1.	C	6.	C	5
2.	B	7.	B	
3.	C	8.	E	
4.	D	9.	B	
5.	A	10.	A	

AB Review 05, Use your calculator ONLY on #2.

1. If $\frac{dy}{dx} = y\sec^2 x$ and $y = 5$ when $x = 0$, then $y =$

   (A) $e^{\tan x} + 4$ (B) $e^{\tan x} + 5$ (C) $5e^{\tan x}$ (D) $\tan x + 5$ (E) $\tan x + 5e^x$

2. (Calculator Permitted) The average value of the function $f(x) = e^{-x^2}$ on the closed interval $[-1,1]$ is

   (A) 0.70 (B) 0.75 (C) 0.80 (D) 0.85 (E) 0.90

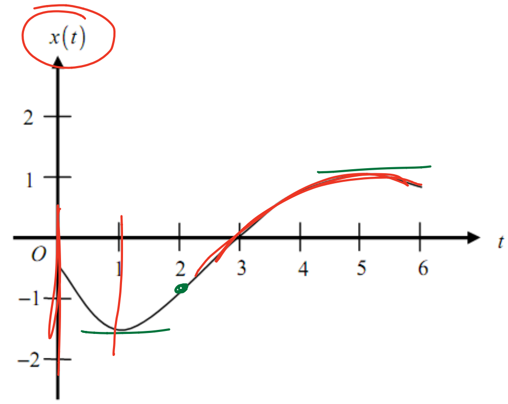
3. If $\frac{dy}{dx} = \frac{1}{x}$, then the average rate of change of y with respect to x on the closed interval $[1,4]$ is

   (A) $-\frac{1}{4}$ (B) $\frac{1}{2}\ln 2$ (C) $\frac{2}{3}\ln 2$ (D) $\frac{2}{5}$ (E) 2

4. Let f be a function with a second derivative given by $f''(x) = x^2(x-3)(x-6)$. What are the x-coordinates of the points of inflection of the graph of f?

   (A) 0 only (B) 3 only (C) 0 and 6 only (D) 3 and 6 only (E) 0, 3, and 6

5. A particle moves along a straight line. The graph of the particle's position $x(t)$ at time t is shown at right for $0<t<6$. The graph has horizontal tangents at $t=1$ and $t=5$ and a point of inflection at $t=2$. For what values of t is the velocity of the particle increasing?



(A) $0<t<2$ (B) $1<t<5$ (C) $2<t<6$
(D) $3<t<5$ only (E) $1<t<2$ and $5<t<6$

$a(t)=v'(t)>0$

vel → slopes
accel → ~~cocavity~~ curvature

$\boxed{\text{CCUP}}$

6. $\lim\limits_{h\to 0}\dfrac{3\left(\frac{1}{2}+h\right)^5-3\left(\frac{1}{2}\right)^5}{h}=$

(A) 0 (B) 1 (C) $\dfrac{15}{16}$ (D) the limit does not exist (E) the limit cannot be determined

7. If f is continuous for $a\le x\le b$ and differentiable for $a<x<b$, which of the following could be false?

(A) $f'(c)=\dfrac{f(b)-f(a)}{b-a}$ for some c such that $a<c<b$.
(B) $f'(c)=0$ for some c such that $a<c<b$.
(C) f has a minimum value on $a\le x\le b$.
(D) f has a maximum value on $a\le x\le b$.
(E) $\int_a^b f(x)\,dx$ exists

8. $\int_1^e \left(\frac{x^2-1}{x}\right)dx = \int_1^e \left(\frac{x^2}{x}-\frac{1}{x}\right)dx = \int_1^e \left(x-\frac{1}{x}\right)dx = \frac{1}{2}x^2 - \ln|x|\Big|_1^e = \left(\frac{e^2}{2}-1\right)-\left(\frac{1}{2}-0\right)$

(A) $e-\frac{1}{e}$ (B) e^2-e (C) $\frac{e^2}{2}-e+\frac{1}{2}$ (D) e^2-2 (E) $\boxed{\frac{e^2}{2}-\frac{3}{2}}$

$\ln e = 1$

$\ln 1 = 0$

$\frac{e^2}{2}-\frac{3}{2}$

$\frac{e^2-3}{2}$

$$f(x)=\begin{cases} cx+d & \text{for } x \le 2 \\ x^2-cx & \text{for } x > 2 \end{cases}$$

9. Let f be the function defined above, where c and d are constants. If f is differentiable at $x=2$, what is the value of $c+d$?

