Taylor Series & Polynomials MC Review

Select the correct capital letter. NO CALCULATOR unless specified otherwise.

A 1. Let $T_5(x) = 3x^2 - 5x^3 + 7x^4 + 3x^5$ be the fifth-degree Taylor polynomial for the function *f* about $x = 0$. What is the value of $f'''(\overset{-3}{\cancel{0}})$?

(A) -30 (B) -15 (C) -5 (D) $-\frac{5}{6}$ (E) $-\frac{1}{6}$

$\frac{(-1)^n}{n}$ $\frac{((-1)^3)^n}{n}$ $\frac{(-1)^{3n}}{n}$ $\frac{((-1)^2)^n}{n} = \frac{1}{n}$

D 2. For what integer k, $k > 1$, will both $\sum_{n=1}^{\infty} \frac{(-1)^{kn}}{n}$ and $\sum_{n=1}^{\infty} \left(\frac{k}{4}\right)^n$ converge? $k = \cancel{2}, 3$ 4

~~(A) 6~~ ~~(B) 5~~ ~~(C) 4~~ (D) 3 (E) 2

C 3. (Calculator Permitted) The Taylor series for $\ln x$, centered at $x = 1$, is $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x-1)^n}{n}$. $= (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}$ Let *f* be the function given by the sum of the first three nonzero terms of this series. The maximum value of $\underbrace{\left|\ln x - f(x)\right|}_{y1}$ for $0.3 \le x \le 1.7$ is which of the following? $= (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3}$

(A) 0.030 (B) 0.039 (C) 0.145 (D) 0.153 (E) 0.529

B 4. What are the values of x for which the series $\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt{n}}$ converges?

(A) $-3 < x < -1$ (B) $-3 \le x < -1$ (C) $-3 \le x \le -1$ (D) $-1 \le x < 1$ (E) $-1 \le x \le 1$

$$\frac{(-1)^n}{n^{1/2}}$$

A 5. (Calculator Permitted) The graph of the function represented by the Maclaurin series $1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots + \frac{(-1)^n x^n}{n!} + \cdots$ intersects the graph of $y = x^3$ at $x =$

(A) 0.773 (B) 0.865 (C) 0.929 (D) 1.000 (E) 1.857

$e^{-x} - x^3 =$

e^{-x}

D 6. Which of the following sequences converge?

I. $\left\{ \frac{5n}{2n-1} \right\}$ $\frac{5}{2}$ II. $\left\{ \cancel{\frac{e^n}{n}} \right\}$ III. $\left\{ \frac{e^n}{1+e^n} \right\}$ $=1$

(A) I only (B) II only (C) I and II only (D) I and III only (E) I, II, and III

E ____ 7. What is the approximation of the value of $\sin 1$ obtained by using the fifth-degree Taylor polynomial about $x=0$ for $\sin x$?

(A) $1-\frac{1}{2}+\frac{1}{24}$ (B) $1-\frac{1}{2}+\frac{1}{4}$ (C) $1-\frac{1}{3}+\frac{1}{5}$ (D) $1-\frac{1}{4}+\frac{1}{8}$ (E) $1-\frac{1}{6}+\frac{1}{120}$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$

$\sin 1 = 1 - \frac{1}{6} + \frac{1}{120}$

B

____ 8. Which of the following series converge?

I. $\sum_{n=1}^{\infty} \boxed{\frac{n}{n+2}}$ 1 nth term   II. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$ ✓   III. $\cancel{\sum_{n=1}^{\infty} \frac{1}{n}}$

(A) None (B) II only (C) III only (D) I and II only (E) I and III only

A

____ 9. If $\lim_{b\to\infty} \int_1^b \frac{dx}{x^p}$ is finite, then which of the following must be true?

$\int_1^\infty \frac{1}{x^p}\,dx$ converges

Integral test

(A) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges (B) $\sum_{n=1}^{\infty} \frac{1}{n^p}$ diverges (C) $\sum_{n=1}^{\infty} \frac{1}{n^{p-2}}$ converges

(D) $\sum_{n=1}^{\infty} \frac{1}{n^{p-1}}$ converges (E) $\sum_{n=1}^{\infty} \frac{1}{n^{p+1}}$ diverges

