1. B
2. C
3. E
4. A
5. E
6. C
7. A
8. B
9. D
10. C

2

BC Review 02, Use your calculator ONLY on #11.

1. If $y = xy + x^2 + 1$, then when $x = -1$, $\frac{dy}{dx}$ is

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) -1 (D) -2 (E) nonexistent

2. $\int_1^{\infty} \frac{x}{\left(1+x^2\right)^2}$ is (A) ~~$-\frac{1}{2}$~~ (B) ~~$-\frac{1}{4}$~~ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) divergent

$\int_1^{\infty} x(1+x^2)^{-2}\,dx$

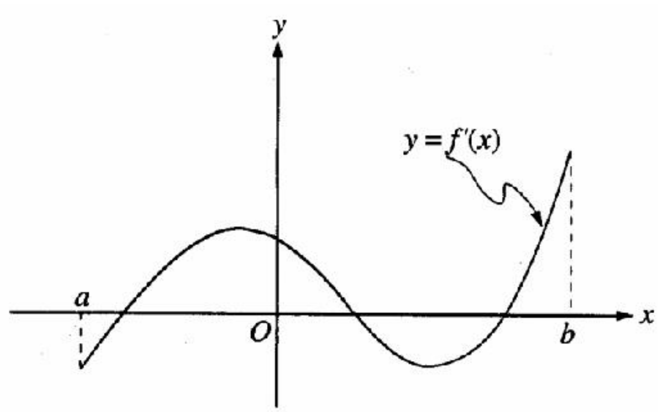
$\lim_{b\to\infty} \frac{1}{2}(-1)(1+x^2)^{-1}\Big|_1^b$

$-\frac{1}{2}\left(\frac{1}{1+x^2}\right)\Big|_1^{\infty} = -\frac{1}{2}\left[0-\frac{1}{2}\right] = \frac{1}{4}$

DNE

3. Let f be the function defined by $f(x) = \begin{cases} x^3 & \text{for } x \le 0 \\ x & \text{for } x > 0 \end{cases}$. Which of the following statements about f is true?

(A) ~~f is an odd function~~ (B) ~~f is discontinuous at $x = 0$~~ (C) f has a relative maximum (D) $f'(0) = 0$ (E) $f'(x) > 0$ for $\boxed{x \neq 0}$



4. The graph of f', the derivative of f, is shown in the figure above. Which of the following describes all relative extrema of f on the open interval (a,b)?

(A) One relative maximum and two relative minima
(B) Two relative maxima and one relative minimum
(C) Three relative maxima and one relative minimum
(D) One relative maximum and three relative minima
(E) Three relative maxima and two relative minima

5. An antiderivative for $\dfrac{1}{x^2-2x+2}$ is

(A) $\cancel{-\left(x^2-2x+2\right)^{-2}}$ (B) $\cancel{\ln\left(x^2-2x+2\right)}$ (C) $\cancel{\ln\left|\dfrac{x-2}{x+1}\right|}$ (D) $\text{Arcsec}\left(x-1\right)$ (E) $\text{Arctan}\left(x-1\right)$

$$\int \frac{1}{(x^2-2x+1)-1+2}\,dx$$

$$\int \frac{1}{(x-1)^2+1}\,dx$$

$u=x-1$ $a=1$

$$1\cdot\frac{1}{1}\arctan\frac{(x-1)}{1}+C$$

$$\frac{1}{\sqrt{a^2-u}} \qquad \frac{1}{u\sqrt{u^2-a^2}}$$

6. The region enclosed by the x-axis, the line $x=3$, and the curve $y=\sqrt{x}$ is rotated about the x-axis. What is the volume of the solid generated?

(A) 3π (B) $3\sqrt{3}\pi$ (C) $\dfrac{9}{2}\pi$ (D) 9π (E) $\dfrac{36\sqrt{3}}{5}\pi$

7. $\displaystyle\int_0^{\sqrt{3}} \frac{1\,dx}{\sqrt{4-x^2}} = dx$

$a=2$ $u=x$

(A) $\dfrac{\pi}{3}$ (B) $\dfrac{\pi}{4}$ (C) $\dfrac{\pi}{6}$ (D) $\dfrac{1}{2}\ln 2$ (E) $-\ln 2$

$\arcsin \frac{x}{2}\Big|_0^{\sqrt{3}}$

$\boxed{\arcsin\frac{\sqrt{3}}{2} - \arcsin 0}$

$\boxed{\frac{\pi}{3}}$

8. If $\frac{dy}{dx} = 2y^2$ and if $y = -1$ when $x = 1$, then when $x = 2$, $y =$

$\int y^{-2}\,dy = \int 2\,dx$ ✓

(A) $-\frac{2}{3}$ (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) $\frac{2}{3}$

$-y^{-1} = 2x + C$

$y^{-1} = -2x + C$

$\frac{1}{y} = -2x + C$

$\boxed{y = \frac{1}{C - 2x}}$ gen

$\boxed{y = \frac{1}{1-2x}}$

$2y^2$

At (1,-1):

