1. A
2. A
3. B
4. A
5. B
6. D
7. E
8. D
9. E
10. B

12

AB Review 12   Calculator Permitted

1.

The velocity, in ft/sec, of a particle moving along the x-axis is given by the function $v(t) = e^t + te^t$. What is the average velocity of the particle from time t=0 to time t=3?

A) 20.086 ft/sec

B) 26.447 ft/sec

C) 32.809 ft/sec

D) 40.671 ft/sec

E) 79.342 ft/sec

Totino's pepperoni

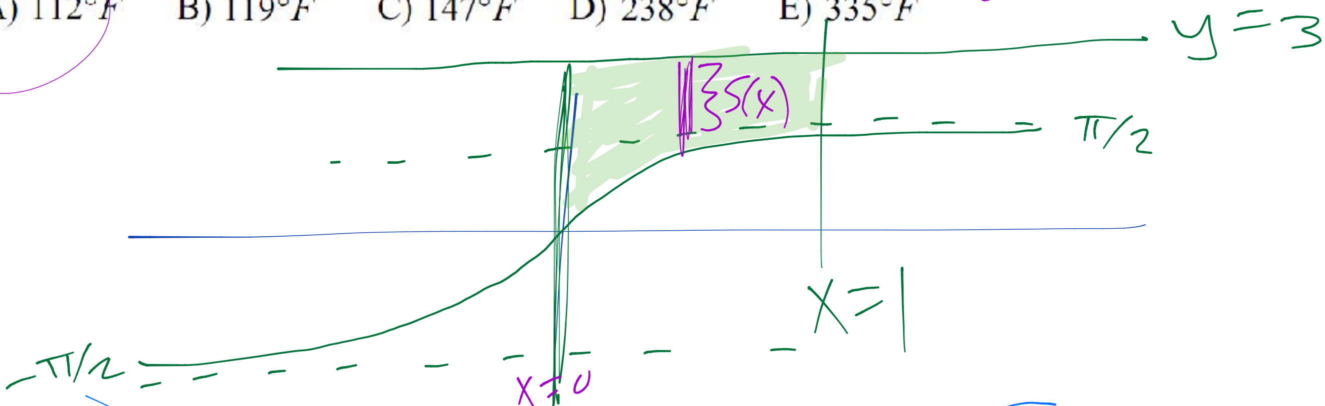
$T(5) = 350° + \int_0^5 -100e^{-0.4t}$

2.

A pizza, heated to a temperature of 350 degrees Fahrenheit $(°F)$, is taken out of an oven and placed in a $75\,°F$ room at time t=0 minutes. The temperature of the pizza is changing at a rate of $\boxed{-110e^{-0.4t}}$ degrees Fahrenheit per minute. To the nearest degree, what is the temperature of the pizza at time t=5 minutes?

$\text{Vol} = 1 \cdot \int_0^1 (3 - \tan^{-1}x)^2\,dx$

A) $112°F$ B) $119°F$ C) $147°F$ D) $238°F$ E) $335°F$



3.

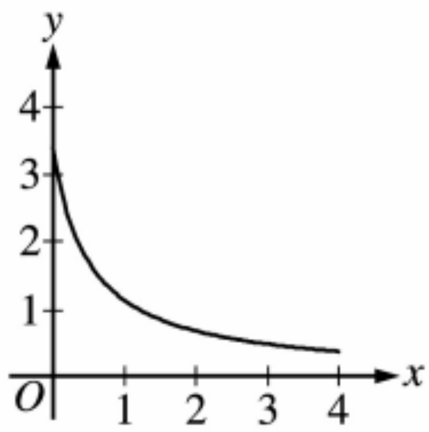
The base of a solid is the region in the first quadrant bounded by the y- axis, the graph of $y = \tan^{-1}(x)$, the horizontal line y=3, and the vertical line x=1. For this solid, each cross section perpendicular to the x-axis is a square. What is the volume of the solid?

A) 2.561   B) 6.612   C) 8.046   D) 8.755   E) 20.773

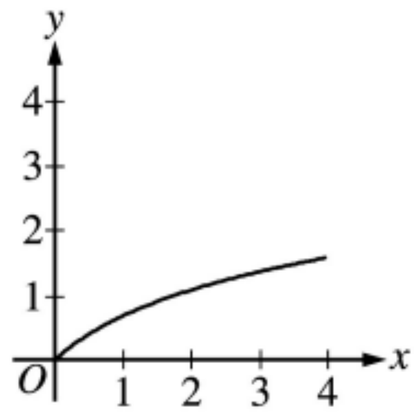
4.

