1. B	6. A
2. B	7. C
3. C	8. B
4. C	9. C
5. E	10. A

1.

A particle moves along a straight line with velocity given by $v(t) = 7 - (1.01)^{-t^2}$ at time $t \geq 0$. What is the acceleration of the particle at time $t = 3$?

(A) -0.914 (B) 0.055 (C) 5.486 (D) 6.086 (E) 18.087

$V'(t) = -(1.01)^{-t^2} \cdot \ln(1.01)(-2t)$

$V'(3) =$

2.

x	-4	-3	-2	-1
$f(x)$	0.75	-1.5	-2.25	-1.5
$f'(x)$	-3	-1.5	0	1.5

The table above gives values of a function f and its derivative at selected values of x. If f' is continuous on the interval $[-4, -1]$, what is the value of $\int_{-4}^{-1} f'(x)\,dx$?

(A) -4.5 (B) -2.25 (C) 0 (D) 2.25 (E) 4.5

3.

The radius of a sphere is decreasing at a rate of 2 centimeters per second. At the instant when the radius of the sphere is 3 centimeters, what is the rate of change, in square centimeters per second, of the surface area of the sphere? (The surface area S of a sphere with radius r is $S = 4\pi r^2$)

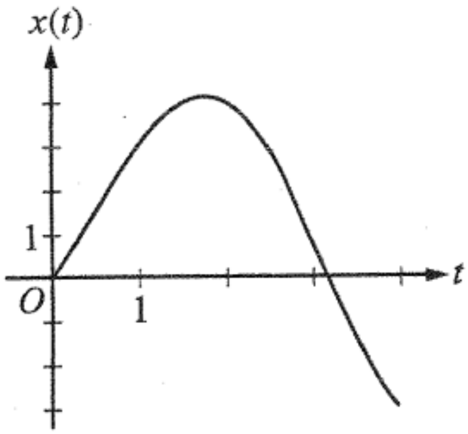
(A) -108π (B) -72π (C) -48π (D) -24π (E) -16π

4.

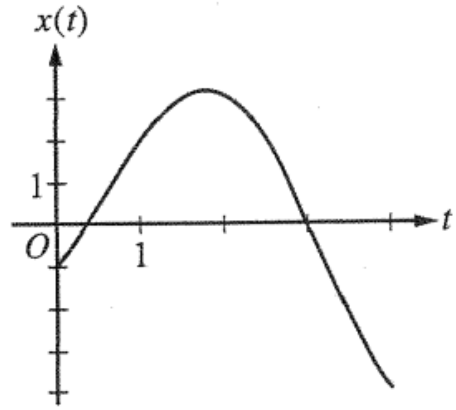
t	0	1	2	3	4
$v(t)$	–1	2	3	0	–4

The table gives selected values of the velocity, $v(t)$, of a particle moving along the x-axis. At time $t = 0$, the particle is at the origin. Which of the following could be the graph of the position, $x(t)$, of the particle for $0 \le t \le 4$?

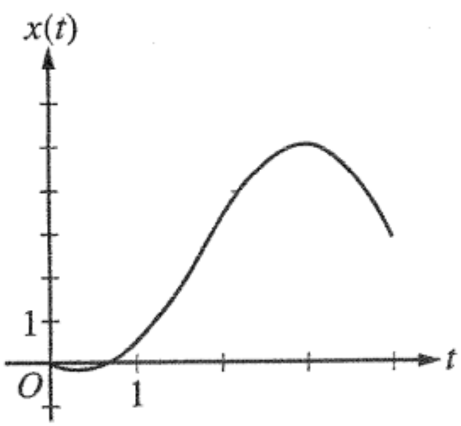
(A)



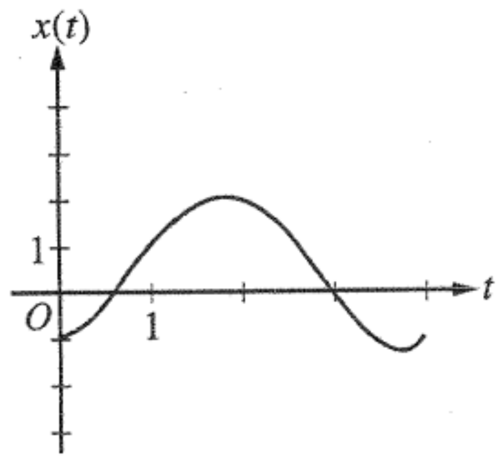
(B)



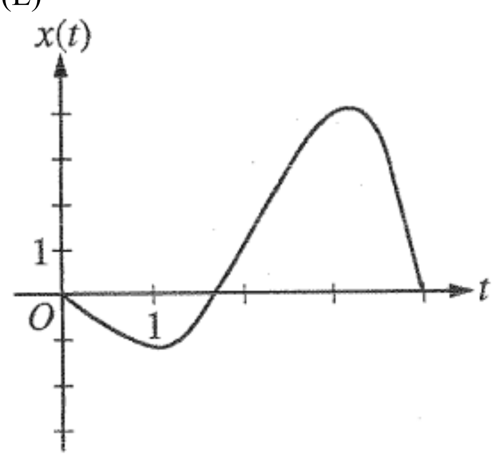
(C)



(D)



(E)



5.