$-\frac{1}{1} = \frac{1}{C-2}$

$y(2) = \frac{1}{1-4} = -\frac{1}{3}$

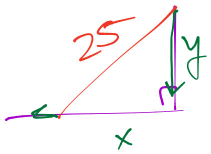
$C - 2 = -1$

$\boxed{C = 1}$

9. The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change, in feet per minute, of the distance between the bottom of the ladder and the wall? $\frac{dx}{dt} = ?$

RR

(A) ~~$-\frac{7}{8}$~~ (B) ~~$-\frac{7}{24}$~~ (C) $\frac{7}{24}$ (D) $\frac{7}{8}$ (E) ~~$\frac{21}{25}$~~



$\frac{dy}{dt} = -3$

$\boxed{y = 7 \quad x = 24}$

$\boxed{x^2 + y^2 = 25^2}$ "pos"

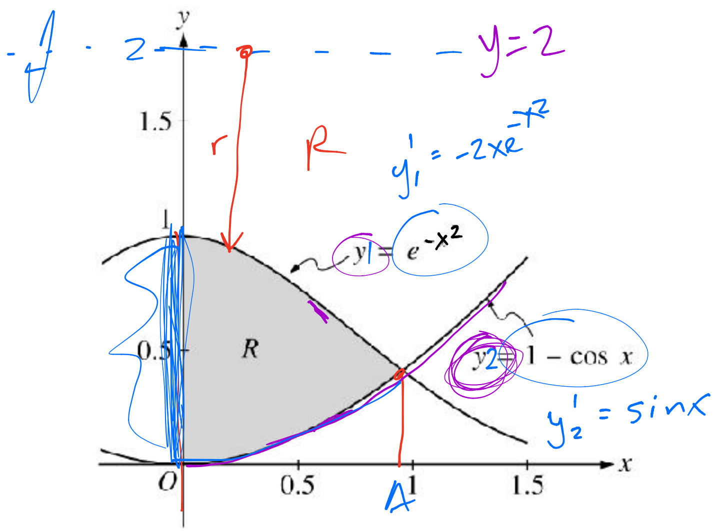
$\frac{d}{dt}$: $\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt} = 0$ rate eq.

When $y = 7$: $(24)\left(\frac{dx}{dt}\right) + (7)(-3) = 0$

$\frac{dx}{dt} = \frac{21}{24} = \frac{7}{8}$

10. At what value of x does the graph of $y = \frac{1}{x^2} - \frac{1}{x^3}$ have a point of inflection?

(A) 0 (B) 1 (C) 2 (D) 3 (E) At no value of x



11. (Calculator Permitted) (2000, AB-1) Let R be the region in the first quadrant enclosed by the graphs of $y_1 = e^{-x^2}$, $y_2 = 1 - \cos x$, and the y-axis, as shown in the figure above.

(a) Find the volume of the solid generated when the region R is revolved about the line $y = 2$.

$y_1 = y_2$

$x = .941 = A$

$$\text{Vol} = \pi \int_0^A \left[(2 - y_2)^2 - (2 - y_1)^2\right] dx$$

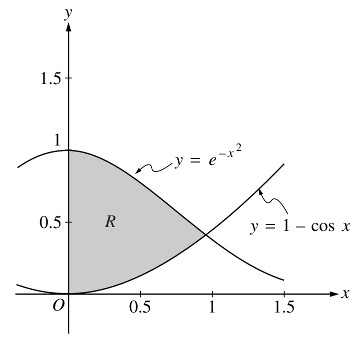
$$= 5.679$$

(b) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a semicircle. Find the volume of this solid.

$$\text{Vol} = \boxed{\frac{\pi}{8}} \int_0^A (y_1 - y_2)^2 \, dx$$

(c) Write, but do not evaluate, an expression involving integrals that could be used to find the perimeter of the region R.

$$\text{Perim} = 1 + \int_0^A \sqrt{1 + (\sin x)^2}\, dx + \int_0^A \sqrt{1 + (-2xe^{-x^2})^2}\, dx$$



Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.

(a) Find the area of the region R.

(b) Find the volume of the solid generated when the region R is revolved about the x-axis.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.

Region R

$e^{-x^2} = 1 - \cos x$ at $x = 0.941944 = A$

1 : Correct limits in an integral in (a), (b), or (c).