If a trapezoidal sum over approximates $\int_0^4 f(x)dx$, and a right Riemann sum under approximates $\int_0^4 f(x)dx$, which of the following could be the graph of $y = f(x)$?

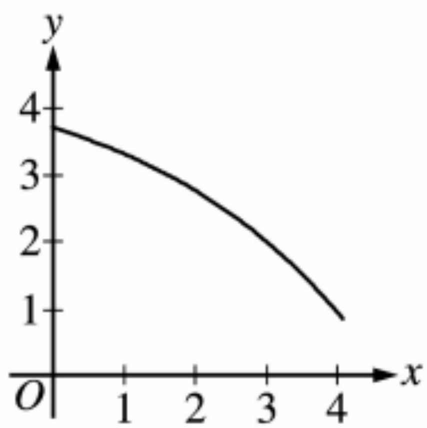
(A)



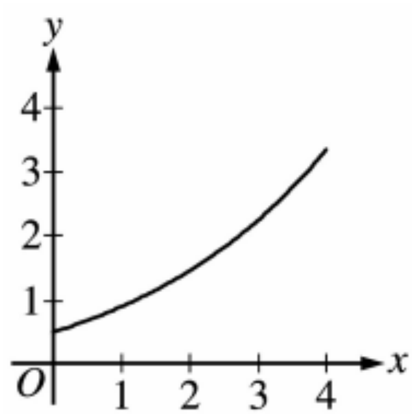
(B)



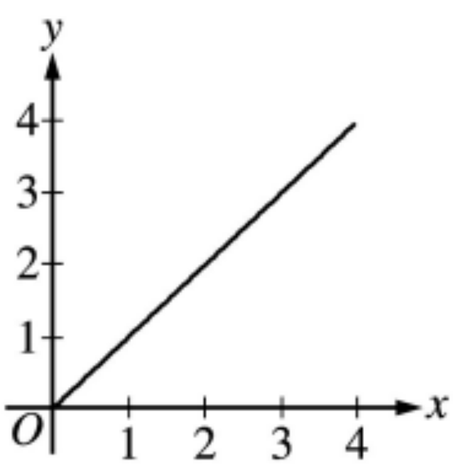
(C)



(D)



(E)



5.

The function f has first derivative given by $f'(x) = \dfrac{\sqrt{x}}{1+x+x^3}$. What is the x-coordinate of the inflection point of the graph of f?

A) 1.008 B) 0.473 C) 0 D) –0.278 E) The graph of f has no inflection point

6.

Let g be the function given by $g(x) = \int_0^x \sin(t^2)\,dt$ for $-1 \le x \le 3$. On which of the following intervals is g decreasing?

A) $-1 \le x \le 0$

B) $0 \le x \le 1.772$

C) $1.253 \le x \le 2.171$

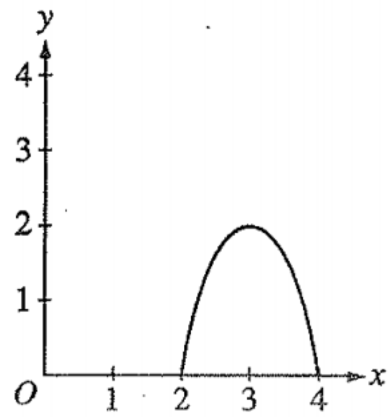
D) $1.772 \le x \le 2.507$

E) $2.802 \le x \le 3$

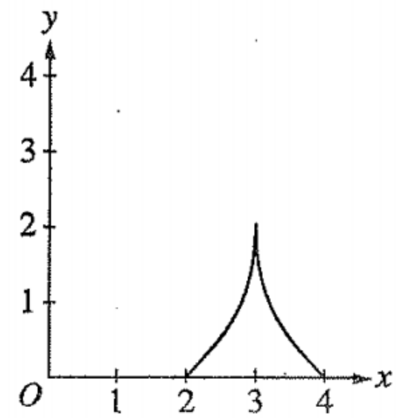
7.

On the closed interval $[2,4]$, which of the following could be the graph of a function f with the property that $\frac{1}{4-2}\int_{2}^{4} f(t)dt = 1$?