D 10. If $\sum_{n=0}^{\infty} a_n x^n$ is a Taylor series that converges to $f(x)$ for all real x, then $f'(1) =$

$f(x) =$

(A) 0 (B) a_1 (C) $\sum_{n=0}^{\infty} a_n$ (D) $\sum_{n=1}^{\infty} n a_n$ (E) $\sum_{n=1}^{\infty} n a_n^{n-1}$

$f'(x) = \sum a_n \cdot n \cdot x^{n-1}$

$f'(1) = \boxed{\sum a_n \cdot n} \cdot 1^{n-1}$

C 11. What is the value of $\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n}$? $= 2\left(\frac{2^n}{3^n}\right)$

$\sum_{n=1} 2\left(\frac{2}{3}\right)^n$

(A) 1 (B) 2 (C) 4 (D) 6 (E) The series diverges

$\frac{\text{first term}}{1-r}$ $\frac{4/3}{1-2/3}$ $\frac{4}{\cancel{3}}\left(\frac{\cancel{3}}{1}\right)$

D 12. The Maclaurin series for $\frac{1}{1-x}$ is $\sum_{n=0}^{\infty} x^n$. Which of the following is a power series expansion for $\frac{x^2}{1-x^2}$? $x^2\left(\boxed{\frac{1}{1-(x^2)}}\right)$

sub x^2 $= 1 + x + x^2 + x^3 + x^4 + \cdots$

$: 1 + x^2 + x^4 + x^6 + x^8$

$x(x^2)$ $x^2 + x^4 + x^6 + x^8 + x^{10}$

(A) $1 + x^2 + x^4 + x^6 + x^8 + \cdots$ (B) $x^2 + x^3 + x^4 + x^5 + \cdots$ (C) $x^2 + 2x^3 + 3x^4 + 4x^5 + \cdots$

(D) $x^2 + x^4 + x^6 + x^8 + \cdots$ (E) $x^2 - x^4 + x^6 - x^8 + \cdots$

D

___ 13. A function f has a Maclaurin series given by $\frac{x^4}{2!}+\frac{x^5}{3!}+\frac{x^6}{4!}+\cdots+\frac{x^{n+3}}{(n+1)!}+\cdots$. Which of the following is an expression for $f(x)$?

(A) $-3x\sin x+3x^2$ (B) $-\cos\left(x^2\right)+1$ (C) $-x^2\cos x+x^2$

(D) $x^2e^x-x^3-x^2$ (E) $e^{x^2}-x^2-1$

e^{x^2}

e^x

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^6}{6!}$

times x^2: $x^2 + x^3 + \frac{x^4}{2!} + \frac{x^5}{3!}$

D

___ 14. Which of the following series diverge?

I. $\sum_{n=0}^{\infty}\left(\frac{\sin 2}{\pi}\right)^n$ CON $|\,| < 1$ II. $\sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n}}$ D $\frac{1}{n^{1/3}}$ $p > 1$ III. $\sum_{n=1}^{\infty}\left(\frac{e^n}{e^n+1}\right)$ D nth term $\lim_{n\to\infty} \frac{e^n}{e^n+1} = 1$

(A) III only (B) I and II only (C) I and III only (D) II and III only (E) I, II, and III

D

___ 15. What is the coefficient of x^2 in the Taylor series for $\frac{1}{(1+x)^2}$ about $x=0$?

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) 1 (D) 3 (E) 6

$f(x) = (1+x)^{-2}$

$f'(x) = -2(1+x)^{-3} \cdot 1$

$f''(x) = 6(1+x)^{-4} \cdot 1$

$\boxed{\frac{f''(0)}{2!}} x^2$

$\frac{6}{2}$

C 16. The sum of the infinite geometric series $\frac{3}{2}+\frac{9}{16}+\frac{27}{128}+\frac{81}{1024}+\cdots$ is

(A) 1.60 (B) 2.35 (C) 2.40 (D) 2.45 (E) 2.50

$\frac{3}{2}\left(1+\frac{3}{8}+\frac{9}{64}\right)$

$(\ )^0+\left(\frac{3}{8}\right)^1$

$\sum_{n=0}^{\infty}\frac{3}{2}\left(\frac{3}{8}\right)^n=\frac{3/2}{1-3/8}=\frac{3}{2}\left(\frac{8}{5}\right)=\frac{24}{10}=2.4$

$10\overline{)24}$

E 17. What are all values of x for which the series $\sum_{n=1}^{\infty}\frac{(x-2)^n}{n3^n}$ converges?