(A) -4 (B) -2 (C) 0 (D) 2 (E) 4

continuity

$2c+d = 4-2c$

$4+d = 4-4$

$\boxed{d=-4}$

$f'(x) = \begin{cases} c, & x \le 2 \\ 2x-c, & x>2 \end{cases}$

slopes

$c = 4-c$

$2c = 4$

$\boxed{c=2}$

10. Determine the y-intercept of the tangent line to the curve $y=\sqrt{x^2+24}$ at $x=5$.

(A) $\frac{24}{7}$ (B) $-\frac{72}{49}$ (C) $\frac{48}{49}$ (D) $\frac{44}{7}$ (E) $\frac{88}{49}$

$(x^2+24)^{1/2}$

$y(5)=7$

$y'(x) = \cancel{\tfrac{1}{2}}(x^2+24)^{-1/2}(\cancel{2}x)$

$y'(x) = \frac{x}{\sqrt{x^2+24}}$

$y'(5) = \frac{5}{7} = m$

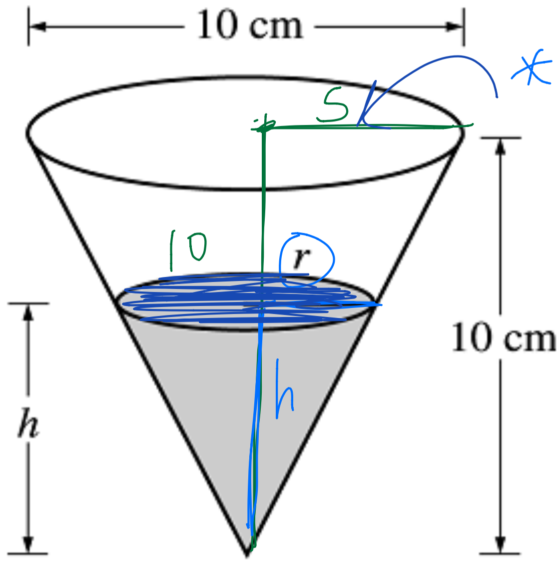
eq: $y = 7+\frac{5}{7}(x-5)$

$y(0) = 7+\frac{5}{7}(-5)$

$= 7-\frac{25}{7}$

$= \frac{49}{7}-\frac{25}{7}$

$= \boxed{\frac{24}{7}}$



**

$\frac{10}{5} = \frac{h}{r}$

$2 = \frac{h}{r}$

$\boxed{h = 2r}$

$r = \frac{1}{2}h$

$\frac{dh}{dt} = 2\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{1}{2}\frac{dh}{dt}$

11. (2002, AB-5) A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm, and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $-\frac{3}{10}$ cm/hr.

$\boxed{\frac{dh}{dt} = -\frac{3}{10}}$

(a) Find the volume V of water in the container when $h = 5$ cm. Indicate units of measure.

$\boxed{V = \frac{\pi}{3}r^2h}$

$V = \frac{\pi}{3}r^2(2r)$

$\boxed{V = \frac{2\pi}{3}r^3}$

$V = \frac{\pi}{3}\left(\frac{1}{2}h\right)^2 h$

$\boxed{V = \frac{\pi}{12}h^3}$

$V(5) = \frac{\pi}{12}(5^3)\text{ cm}^3$

When $h=5$: $V = \frac{\pi}{3}\left(\frac{5}{2}\right)^2(5)\text{ cm}^3$

(b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.

$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{\pi}{12}h^3\right]$

$\boxed{\frac{dV}{dt} = \frac{3\pi}{12}h^2 \cdot \frac{dh}{dt}}$ rate eq.

