
(E) $\frac{1}{2}$

35. A spherical balloon is being inflated at a rate of 3 cubic inches per second. Determine the rate of change of the radius of the balloon when the balloon's radius is 5 inches, accurate to three decimal places. The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

(A) 3.000 inches per second
(B) 1.667 inches per second
(C) 0.010 inches per second
(D) −2.000 inches per second
(E) 0.120 inches per second

36. A gun is fired vertically upward from a position 100 feet above ground at an initial velocity of 400 feet per second. Determine the maximum height of the projectile. The acceleration of gravity is -32ft/sec^2.

(A) 3,000 feet
(B) 2,600 feet
(C) 2,200 feet
(D) 1,800 feet
(E) 1,400 feet

37. The table below provides data points for the continuous function $y = h(x)$.

x	0	2	4	6	8	10
$h(x)$	9	25	30	16	25	32

Use a right Riemann sum with 5 subdivisions to approximate the area under the curve of $y = h(x)$ on the interval [0, 10].

(A) 256
(B) 235
(C) 210
(D) 206
(E) 242

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38. Using the trapezoid rule, approximate the area under the curve $f(x) = x^2 + 4$ between $x = 0$ and $x = 3$ using $n = 6$ subintervals.

(A) 18.875
(B) 20.938
(C) 21
(D) 21.125
(E) 23.375

39. The area bounded by the curves $y = x^2 + 4$ and $y = -2x + 1$ between $x = -2$ and $x = 5$ equals

(A) 86.500
(B) 86.425
(C) 86.333
(D) 86.125
(E) 86.000

40. The best approximation given below for the area of the region bounded by the graphs of $f(x) = 3x^3 - 12x^2 + 9x$ and $g(x) = -x^3 + 4x^2 - 3x$ is

(A) 12.000
(B) 12.333
(C) 12.400
(D) 12.525
(E) 12.666

41. Let R be the region in the first quadrant bounded by the graph of $y = 4 - x^2$, the x-axis, and the y-axis. Determine the value of k such that the vertical line $x = k$ cuts the region R into two smaller regions whose areas are equal.

(A) 0.520
(B) 0.695
(C) 0.875
(D) 1.000
(E) 1.135

42. The acceleration of an object at any time $t \geq 0$ is given by $a(t) = t^2 + \sqrt{t+9} + e^{-t}$. The velocity of the object at time $t = 0$ is 4 and the position of the object at time $t = 0$ is 5. Then the position of the object at time $t = 7$ is approximately equal to

(A) −14.714
(B) 15.507
(C) 103.668
(D) 246.752
(E) 321.351

43. The average value of the function $f(x) = \frac{2x}{x^2-4}$ on the interval $[5, 8]$ is approximately

(A) 0.350
(B) 1.050
(C) 0.743
(D) 0.248
(E) 0.201

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44. Which of the following is a solution of $\frac{dy}{dx} = \frac{5}{y(x+4)}$ given that $y(e-4) = 0$?

(A) $y = (t+4)^2 - e^2$

(B) $y^2 = 5\ln|x+4| - 5$

(C) $y = 2\sqrt{t+4} - 2\sqrt{e}$

(D) $y^2 = 10\ln|x+4| - 10$

(E) No solution exists.

45. Which of the following is true of the function $f(x) = \sqrt{x^2+1}$?

(A) $\lim_{x\to\infty}(f(x)-x) = 0$ and $\lim_{x\to-\infty}(f(x)-x) = 0$

(B) $\lim_{x\to\infty}(f(x)+x) = 0$ and $\lim_{x\to-\infty}(f(x)-x) = 0$

(C) $\lim_{x\to\infty}(f(x)-x) = 0$ and $\lim_{x\to-\infty}(f(x)+x) = 0$

(D) $\lim_{x\to\infty}(f(x)+x) = 0$ and $\lim_{x\to-\infty}(f(x)+x) = 0$

(E) None of the above

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON THIS SECTION ONLY. DO NOT TURN TO ANY OTHER SECTION IN THE TEST.

STOP