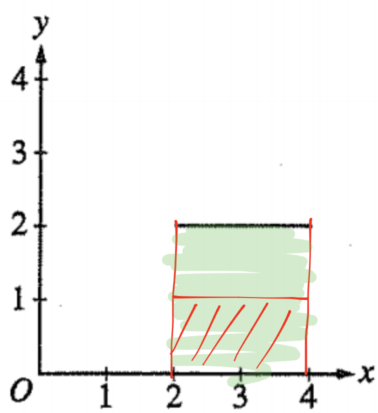
(A)



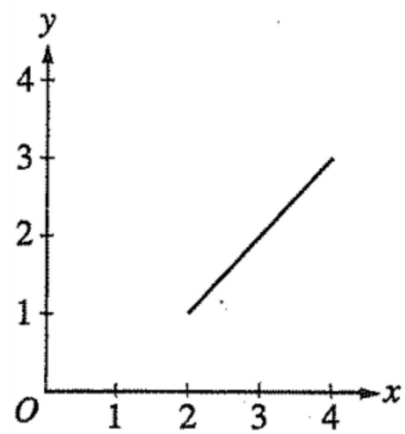
(B)



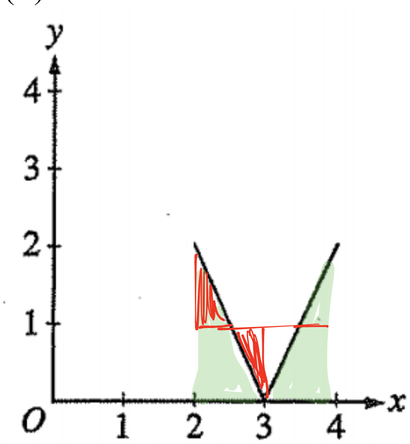
(C)



(D)



(E)



8.

Let f be a differentiable function with $f(2)=3$ and $f'(2)=-5$, and let g be the function defined by $g(x)=xf(x)$. Which of the following is an equation of the line tangent to the graph of g at the point where x=2?

A) $y=3x$

B) $y-3=-5(x-2)$

C) $y-6=-5(x-2)$

D) $y-6=-7(x-2)$

E) $y-6=-10(x-2)$

9.

A particle moves along the x-axis so that at any time $t>0$, its acceleration is given by $a(t)=\ln(1+2^t)$. If the velocity of the particle is 2 at time t=1, then the velocity of the particle at time t=2 is

A) 0.462 B) 1.609 C) 2.555 D) 2.886 E) 3.346

10.

For all x in the closed interval $[2,5]$, the function f has a positive first derivative and a negative second derivative. Which of the following could be a table of values for f?

A)

x	f(x)
2	7
3	9
4	12
5	16

B)

x	f(x)
2	7
3	11
4	14
5	16

C)

X	f(x)
2	16
3	12
4	9
5	7

D)

x	f(x)
2	16
3	14
4	11
5	7

E)

x	f(x)
2	16
3	13
4	10
5	7

11. (2012, AB-1)

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

Question 1

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is $55°\text{F}$. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

(a) $$W'(12) \approx \frac{W(15) - W(9)}{15 - 9} = \frac{67.9 - 61.8}{6} = 1.017 \text{ (or 1.016)}$$

The water temperature is increasing at a rate of approximately $1.017\ °\text{F}$ per minute at time $t = 12$ minutes.

$$2 : \begin{cases} 1 : \text{estimate} \\ 1 : \text{interpretation with units} \end{cases}$$

(b) $$\int_0^{20} W'(t)\,dt = W(20) - W(0) = 71.0 - 55.0 = 16$$

The water has warmed by $16\ °\text{F}$ over the interval from $t = 0$ to $t = 20$ minutes.

$$2 : \begin{cases} 1 : \text{value} \\ 1 : \text{interpretation with units} \end{cases}$$

(c) $$\begin{aligned} \frac{1}{20}\int_0^{20} W(t)\,dt &\approx \frac{1}{20}(4 \cdot W(0) + 5 \cdot W(4) + 6 \cdot W(9) + 5 \cdot W(15)) \\ &= \frac{1}{20}(4 \cdot 55.0 + 5 \cdot 57.1 + 6 \cdot 61.8 + 5 \cdot 67.9) \\ &= \frac{1}{20} \cdot 1215.8 = 60.79 \end{aligned}$$