When $h=5$:

$\frac{dV}{dt} = \left(\frac{3\pi}{12}\right)(5^2)\left(-\frac{3}{10}\right)\text{ cm}^3/\text{hr}$

(c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

Show

$\boxed{\frac{dV}{dt} = k \cdot \pi r^2}$

$V = \frac{2\pi}{3}r^3$

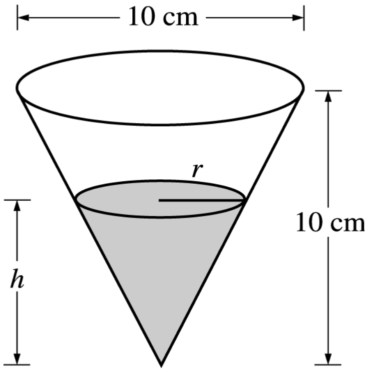
$\frac{d}{dt}[V] = \frac{d}{dt}\left[\frac{2\pi}{3}r^3\right]$

$\frac{dV}{dt} = 2\pi r^2 \cdot \frac{dr}{dt}$

$\frac{dV}{dt} = \left(2\frac{dr}{dt}\right) \cdot \pi r^2$

$k = 2\frac{dr}{dt} = \frac{dh}{dt} = -\frac{3}{10}\text{ cm/hr}$

Question 5



A container has the shape of an open right circular cone, as shown in the figure above. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth h is changing at the constant rate of $\frac{-3}{10}$ cm/hr.

(The volume of a cone of height h and radius r is given by $V = \frac{1}{3}\pi r^2 h.$)

(a) Find the volume V of water in the container when $h = 5$ cm. Indicate units of measure.

(b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 5$ cm. Indicate units of measure.

(c) Show that the rate of change of the volume of water in the container due to evaporation is directly proportional to the exposed surface area of the water. What is the constant of proportionality?

(a) When $h = 5$, $r = \frac{5}{2}$; $V(5) = \frac{1}{3}\pi\left(\frac{5}{2}\right)^2 5 = \frac{125}{12}\pi$ cm^3

1 : V when $h = 5$

(b) $\frac{r}{h} = \frac{5}{10}$, so $r = \frac{1}{2}h$

$$V = \frac{1}{3}\pi\left(\frac{1}{4}h^2\right)h = \frac{1}{12}\pi h^3;\ \frac{dV}{dt} = \frac{1}{4}\pi h^2\frac{dh}{dt}$$

$$\left.\frac{dV}{dt}\right|_{h=5} = \frac{1}{4}\pi(25)\left(-\frac{3}{10}\right) = -\frac{15}{8}\pi \ \ \text{cm}^3/\text{hr}$$

OR

$$\frac{dV}{dt} = \frac{1}{3}\pi\left(r^2\frac{dh}{dt} + 2rh\frac{dr}{dt}\right);\ \frac{dr}{dt} = \frac{1}{2}\frac{dh}{dt}$$

$$\left.\frac{dV}{dt}\right|_{h=5,r=\frac{5}{2}} = \frac{1}{3}\pi\left(\left(\frac{25}{4}\right)\left(-\frac{3}{10}\right) + 2\left(\frac{5}{2}\right)5\left(-\frac{3}{20}\right)\right)$$

$$= -\frac{15}{8}\pi \ \text{cm}^3/\text{hr}$$

5 {
- 1 : $r = \frac{1}{2}h$ in (a) or (b)
- 1 : { V as a function of one variable in (a) or (b) OR $\frac{dr}{dt}$ }
- 2 : $\frac{dV}{dt}$
  < -2 > chain rule or product rule error
- 1 : evaluation at $h = 5$

(c) $\frac{dV}{dt} = \frac{1}{4}\pi h^2\frac{dh}{dt} = -\frac{3}{40}\pi h^2$

$$= -\frac{3}{40}\pi(2r)^2 = -\frac{3}{10}\pi r^2 = -\frac{3}{10}\cdot\text{area}$$

The constant of proportionality is $-\frac{3}{10}$.

2 {
- 1 : shows $\frac{dV}{dt} = k\cdot\text{area}$
- 1 : identifies constant of proportionality

---

units of cm^3 in (a) and cm^3/hr in (b)

1 : correct units in (a) and (b)

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

12. (2002, AB-6) Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

cc up, slopes inc

(a) Evaluate $\int_0^{1.5} \left(3f'(x)+4\right)dx$. Show the work that leads to your answer.

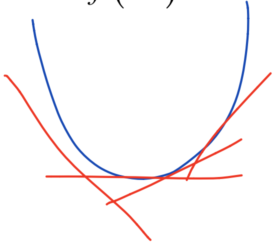
$3\cdot f(x) + 4x \Big|_0^{1.5}$

$\left(3f(1.5) + 4(1.5)\right) - \left(3\cdot f(0) + 0\right)$

$3(-1) + 4(1.5) - 3(-7)$

(b) Write an equation of the line tangent to the graph of f at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.

