1. E
2. A
3. D
4. A
5. D
6. A
7. D
8. E
9. B
10. B

14

Tue

7:00

TACO → Bacon/Egg

→ Beann cheese

7:29

7:30 AM.

BYOB

AB Review 14 No Calculator permitted (except on #11)

1.

If $y = x^2 \sin(2x)$, then $\dfrac{dy}{dx} =$

A) $2x\cos(2x)$

B) $4x\cos(2x)$

C) $2x[\sin(2x) + \cos(2x)]$

D) $2x[\sin(2x) - x\cos(2x)]$

E) $2x[\sin(2x) + x\cos(2x)]$

2.

Let f be the function given by $f(x) = 2xe^x$. The graph of f is concave down when

$f'' < 0$

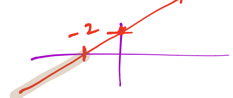
A) $x < -2$ B) $x > -2$ C) $x < -1$ D) $x > -1$ E) $x < 0$

$f'(x) = 2 \cdot e^x + (2x) e^x$

$f''(x) = 2e^x + 2e^x + 2x \cdot e^x \leq 0$ $= 0$

$= 4e^x + 2xe^x$

$2e^x(2+x) = 0$ $= 0$ $x = -2$



3.

A curve has a slope $y' = 2x + 3$ at each point (x,y) on the curve. Which of the following is an equation for this curve if it passes through the point (1,2)?

~~A) $y = 5x - 3$~~

~~B) $y = x^2 + 1$~~

C) $y = x^2 + 3x$

D) $y = x^2 + 3x - 2$

~~E) $y = 2x^2 + 3x - 3$~~

$\int (2x+3)\,dx$

$y = x^2 + 3x + C$

At $(1,2)$: $2 = 1 + 3 + C$

$2 = 4 + C$

$C = -2$

4.

x	-4	-3	-2	-1	0	1	2	3	4
$g'(x)$	2	3	0	-3	-2	-1	0	3	2

The derivative g' of a function g is continuous and has exactly two zeros. Selected values of g' are given in the table above. If the domain of g is the set of all real numbers, then g is decreasing on which of the following intervals?

A) $-2 \le x \le 2$ only

B) $-1 \le x \le 1$ only

C) $x \ge -2$

D) $x \ge 2$ only

E) $x \le -2$ or $x \ge 2$

5.

$$f(x) = \begin{cases} x+2 & \text{if } x \le 3 \\ 4x-7 & \text{if } x > 3 \end{cases}$$

Let f be the function given above. Which of the following statements are true about f?

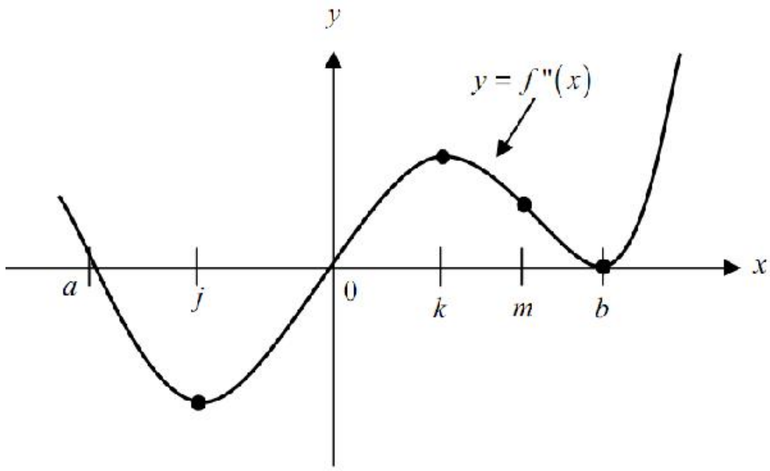
I. $\lim_{x \to 3} f(x)$ exists

II. f is continuous at x=3

III. f is differentiable at x=3

(A) None (B) I only (C) II only (D) I and II only (E) I, II, and III

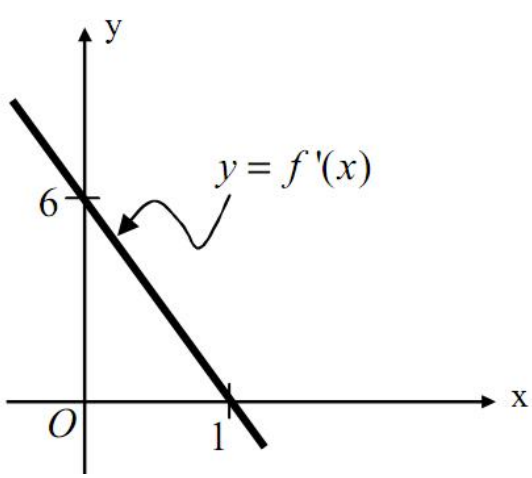
6.



