1. B
2. E
3. B
4. D
5. E
6. B
7. D
8. D
9. C
10. D

9

AB Review 09 Calculator Permitted

1. A particle moves along the y-axis with velocity given by $v(t)=3t^2+6t$ for time $t\geq 0$. If the particle is at position $y=2$ at time $t=0$, what is the position of the particle at $t=1$?

   (A) 4 (B) 6 (C) 9 (D) 11 (E) 12

2. If $f(x)=\cos(3x)$, then $f'\left(\frac{\pi}{9}\right)=$

   (A) $\frac{3\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $-\frac{\sqrt{3}}{2}$ (D) $-\frac{3}{2}$ (E) $-\frac{3\sqrt{3}}{2}$

3. If $f(x)=\begin{cases}\dfrac{\sqrt{2x+5}-\sqrt{x+7}}{x-2}, & x\neq 2\\ k, & x=2\end{cases}$, and if f is continuous at $x=2$, then $k=$

   (A) 0 (B) $\frac{1}{6}$ (C) $\frac{1}{3}$ (D) 1 (E) $\frac{7}{5}$

$0/0$

$$\lim_{x\to 2}\frac{\sqrt{2x+5}-\sqrt{x+7}}{(x-2)}\left(\frac{\sqrt{2x+5}+\sqrt{x+7}}{\sqrt{2x+5}+\sqrt{x+7}}\right)$$

$$\lim_{x\to 2}\frac{2x+5-x+7}{(x-2)(\sqrt{2x-5}+\sqrt{x+7})}$$

$$\lim_{x\to 2}\frac{\cancel{(x-2)}}{\cancel{(x-2)}(\sqrt{2x-5}+\sqrt{x+7})}$$

$$\frac{1}{6}$$

4. $\displaystyle\int_0^8 \frac{dx}{\sqrt{1+x}} =$

(A) 1 (B) $\frac{3}{2}$ (C) 2 (D) 4 (E) 6

5. If $3x^2 + 2xy + y^2 = 2$, then the value of $\frac{dy}{dx}$ at $x = 1$ is

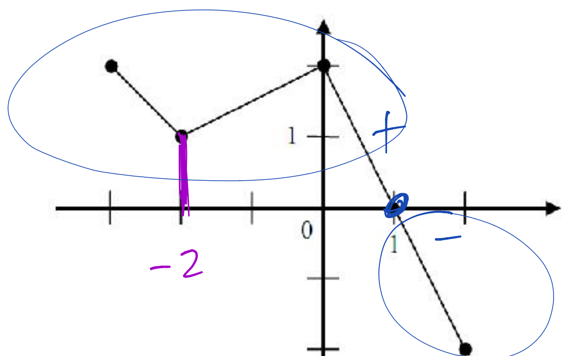
(A) -2 (B) (C) 2 (D) 4 (E) not defined

6. What is $\displaystyle\lim_{h\to 0} \frac{8\left(\frac{1}{2}+h\right)^8 - 8\left(\frac{1}{2}\right)^8}{h}$?

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) DNE (E) Cannot be determined from info given

7. For what value of k will $x + \frac{k}{x}$ have a relative maximum at $x = -2$?

(A) -4 (B) -2 (C) 2 (D) 4 (E) None of these



Graph of f

$g'(x) = f(x) = 0$

8. The graph of the piecewise linear function f is shown in the figure above. If $g(x) = \int_{-2}^{x} f(t)\,dt$, which of the following values is greatest?

(A) $g(-3)$ (B) $g(-2)$ (C) $g(0)$ (D) $g(1)$ (E) $g(2)$

9. When the area in square units of an expanding circle is increasing twice as fast as its radius in linear units, the radius is

(A) $\frac{1}{4\pi}$ (B) $\frac{1}{4}$ (C) $\frac{1}{\pi}$ (D) 1 (E) π

$\boxed{\frac{dA}{dt} = \boxed{2\frac{dr}{dt}}}$

$A = \pi r^2$

$\frac{d}{dt}: \boxed{\frac{dA}{dt} = \boxed{2\pi r \frac{dr}{dt}}}$

$r = ?$

$\cancel{2}\cancel{\frac{dr}{dt}} = \cancel{2}\pi r \cancel{\frac{dr}{dt}}$

$1 = \pi r$

$\boxed{r = \frac{1}{\pi}}$

10. If $f(x) = e^{(2/x)}$, then $f'(x) =$

(A) $2e^{(2/x)} \ln x$ (B) $e^{(2/x)}$ (C) $e^{\left(-2/x^2\right)}$ (D) $-\frac{2}{x^2}e^{(2/x)}$ (E) $-2x^2e^{(2/x)}$

w/o cal

11. (2000, AB-4) Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

(a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

$$H_2O = \int_0^3 1\cdot(t+1)^{1/2}\,dt$$

$$\frac{2}{3}(t+1)^{3/2}\Big|_0^3$$

$$\frac{2}{3}\left[4^{3/2} - 1^{3/2}\right] \text{gallons}, \quad \frac{2}{3}(7) = \boxed{\frac{14}{3}\text{gal}}$$