$f'' > 0$



Since f is concave up on $-1.5 \le x \le 1.5$, $\mathcal{L}(1.2)$ underapproximates $f(1.2)$

$f(1) = -4$

$f'(1) = 5$

eq: $y = \boxed{-4 + 5(x-1) = \mathcal{L}(x)}$

$f(1.2) \approx \mathcal{L}(1.2) = -4 + 5(1.2-1)$

(c) Find a positive real number r having the property that there must exist a value c with $0 < c < 0.5$ and $f''(x) = r$. Give a reason for your answer.

(d) Let g be the function given by $g(x) = \begin{cases} 2x^2 - x - 7 & \text{for } x < 0 \\ 2x^2 + x - 7 & \text{for } x \ge 0 \end{cases}$. The graph of g passes through each of the points $(x, f(x))$ given in the table above. Is it possible that f and g are the same function? Give a reason for your answer.

Question 6

x	-1.5	-1.0	-0.5	0	0.5	1.0	1.5
$f(x)$	-1	-4	-6	-7	-6	-4	-1
$f'(x)$	-7	-5	-3	0	3	5	7

Let f be a function that is differentiable for all real numbers. The table above gives the values of f and its derivative f' for selected points x in the closed interval $-1.5 \le x \le 1.5$. The second derivative of f has the property that $f''(x) > 0$ for $-1.5 \le x \le 1.5$.

(a) Evaluate $\int_0^{1.5} \left(3f'(x) + 4\right) dx$. Show the work that leads to your answer.

(b) Write an equation of the line tangent to the graph of f at the point where $x = 1$. Use this line to approximate the value of $f(1.2)$. Is this approximation greater than or less than the actual value of $f(1.2)$? Give a reason for your answer.

(c) Find a positive real number r having the property that there must exist a value c with $0 < c < 0.5$ and $f''(c) = r$. Give a reason for your answer.

(d) Let g be the function given by $g(x) = \begin{cases} 2x^2 - x - 7 & \text{for } x < 0 \\ 2x^2 + x - 7 & \text{for } x \ge 0. \end{cases}$

The graph of g passes through each of the points $\left(x, f(x)\right)$ given in the table above. Is it possible that f and g are the same function? Give a reason for your answer.

(a) $\int_0^{1.5} \left(3f'(x) + 4\right) dx = 3\int_0^{1.5} f'(x)\,dx + \int_0^{1.5} 4\,dx$

$= 3f(x) + 4x\Big|_0^{1.5} = 3(-1 - (-7)) + 4(1.5) = 24$

$2 \begin{cases} 1: \text{ antiderivative} \\ 1: \text{ answer} \end{cases}$

(b) $y = 5(x - 1) - 4$

$f(1.2) \approx 5(0.2) - 4 = -3$

The approximation is less than $f(1.2)$ because the graph of f is concave up on the interval $1 < x < 1.2$.

$3 \begin{cases} 1: \text{ tangent line} \\ 1: \text{ computes } y \text{ on tangent line at } x = 1.2 \\ 1: \text{ answer with reason} \end{cases}$

(c) By the Mean Value Theorem there is a c with $0 < c < 0.5$ such that

$$f''(c) = \frac{f'(0.5) - f'(0)}{0.5 - 0} = \frac{3 - 0}{0.5} = 6 = r$$

$2 \begin{cases} 1: \text{ reference to MVT for } f' \text{ (or differentiability of } f'\text{)} \\ 1: \text{ value of } r \text{ for interval } 0 \le x \le 0.5 \end{cases}$

(d) $\lim_{x \to 0^-} g'(x) = \lim_{x \to 0^-} (4x - 1) = -1$

$\lim_{x \to 0^+} g'(x) = \lim_{x \to 0^+} (4x + 1) = +1$

Thus g' is not continuous at $x = 0$, but f' is continuous at $x = 0$, so $f \ne g$.

OR

$g''(x) = 4$ for all $x \ne 0$, but it was shown in part (c) that $f''(c) = 6$ for some $c \ne 0$, so $f \ne g$.

$2 \begin{cases} 1: \text{ answers "no" with reference to } g' \text{ or } g'' \\ 1: \text{ correct reason} \end{cases}$