The second derivative of the function f is given by $f''(x) = x(x-a)(x-b)^2$. The graph of f'' is shown above. For what values of x does the graph of f have a point of inflection?

A) 0 and a only B) 0 and m only C) b and j only D) 0, a, and b E) b, j, and k

7.



The graph of f', the derivative of f, is the line shown in the figure above. If $f(0) = 5$, then $f(1) =$

A) 0 B) 3 C) 6 D) 8 E) 11

8.

$$\frac{d}{dx}\left(\int_{3^x}^{x^2} \sin(t^3)\,dt\right) =$$

$$= \sin\left((x^2)^3\right)\cdot(2x) - \sin\left((3^x)^3\right)\cdot 3^x \cdot \ln 3$$

A) $-\cos(x^6)$ B) $\sin(x^3)$ C) $\sin(x^6)$ D) $2x\sin(x^3)$ E) $2x\sin(x^6)$

9.

What is the slope of the line tangent to the curve $3y^2-2x^2 = 6-2xy$ at the point (3,2)?

A)0 B) $\frac{4}{9}$ C) $\frac{7}{9}$ D) $\frac{6}{7}$ E) $\frac{5}{3}$

10.

Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and g(2)=1, what is the value of $g'(2)$?

A) $\frac{1}{13}$ B) $\frac{1}{4}$ C) $\frac{7}{4}$ D) 4 E) 13

$\frac{1}{10}\int_0^{10} H(t)\,dt \approx \left(\frac{1}{10}\right)\left(\frac{1}{2}\right)\left[2(66+60)+3(60+52)+4(52+44)+1(44+43)\right]$ °C

11. (2011, AB-2) (**Calculator Permitted**)

2 3 4 1

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

This gives the tea's avg temp in °C from t = 0 min to t = 10 min.

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

(c) Evaluate $\int_0^{10} H'(t)\,dt$. Using correct units, explain the meaning of the expression in the context of this problem.

$= H(t)\Big|_0^{10} = H(10) - H(0) = 43 - 66$ °C $= -23$ °C

From t = 0 min to t = 10 min, the tea's temp decreased by 23°C.

(d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $\boxed{B'(t) = -13.84e^{-0.173t}}$. Using the given models, at time $\boxed{t = 10}$, how much cooler are the biscuits than the tea?

°C/min

tea's temp @ t = 10: 43°C

Biscuit's temp @ t = 10: $100 + \int_0^{10} B'(t)\,dt = 34.182$ °C

So the biscuits are $43 - 34.182 = 8.817$ °C cooler.

Question 2

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \le t \le 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

(a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.

(b) Using correct units, explain the meaning of $\frac{1}{10}\int_0^{10} H(t)\,dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10}\int_0^{10} H(t)\,dt$.

(c) Evaluate $\int_0^{10} H'(t)\,dt$. Using correct units, explain the meaning of the expression in the context of this problem.

(d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

(a) $H'(3.5) \approx \dfrac{H(5) - H(2)}{5 - 2}$

$= \dfrac{52 - 60}{3} = -2.666$ or -2.667 degrees Celsius per minute

1 : answer

(b) $\frac{1}{10}\int_0^{10} H(t)\,dt$ is the average temperature of the tea, in degrees Celsius, over the 10 minutes.

$$\frac{1}{10}\int_0^{10} H(t)\,dt \approx \frac{1}{10}\left(2 \cdot \frac{66+60}{2} + 3 \cdot \frac{60+52}{2} + 4 \cdot \frac{52+44}{2} + 1 \cdot \frac{44+43}{2}\right)$$
$$= 52.95$$

3 : $\begin{cases} 1 : \text{meaning of expression} \\ 1 : \text{trapezoidal sum} \\ 1 : \text{estimate} \end{cases}$

(c) $\int_0^{10} H'(t)\,dt = H(10) - H(0) = 43 - 66 = -23$

The temperature of the tea drops 23 degrees Celsius from time $t = 0$ to time $t = 10$ minutes.