(b) How many gallons of water are in the tank at time $t = 3$ minutes?

$$H_2O \text{ at } t=3 = 30 + 3(8) - \frac{14}{3}\text{ gal}$$

(c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

$$A(t) = 30 + 8t - \int_0^t \sqrt{z+1}\,dz$$

$$A'(t) = 8 - \sqrt{t+1} = 0$$

$$\sqrt{t+1} = 8$$

$$t+1 = 64$$

$$\boxed{t = 63\text{min}}$$

(d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

$A(63)$

EVT

$$A(0) = 30$$

$$A(120) = 30 + 8(120) - \int_0^{120} (z+1)^{1/2}\,dz$$

$$= 30 + 960 - \frac{2}{3}(z+1)^{3/2}\Big|_0^{120}$$

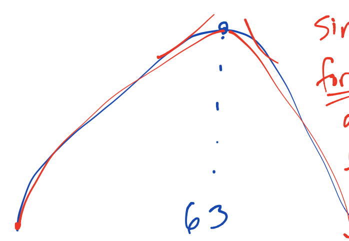
$$= 990 - \frac{2}{3}\left[121^{3/2} - 1^{3/2}\right]$$

$$= 990 - \frac{2}{3}(11^3 - 1)$$

$$A(63) = 30 + 8(63) - \int_0^{63} (z+1)^{1/2}\,dz$$

$$= 30 + 504 - \frac{2}{3}(z+1)^{3/2}\Big|_0^{63}$$

$$= 534 - \frac{2}{3}\left[8^3 - 1\right]$$



Since $A'(t) > 0$ for all $0 < t < 63$ and $A'(t) < 0$ for all $63 < t < 120$ min the max occurs at $t = 63$ min.

So the max is at $t = 63$ min

Water is pumped into an underground tank at a constant rate of 8 gallons per minute. Water leaks out of the tank at the rate of $\sqrt{t+1}$ gallons per minute, for $0 \le t \le 120$ minutes. At time $t = 0$, the tank contains 30 gallons of water.

(a) How many gallons of water leak out of the tank from time $t = 0$ to $t = 3$ minutes?

(b) How many gallons of water are in the tank at time $t = 3$ minutes?

(c) Write an expression for $A(t)$, the total number of gallons of water in the tank at time t.

(d) At what time t, for $0 \le t \le 120$, is the amount of water in the tank a maximum? Justify your answer.

(a) Method 1: $\displaystyle\int_0^3 \sqrt{t+1}\,dt = \frac{2}{3}(t+1)^{3/2}\Big|_0^3 = \frac{14}{3}$

– or –

Method 2: $L(t)$ = gallons leaked in first t minutes

$$\frac{dL}{dt} = \sqrt{t+1}; \quad L(t) = \frac{2}{3}(t+1)^{3/2} + C$$

$$L(0) = 0; \quad C = -\frac{2}{3}$$

$$L(t) = \frac{2}{3}(t+1)^{3/2} - \frac{2}{3}; \quad L(3) = \frac{14}{3}$$

Method 1:

$$3\begin{cases} 2: \text{definite integral} \\ \quad 1: \text{limits} \\ \quad 1: \text{integrand} \\ 1: \text{answer} \end{cases}$$

– or –

Method 2:

$$3\begin{cases} 1: \text{antiderivative with } C \\ 1: \text{solves for } C \text{ using } L(0) = 0 \\ 1: \text{answer} \end{cases}$$

(b) $30 + 8 \cdot 3 - \dfrac{14}{3} = \dfrac{148}{3}$

1 : answer

(c) Method 1:

$$A(t) = 30 + \int_0^t \left(8 - \sqrt{x+1}\right)dx$$
$$= 30 + 8t - \int_0^t \sqrt{x+1}\,dx$$

– or –

Method 2:

$$\frac{dA}{dt} = 8 - \sqrt{t+1}$$

$$A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + C$$

$$30 = 8(0) - \frac{2}{3}(0+1)^{3/2} + C; \quad C = \frac{92}{3}$$

$$A(t) = 8t - \frac{2}{3}(t+1)^{3/2} + \frac{92}{3}$$

Method 1:

$$2\begin{cases} 1: 30 + 8t \\ 1: -\displaystyle\int_0^t \sqrt{x+1}\,dx \end{cases}$$

– or –

Method 2:

$$2\begin{cases} 1: \text{antiderivative with } C \\ 1: \text{answer} \end{cases}$$