The function f is continuous for $-2 \le x \le 2$ and $f(-2) = f(2) = 0$. If there is no c, where $-2 < c < 2$, for which $f'(c) = 0$, which of the following statements must be true?

(A) For $-2 < k < 2$, $f'(k) > 0$.

(B) For $-2 < k < 2$, $f'(k) < 0$.

(C) For $-2 < k < 2$, $f'(k)$ exists.

(D) For $-2 < k < 2$, $f'(k)$ exists, but f' is not continuous.

(E) For some k, where $-2 < k < 2$, $f'(k)$ does not exist.

6.

The function f is continuous on the closed interval [2, 4] and twice differentiable on the open interval (2, 4). If $f'(3) = 2$ and $f''(x) < 0$ on the open interval $(2, 4)$, which of the following could be a table of values for f?

(A)

x	$f(x)$
2	2.5
3	5
4	6.5

(B)

x	$f(x)$
2	2.5
3	5
4	7

(C)

x	$f(x)$
2	3
3	5
4	6.5

(D)

x	$f(x)$
2	3
3	5
4	7

(E)

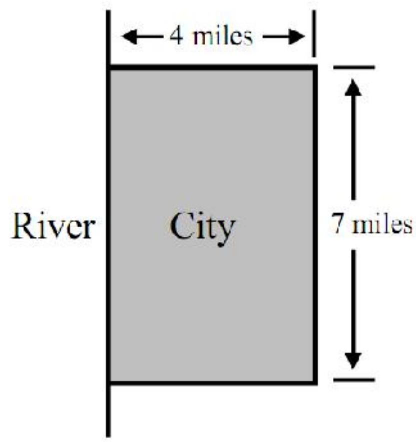
x	$f(x)$
2	3.5
3	5
4	7.5

7.

What is the average value of $y = \dfrac{\cos x}{x^2 + x + 2}$ on the closed interval $[-1, 3]$?

(A) -0.085 (B) 0.090 (C) 0.183 (D) 0.244 (E) 0.732

8.



A city located beside a river has a rectangular boundary as shown in the figure above. The population density of the city at any point along a strip x miles from the river's edge is $f(x)$ persons per square mile. Which of the following expressions gives the population of the city?

(A) $\int_0^4 f(x)\,dx$

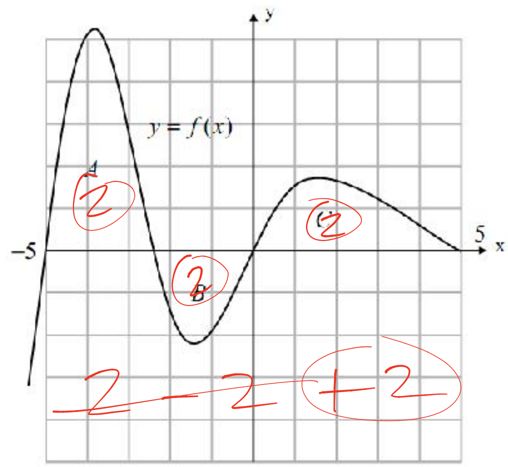
(B) $7\int_0^4 f(x)\,dx$

(C) $28\int_0^4 f(x)\,dx$

(D) $\int_0^7 f(x)\,dx$

(E) $4\int_0^7 f(x)\,dx$

9.



The regions A, B, and C in the figure above are bounded by the graph of the function f and the x-axis. If the area of each region is 2, what is the value of $\int_{5}^{-5} \left(f(x)-5\right)dx$?

(A) 24 (B) 36 (C) 48 (D) 60 (E) 72

$\int_5^{-5} f(x)dx - \int_5^{-5} 5\,dx$

$-\int_{-5}^{5} f(x)dx + \boxed{\int_{-5}^{5} 5\,dx}$ + -5 10 5

$-(2)$ + $5x\Big|_{-5}^{5}$

$25-(-25)$

50

10.

The rate of change of the altitude of a hot-air balloon is given by $r(t)=t^3-4t^2+6$ for $0 \le t \le 8$. Which of the following expressions gives the change in altitude of the balloon during the time the altitude is decreasing?

A) $\int_{1.572}^{3.514} r(t)dt$

B) $\int_{0}^{8} r(t)dt$

C) $\int_{0}^{2.667} r(t)dt$

D) $\int_{1.572}^{3.514} r'(t)dt$

E) $\int_{0}^{2.667} r'(t)dt$

*

11. (2013, AB-1)

On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday ($t = 0$), the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

$G'(5) = -24.587$ tons/hr^2, At $t=5$ hrs, the rate at which gravel arrives is decreasing by 24.587 tons/hr per hour.

(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday. Gravel $= \int_0^8 G(t)\,dt = 825.551$ tons

(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

(c) arrive: $G(5) = 98.140$ Tons/hr

processed: 100 tons/hr

Since $100 > 98.140$, the amount of gravel is decreasing at $t = 5$hrs.