This approximation is an underestimate, because a left Riemann sum is used and the function W is strictly increasing.

$$3 : \begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{approximation} \\ 1 : \text{underestimate with reason} \end{cases}$$

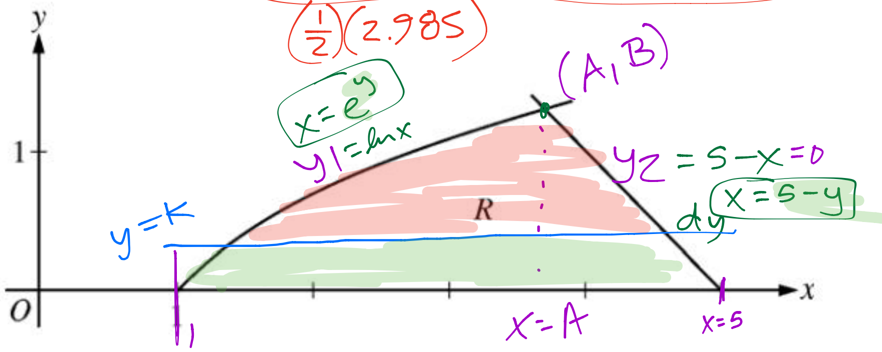
(d) $$\begin{aligned} W(25) &= 71.0 + \int_{20}^{25} W'(t)\,dt \\ &= 71.0 + 2.043155 = 73.043 \end{aligned}$$

$$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$$

$$\int_0^K (5-y-e^y)\,dy = \int_K^B (5-y-e^y)\,dy$$

$$= \left(\tfrac{1}{2}\right)(2.985)$$

12. (2012, AB-2)



Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y_1 = \ln x$ and $y_2 = 5 - x$, as shown in the figure above.

$$Vol = \frac{\pi}{8}\int_1^A (y1 - 0)^2\,dx + \frac{\pi}{8}\int_A^5 (y2 - 0)^2\,dx$$

(a) Find the area of R.

(b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

(c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

(a) $y1 = y2$

$(A, B) = (3.693, 1.306)$

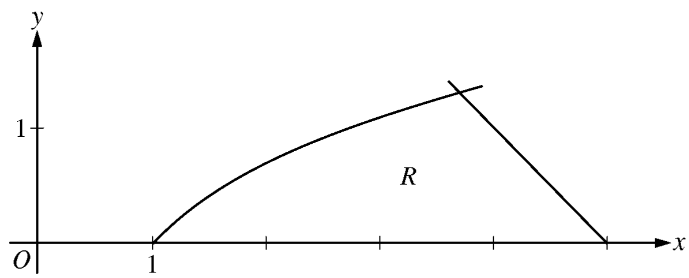
$$\text{Area} = \int_0^B \big((5-y) - (e^y)\big)\,dy = 2.985$$

(b)

$Vol =$

Question 2

Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.



(a) Find the area of R.

(b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

(c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

$\ln x = 5 - x \quad \Rightarrow \quad x = 3.69344$

Therefore, the graphs of $y = \ln x$ and $y = 5 - x$ intersect in the first quadrant at the point $(A, B) = (3.69344, 1.30656)$.

(a) $$\text{Area} = \int_0^B \left(5 - y - e^y\right) dy$$
$$= 2.986 \text{ (or } 2.985)$$

OR

$$\text{Area} = \int_1^A \ln x\, dx + \int_A^5 (5 - x)\, dx$$
$$= 2.986 \text{ (or } 2.985)$$

$3: \begin{cases} 1: \text{integrand} \\ 1: \text{limits} \\ 1: \text{answer} \end{cases}$

(b) $$\text{Volume} = \int_1^A (\ln x)^2\, dx + \int_A^5 (5 - x)^2\, dx$$

$3: \begin{cases} 2: \text{integrands} \\ 1: \text{expression for total volume} \end{cases}$

(c) $$\boxed{\int_0^k \left(5 - y - e^y\right)} dy = \boxed{\frac{1}{2} \cdot 2.986} \left(\text{or } \frac{1}{2} \cdot 2.985\right)$$

$3: \begin{cases} 1: \text{integrand} \\ 1: \text{limits} \\ 1: \text{equation} \end{cases}$