(A) $-3\le x\le 3$ (B) $-3<x<3$ (C) $-1<x\le 3$ (D) $-1\le x\le 5$ (E) $-1\le x<5$

$R=3$

$\sum_{n=1}^{\infty}\frac{\cancel{3^n}}{n\cdot\cancel{3^n}}$ $\frac{1}{n}$

$(-3)^n$

$\frac{(-1)^n\cancel{(3)^n}}{n\cdot\cancel{3^n}}$

D 18. The Taylor series for $\sin x$ about $x=0$ is $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\cdots$. If f is a function such that $f'(x)=\sin\left(x^2\right)$, then the coefficient of x^7 in the Taylor series for $f(x)$ about $x=0$ os

(A) $\frac{1}{7!}$ (B) $\frac{1}{7}$ (C) 0 (D) $-\frac{1}{42}$ (E) $-\frac{1}{7!}$

$\int f'=\int x^2-\frac{x^6}{3!}+\frac{x^{10}}{5!}$

$f=C+\frac{1}{3}x^3\boxed{-\frac{1x^7}{7\cdot 3!}}$

$-\frac{1}{7\cdot 6}=-\frac{1}{42}$

D 19. $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$ is the Taylor series about zero for which of the following functions?

(A) ~~$\sin x$~~ (B) ~~$\cos x$~~ (C) e^x (D) e^{-x} (E) $\ln(1+x)$

$\frac{x^n}{n!}$

$\cancel{\frac{1}{n^{-7}}}$ n^7 $n^{-2} = \frac{1}{n^2}$

B 20. For what values of x does the series $1+2^x+3^x+4^x+\cdots+n^x+\cdots$ converge?

(A) No values of x (B) $x<-1$ (C) ~~$x \geq -1$~~ (D) ~~$x>-1$~~ (E) ~~All values of x~~

$C=-1$

E 21. The complete interval of convergence of the series $\sum_{k=1}^{\infty} \frac{(x+1)^k}{k^2}$ is $\sum \frac{1}{k^2}$ $\frac{1}{n^2}$

(A) ~~$0<x<2$~~ (B) ~~$0 \leq x \leq 2$~~ (C) $-2<x \leq 0$ (D) ~~$-2 \leq x<0$~~ (E) $-2 \leq x \leq 0$

-1 -1 $+1$

$\frac{\cancel{(-1)^k}}{k^2}$ $\frac{1}{k^2}$

A 21. For $-1<x<1$, if $f(x)=\sum_{n=1}^{\infty}\boxed{\frac{(-1)^{n+1}x^{2n-1}}{2n-1}}$, then $f'(x)=$

$= x - \frac{x^3}{3} + \cdots$

$= 1 - x^2$

(A) $\sum_{n=1}^{\infty}(-1)^{n+1}x^{2n-2}$ (B) $\sum_{n=1}^{\infty}(-1)^{n}x^{2n-2}$ (C) $\sum_{n=1}^{\infty}(-1)^{2n}x^{2n}$ (D) $\sum_{n=1}^{\infty}(-1)^{n}x^{2n}$ (E) $\sum_{n=1}^{\infty}(-1)^{n+1}x^{2n}$

$$\sum \frac{(-1)^{n+1}\cancel{(2n-1)}x^{2n-2}}{\cancel{(2n-1)}}$$

$$\sum (-1)^{n+1}x^{2n-2}$$

E 22. The coefficient of x^3 in the Taylor series for e^{3x} about $x=0$ is

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{2}$ (E) $\frac{9}{2}$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$$

$$\frac{(3x)^3}{3!} = \boxed{\frac{27x^3}{6}} \qquad \frac{9}{2}$$

B 23. $\sum_{i=n}^{\infty}\left(\frac{1}{3}\right)^{i} = \frac{(1/3)^n}{1-1/3} = \left(\frac{1}{3}\right)^n\cdot\left(\frac{3}{2}\right)$