$G(5) - 100 = -1.86 < 0$

so

(d) Let $R(t)$ = grrravel at t

$R(t) = 500 + \int_0^t G(q)\,dq - 100t$

$R(0) = 500$ tons

$R(8) = 525.551$ tons

$R(A) = 635.376$ ton

$R'(t) = 0$

$G(t) - 100 = 0$

$t = 4.923 = A$

so, Max amt of gravel is 635.376 tons.

as/hr

Question 1

On a certain workday, the rate, in tons per hour, at which unprocessed gravel arrives at a gravel processing plant is modeled by $G(t) = 90 + 45\cos\left(\frac{t^2}{18}\right)$, where t is measured in hours and $0 \le t \le 8$. At the beginning of the workday $(t = 0)$, the plant has 500 tons of unprocessed gravel. During the hours of operation, $0 \le t \le 8$, the plant processes gravel at a constant rate of 100 tons per hour.

(a) Find $G'(5)$. Using correct units, interpret your answer in the context of the problem.

(b) Find the total amount of unprocessed gravel that arrives at the plant during the hours of operation on this workday.

(c) Is the amount of unprocessed gravel at the plant increasing or decreasing at time $t = 5$ hours? Show the work that leads to your answer.

(d) What is the maximum amount of unprocessed gravel at the plant during the hours of operation on this workday? Justify your answer.

(a) $G'(5) = -24.588$ (or -24.587)

The rate at which gravel is arriving is decreasing by 24.588 (or 24.587) tons per hour per hour at time $t = 5$ hours.

$2 : \begin{cases} 1 : G'(5) \\ 1 : \text{interpretation with units} \end{cases}$

(b) $\int_0^8 G(t)\,dt = 825.551$ tons

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) $G(5) = 98.140764 < 100$

At time $t = 5$, the rate at which unprocessed gravel is arriving is less than the rate at which it is being processed. Therefore, the amount of unprocessed gravel at the plant is decreasing at time $t = 5$.

$2 : \begin{cases} 1 : \text{compares } G(5) \text{ to } 100 \\ 1 : \text{conclusion} \end{cases}$

(d) The amount of unprocessed gravel at time t is given by $A(t) = 500 + \int_0^t (G(s) - 100)\,ds.$

$A'(t) = G(t) - 100 = 0 \Rightarrow t = 4.923480$

t	$A(t)$
0	500
4.92348	635.376123
8	525.551089

The maximum amount of unprocessed gravel at the plant during this workday is 635.376 tons.

$3 : \begin{cases} 1 : \text{considers } A'(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

12. (2013, AB-2) (Calculator Permitted)

A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

(a) $\text{Speed} = |v(t)| = 2$

$|v(t)| - 2 = 0$

$t = 3.127$

$t = 3.43$

(b) $s(t) = 10 + \int_0^t v(x)\,dx$

$s(5) = 10 + \int_0^5 v(x)\,dx$

$= -9.207$

(c) $v(t) = 0$

$v(4) = y1(4) = -\_\_\_\ldots < 0$

$v'(4) = \boxed{\text{MATH } 8} = -\_\_\_\ldots < 0$

So, At $t=4$, speed is inc

Question 2

A particle moves along a straight line. For $0 \le t \le 5$, the velocity of the particle is given by $v(t) = -2 + \left(t^2 + 3t\right)^{6/5} - t^3$, and the position of the particle is given by $s(t)$. It is known that $s(0) = 10$.

(a) Find all values of t in the interval $2 \le t \le 4$ for which the speed of the particle is 2.

(b) Write an expression involving an integral that gives the position $s(t)$. Use this expression to find the position of the particle at time $t = 5$.

(c) Find all times t in the interval $0 \le t \le 5$ at which the particle changes direction. Justify your answer.

(d) Is the speed of the particle increasing or decreasing at time $t = 4$? Give a reason for your answer.

(a) Solve $\lvert v(t) \rvert = 2$ on $2 \le t \le 4$. $t = 3.128$ (or 3.127) and $t = 3.473$	$2 : \begin{cases} 1 : \text{considers } \lvert v(t) \rvert = 2 \\ 1 : \text{answer} \end{cases}$
(b) $s(t) = 10 + \int_0^t v(x)\,dx$ $s(5) = 10 + \int_0^5 v(x)\,dx = -9.207$	$2 : \begin{cases} 1 : s(t) \\ 1 : s(5) \end{cases}$
(c) $v(t) = 0$ when $t = 0.536033,\ 3.317756$ $v(t)$ changes sign from negative to positive at time $t = 0.536033$. $v(t)$ changes sign from positive to negative at time $t = 3.317756$. Therefore, the particle changes direction at time $t = 0.536$ and time $t = 3.318$ (or 3.317).	$3 : \begin{cases} 1 : \text{considers } v(t) = 0 \\ 2 : \text{answers with justification} \end{cases}$
(d) $v(4) = -11.475758 < 0,\ a(4) = v'(4) = -22.295714 < 0$ The speed is increasing at time $t = 4$ because velocity and acceleration have the same sign.	2 : conclusion with reason