(d) $A'(t) = 8 - \sqrt{t+1} = 0$ when $t = 63$

$A'(t)$ is positive for $0 < t < 63$ and negative for $63 < t < 120$. Therefore there is a maximum at $t = 63$.

$$3\begin{cases} 1: \text{sets } A'(t) = 0 \\ 1: \text{solves for } t \\ 1: \text{justification} \end{cases}$$

when $y=\frac{1}{2}x^2$:

$x\left(\frac{1}{2}x^2\right)^2 - x^3\left(\frac{1}{2}x^2\right) = 6$

$\frac{1}{4}x^5 - \frac{1}{2}x^5 = 6$

$-\frac{1}{4}x^5 = 6$

$x^5 = -24$

$\boxed{x = \sqrt[5]{-24}}$

12. (2000, AB/BC-5) Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$.

$\frac{d}{dx}\left[xy^2 - x^3y\right] = \frac{d}{dx}[6]$

$1 \cdot y^2 + x \cdot 2y\frac{dy}{dx} - 3x^2y - x^3\frac{dy}{dx} = 0$

$\frac{dy}{dx}(2xy - x^3) = 3x^2y - y^2$

$\boxed{\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}}$

$\left.\frac{dy}{dx}\right|_{(1,3)} = \frac{9-9}{6-1} = \frac{0}{5} = 0$

$\left.\frac{dy}{dx}\right|_{(1,-2)} = \frac{-6-4}{-4-1} = \frac{-10}{-5} = 2$

(b) Find all the points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

$xy^2 - x^3y = 6$

when $x=1$:

$y^2 - y = 6$

$y^2 - y - 6 = 0$

$(y-3)(y+2) = 0$

$\boxed{y = 3, y - 2}$

pt 1: $(1,3)$

$\left.\frac{dy}{dx}\right|_{(1,3)} = 0$

$\boxed{\text{eq: } y = 3 + 0(x-1)}$

pt 2: $(1,-2)$

$\left.\frac{dy}{dx}\right|_{(1,-2)} = 2$

$\boxed{\text{eq: } y = -2 + 2(x-1)}$

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

$\frac{dy}{dx} = \frac{a}{b}$

Horz tangent $a = 0$

Vert tangent $b = 0$

$2xy - x^3 = 0$

$2xy = x^3$

$y = \frac{x^3}{2x}$

$\boxed{y = \frac{1}{2}x^2}$

Consider the curve given by $xy^2 - x^3y = 6$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{3x^2y - y^2}{2xy - x^3}$.

(b) Find all points on the curve whose x-coordinate is 1, and write an equation for the tangent line at each of these points.

(c) Find the x-coordinate of each point on the curve where the tangent line is vertical.

(a) $y^2 + 2xy\dfrac{dy}{dx} - 3x^2y - x^3\dfrac{dy}{dx} = 0$

$$\frac{dy}{dx}\left(2xy - x^3\right) = 3x^2y - y^2$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$$

$2\begin{cases} 1: \text{implicit differentiation} \\ 1: \text{verifies expression for } \dfrac{dy}{dx} \end{cases}$

(b) When $x = 1$, $y^2 - y = 6$

$$y^2 - y - 6 = 0$$

$$(y-3)(y+2) = 0$$

$$y = 3,\ y = -2$$

At $(1,3)$, $\dfrac{dy}{dx} = \dfrac{9-9}{6-1} = 0$

Tangent line equation is $y = 3$

At $(1,-2)$, $\dfrac{dy}{dx} = \dfrac{-6-4}{-4-1} = \dfrac{-10}{-5} = 2$

Tangent line equation is $y + 2 = 2(x-1)$

$4\begin{cases} 1: \ y^2 - y = 6 \\ 1: \ \text{solves for } y \\ 2: \ \text{tangent lines} \end{cases}$

Note: 0/4 if not solving an equation of the form $y^2 - y = k$

(c) Tangent line is vertical when $2xy - x^3 = 0$

$x\left(2y - x^2\right) = 0$ gives $x = 0$ or $y = \dfrac{1}{2}x^2$

There is no point on the curve with x-coordinate 0.

When $y = \dfrac{1}{2}x^2$, $\dfrac{1}{4}x^5 - \dfrac{1}{2}x^5 = 6$

$$-\frac{1}{4}x^5 = 6$$

$$x = \sqrt[5]{-24}$$

$3\begin{cases} 1: \ \text{sets denominator of } \dfrac{dy}{dx} \text{ equal to } 0 \\ 1: \ \text{substitutes } y = \dfrac{1}{2}x^2 \text{ or } x = \pm\sqrt{2y} \\ \quad\ \text{into the equation for the curve} \\ 1: \ \text{solves for } x\text{-coordinate} \end{cases}$