$n=1$

(A) $\frac{3}{2}-\left(\frac{1}{3}\right)^{n}$ (B) $\frac{3}{2}\left[1-\left(\frac{1}{3}\right)^{n}\right]$ (B) $\frac{3}{2}\left(\frac{1}{3}\right)^{n}$ (D) $\frac{2}{3}\left(\frac{1}{3}\right)^{n}$ (E) $\frac{2}{3}\left(\frac{1}{3}\right)^{n+1}$

A 24. Which of the followign series converge?

I. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{2n+1}$ II. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n}\left(\frac{3}{2}\right)^{n}$ III. $\displaystyle\sum_{n=2}^{\infty}\frac{1}{n\ln n}$

(A) I only (B) II only (C) III only (D) I and III only (E) I, II, and III

Integral test

$\int_2^{\infty}\frac{1}{n}\left(\frac{1}{(\ln n)'}\right)dn$

$\ln|\ln n|\Big|_2^{\infty}$

$= \ln|\ln\infty| - \ln|\ln 2|$

$\infty - \ln|\ln 2|$

$= \infty$

Diverges

$\dfrac{(n+5)^{100}}{5(n+4)^{100}}$ $\lim_{n\to\infty} = \dfrac{1n^{100}+\dots}{5n^{100}+\dots}$

A 25. If $s_n = \left(\dfrac{(5+n)^{100}}{5^{n+1}}\right)\left(\dfrac{5^n}{(4+n)^{100}}\right)$, to what number does the sequence $\{s_n\}$ converge?

(A) $\dfrac{1}{5}$ (B) 1 (C) $\dfrac{5}{4}$ (D) $\left(\dfrac{5}{4}\right)^{100}$ (E) The sequence does not converge

$f(1) = \sum_{n=1}^{\infty}\left(\sin^2 1\right)^n$

$\dfrac{\sin^2 1}{1-}$

D 26. (<u>Calculator Permitted</u>) If $f(x)=\sum_{k=1}^{\infty}\left(\sin^2 x\right)^{k}$, then $f(1)$ is

(A) 0.369 (B) 0.585 (C) 2.400 (D) 2.426 (E) 3.426

=

B 27. Let the function given by $f(x)=\ln(3-x)$. The third-degree Taylor polynomial for f about $x=2$ is

(A) $-(x-2)+\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$ (B) $-(x-2)-\frac{(x-2)^2}{2}-\frac{(x-2)^3}{3}$ (C) ~~$(x-2)+(x-2)^2+(x-2)^3$~~

$\boxed{c=2}$

(D) ~~$(x-2)+\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$~~ (E) ~~$(x-2)-\frac{(x-2)^2}{2}+\frac{(x-2)^3}{3}$~~

$f^{0}(x)=\ln(3-x),\ f^{0}(2)=\ln 1=0$

$f'(x)=\frac{-1}{3-x},\ f'(2)=-1$

$f''(x)=1(3-x)^{-2}\cdot(-1),\ f''(2)=-1$

$=\frac{-1}{(3-x)^2}$

Term: $n=0: 0$

$n=1: -(x-2)$

$n=2: \frac{-1}{2!}(x-2)^2$

$=-\frac{1}{2}(x-2)^2$

E 28. (<u>Calculator Permitted</u>) Suppose a function f is approximated with a fourth-degree Taylor polynomial about $x=1$. If the maximum value of the fifth derivative between $x=1$ and $x=3$ is 0.01, that is, $\left|f^{(5)}(x)\right|<0.01$, then the maximum error incurred using this approximation to compute $f(3)$ is

(A) 0.054 (B) 0.0054 (C) 0.26667 (D) 0.02667 (E) 0.00267

$$R_4(3)=\left|\frac{|f^{(5)}(z)|}{5!}(3-1)^5\right|$$

$$\leq\left|\frac{0.01}{5!}(2^5)\right|=$$