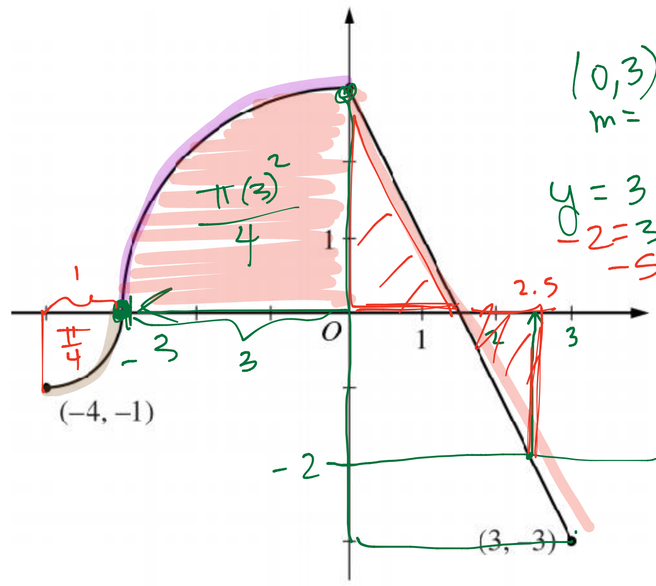
2 : $\begin{cases} 1 : \text{value of integral} \\ 1 : \text{meaning of expression} \end{cases}$

(d) $B(10) = 100 + \int_0^{10} B'(t)\,dt = 34.18275;\quad H(10) - B(10) = 8.817$

The biscuits are 8.817 degrees Celsius cooler than the tea.

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{uses } B(0) = 100 \\ 1 : \text{answer} \end{cases}$

12. (2011, AB-4)



Graph of f

$(0,3)$

$m = -\frac{6}{3} = -2$

$y = 3 - 2(x-0)$

$-2 = 3 - 2x$

$-5 = -2x$

$x = \frac{5}{2} = 2.5$

$g'(x) = DNE$

$g'(x) = 0$

$2 + f(x) = 0$

$f(x) = -2$

The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above. Let $g(x) = 2x + \int_0^x f(t)\,dt$.

$g'(x) = 2 + f(x)$, $g'(-3) = 2 + f(-3) = \boxed{2+0} = \boxed{2}$

(a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

$g(-3) = 2(-3) + \int_0^{-3} f(t)\,dt = -6 - \frac{9\pi}{4}$

(b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

$g(-4) = 2(-4) + \int_0^{-4} f(t)\,dt = -8 - \frac{9\pi}{4} + \frac{\pi}{4}$

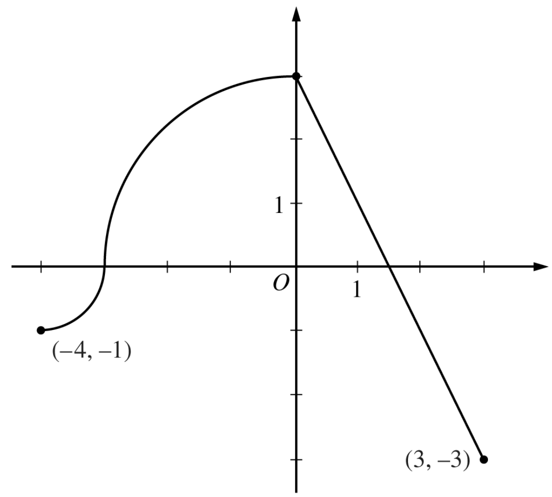
$g(3) =$

$g(2.5) =$

(c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

(d) Find the average rate of change of f on the interval $-4 \le x \le 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

Question 4



Graph of f

The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above.

Let $g(x) = 2x + \int_0^x f(t)\,dt$.

(a) Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.

(b) Determine the x-coordinate of the point at which g has an absolute maximum on the interval $-4 \le x \le 3$. Justify your answer.

(c) Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.

$\frac{f(3)-f(-4)}{3-(-4)} = f'(x)$

(d) Find the average rate of change of f on the interval $-4 \le x \le 3$. There is no point c, $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

(a) $g(-3) = 2(-3) + \int_0^{-3} f(t)\,dt = -6 - \frac{9\pi}{4}$

$g'(x) = 2 + f(x)$

$g'(-3) = 2 + f(-3) = 2$

3 : { 1 : $g(-3)$; 1 : $g'(x)$; 1 : $g'(-3)$ }

(b) $g'(x) = 0$ when $f(x) = -2$. This occurs at $x = \frac{5}{2}$.

$g'(x) > 0$ for $-4 < x < \frac{5}{2}$ and $g'(x) < 0$ for $\frac{5}{2} < x < 3$.

Therefore g has an absolute maximum at $x = \frac{5}{2}$.

3 : { 1 : considers $g'(x) = 0$; 1 : identifies interior candidate ; 1 : answer with justification }

(c) $g''(x) = f'(x)$ changes sign only at $x = 0$. Thus the graph of g has a point of inflection at $x = 0$.