(a) Area $= \displaystyle\int_0^A \left(e^{-x^2} - (1 - \cos x)\right) dx$

$= 0.590$ or 0.591

constant & interval

2 $\begin{cases} 1: \text{ integrand} \\ 1: \text{ answer} \end{cases}$

(b) Volume $= \pi \displaystyle\int_0^A \left(\left(e^{-x^2}\right)^2 - (1 - \cos x)^2\right) dx$

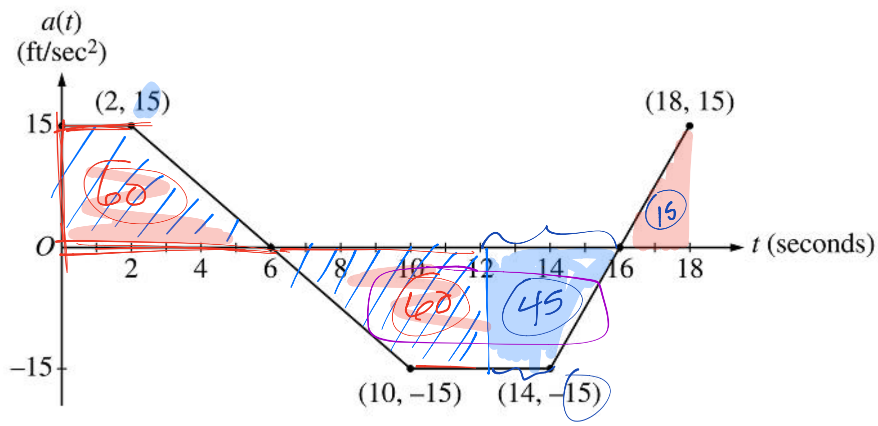
$= 0.55596\pi = 1.746$ or 1.747

3 $\begin{cases} 2: \text{ integrand and constant} \\ \quad < -1 > \text{ each error} \\ 1: \text{ answer} \end{cases}$

(c) Volume $= \displaystyle\int_0^A \left(e^{-x^2} - (1 - \cos x)\right)^2 dx$

$= 0.461$

3 $\begin{cases} 2: \text{ integrand} \\ \quad < -1 > \text{ each error} \\ \quad \text{Note: } 0/2 \text{ if not of the form} \\ \qquad k\displaystyle\int_c^d (f(x) - g(x))^2\, dx \\ 1: \text{ answer} \end{cases}$



12. (2001, AB-3) A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is a piecewise linear function defined by the graph above.

(a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

$V'(2) = a(2) = 15 > 0$
So velocity is increasing at $t = 2$ seconds.

(b) At what time in the interval $0 \le t \le 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

The car is going 55 ft/sec at $t = 12$ sec since
$* \int_0^{12} a(t)\,dt = 0$
$* \int_0^{6} a(t)\,dt = -\int_6^{12} a(t)\,dt$

(c) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.

$V(0) = 55$ ft/sec
$V(6) = 55 + 60 = 115$ ft/sec
$V(16) = 115 - 105 = 10$ ft/sec
$V(18) = 10 + 15 = 25$ ft/sec

$V' = a = 0$ c.v.

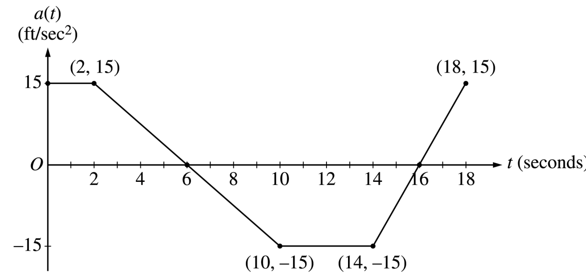
So max velocity is 115 ft/sec at $t = 6$ sec

(d) At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

The car is never zero ft/sec since, from part (c), the min velocity is 10 ft/sec.

Question 3

A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration $a(t)$, in ft/sec^2, is the piecewise linear function defined by the graph above.



(a) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?

(b) At what time in the interval $0 \le t \le 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

(c) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify your answer.

(d) At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify your answer.

(a) Since $v'(2) = a(2)$ and $a(2) = 15 > 0$, the velocity is increasing at $t = 2$.

1 : answer and reason

(b) At time $t = 12$ because

$$v(12) - v(0) = \int_0^{12} a(t)\,dt = 0.$$

$2 : \begin{cases} 1 : t = 12 \\ 1 : \text{reason} \end{cases}$

(c) The absolute maximum velocity is 115 ft/sec at $t = 6$.

The absolute maximum must occur at $t = 6$ or at an endpoint.

$$v(6) = 55 + \int_0^6 a(t)\,dt$$
$$= 55 + 2(15) + \frac{1}{2}(4)(15) = 115 > v(0)$$
$$\int_6^{18} a(t)\,dt < 0 \text{ so } v(18) < v(6)$$

$4 : \begin{cases} 1 : t = 6 \\ 1 : \text{absolute maximum velocity} \\ 1 : \text{identifies } t = 6 \text{ and } \\ \quad t = 18 \text{ as candidates} \\ \quad \text{or} \\ \quad \text{indicates that } v \text{ increases,} \\ \quad \text{decreases, then increases} \\ 1 : \text{eliminates } t = 18 \end{cases}$

(d) The car's velocity is never equal to 0. The absolute minimum occurs at $t = 16$ where

$$v(16) = 115 + \int_6^{16} a(t)\,dt = 115 - 105 = 10 > 0.$$

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{reason} \end{cases}$