1 : answer with reason

(d) The average rate of change of f on the interval $-4 \le x \le 3$ is $\frac{f(3) - f(-4)}{3 - (-4)} = -\frac{2}{7}$.

To apply the Mean Value Theorem, f must be differentiable at each point in the interval $-4 < x < 3$. However, f is not differentiable at $x = -3$ and $x = 0$.

2 : { 1 : average rate of change ; 1 : explanation }

Bonus Probs

$$\frac{d}{dx}\int_{e^x}^{2x} \cos t \, dt$$

$$= \cos(2x)\cdot 2 - \cos(e^x)\cdot e^x$$

$$F(x) = \int_{\tan x}^{4} \sqrt{t^2+1}\, dt$$

find $F'(x)$

$$F'(x) = \left(\sqrt{4^2+1}\right)\cdot 0 - \left(\sqrt{\tan^2 x+1}\right)\cdot \sec^2 x$$

$(\tan x)^2$
$\neq \tan x^2$

$$-\sec^2 x\sqrt{\tan^2 x+1}$$

$$-\sec^2 x \cdot \sec x$$

$$\boxed{-\sec^3 x}$$

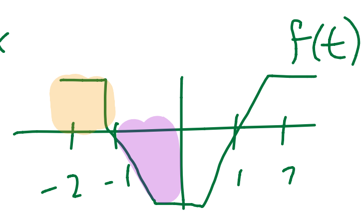
$*\ \int \tan^2 x\, dx$
$= \int (\sec^2 x - 1)\, dx$
$= \tan x - x + C$

$$\boxed{\begin{aligned} &\cos^2 x + \sin^2 x = 1 \\ &1 + \tan^2 x = \sec^2 x \\ &1 + \cot^2 x = \csc^2 x \end{aligned}}$$

$*\ \sin 2x = 2\sin x\cos x$

$*\ \cos 2x = \cos^2 x - \sin^2 x$

$$F(x) = \int_{-1}^{x} f(t)\, dt$$



$$F'(x) = f(x)$$

$F(0) = \text{neg}$

$F(-1) = 0$

$F(-2) = \text{neg}$

Bonus Prob

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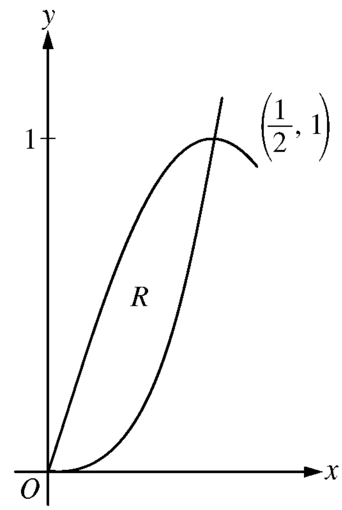
Question 3

Let R be the region in the first quadrant enclosed by the graphs of $f(x) = 8x^3$ and $g(x) = \sin(\pi x)$, as shown in the figure above.



(a) Write an equation for the line tangent to the graph of f at $x = \frac{1}{2}$.

(b) Find the area of R.

(c) Write, but do not evaluate, an integral expression for the volume of the solid generated when R is rotated about the horizontal line $y = 1$.

(a) $f\left(\frac{1}{2}\right) = 1$

$f'(x) = 24x^2$, so $f'\left(\frac{1}{2}\right) = 6$

An equation for the tangent line is $y = 1 + 6\left(x - \frac{1}{2}\right)$.

$$2 : \begin{cases} 1 : f'\left(\frac{1}{2}\right) \\ 1 : \text{answer} \end{cases}$$

(b)
$$\begin{aligned} \text{Area} &= \int_0^{1/2} (g(x) - f(x))\, dx \\ &= \int_0^{1/2} \left(\sin(\pi x) - 8x^3\right) dx \\ &= \left[-\frac{1}{\pi}\cos(\pi x) - 2x^4\right]_{x=0}^{x=1/2} \\ &= -\frac{1}{8} + \frac{1}{\pi} \end{aligned}$$

$$4 : \begin{cases} 1 : \text{integrand} \\ 2 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$$

(c)
$$\begin{aligned} &\pi\int_0^{1/2} \left((1 - f(x))^2 - (1 - g(x))^2\right) dx \\ &= \pi\int_0^{1/2} \left(\left(1 - 8x^3\right)^2 - (1 - \sin(\pi x))^2\right) dx \end{aligned}$$

$$3 : \begin{cases} 1 : \text{limits and constant} \\ 2 : \text{integrand} \end{cases}$$