Korpi's Last-Minute AP Things to Not Forget to Remember

- Integration by *u*-substitution
  - When your "inside" function is linear, and it's derivative is "off" by more than a constant.
  - Let $u = \text{"inside" function}$ and change all variables from *x*, *dx* to *u*, *du*.
  - Example: $\int_0^1 \frac{2x}{\sqrt{x+2}}dx$

    $u = x+2$, $x = u-2$

    $\frac{du}{dx} = 1$, $dx = du$

    $\int_2^3 \frac{2(u-2)}{\sqrt{u}}du$

    $2\int_2^3 (u-2)u^{-1/2}\,du$

    $2\int_2^3 \left(u^{1/2} - 2u^{-1/2}\right)du$

    $2\left[\frac{2}{3}(x+2)^{3/2} - 2(2)(x+2)^{1/2}\right]\Big|_0^1$

- Logarithmic Differentiation (LOG DIFF)
  - When you are taking the derivative of a variable function raised to a power of a variable function.
  - Take the natural log of both sides, differentiate implicitly, then resolve for *y*.
  - Example: $\frac{d}{dx}\left[x^{\sin x}\right] =$

    $y = x^{\sin x}$

    $\ln y = (\sin x)\ln x$

    $\frac{d}{dx}\cdot \frac{1}{y}\frac{dy}{dx} = \cos x \cdot \ln x + \sin x \cdot \frac{1}{x}$

    $\frac{dy}{dx} = \left[\cos x \cdot \ln x + \frac{\sin x}{x}\right]x^{\sin x}$

- Integral of Trig functions
  - $\int \sin x\,dx = -\cos x + C$ $\quad$ $\int \cos x\,dx = \sin x + C$ $\quad$ $\int \tan x\,dx = -\ln|\cos x| + C$ $\quad$ $\int \cot x\,dx = \ln|\sin x| + C$ $\left(\frac{\cos x}{\sin x}\right)$
  - $\int \sec x\,dx = \ln|\sec x + \tan x| + C$ $\quad$ $\int \csc x\,dx = -\ln|\csc x + \cot x| + C$ $\quad$ $\int \sec^2 x\,dx = \tan x + C$ $\quad$ $\int \sec x \tan x\,dx = \sec x + C$

    $\cos^2 x + \sin^2 x = 1$

    $1 + \tan^2 x = \sec^2 x$

    $1 + \cot^2 x = \csc^2 x$

  - Examples: $\int \tan^2 x\,dx =$ $\quad$ $\int (\sec^2 x - 1)\,dx$ $\quad$ $\tan x - x + C$

    $\int \frac{\cot(\sqrt{x})}{\sqrt{x}}dx =$ $\quad$ $\frac{1}{2}x^{-1/2} = \frac{1}{2}\cdot\frac{1}{\sqrt{x}}$ $\quad$ $2\ln|\sin(\sqrt{x})| + C$

- Finding extrema vs. finding the location of extrema
  - An extreme value is a y-value. It occurs at an x-value
  - Example: Find the maximum value of $f(x)=x^2-5$ on $[-1,2]$

y-value

$f' = 2x = 0$
$x = 0$

$f(-1) = -4$
$f(2) = -1$
$f(0) = -5$

So f has a maximum value of -1 (at $x=2$)

- Finding slopes of inverse functions
  - Inverse functions, at corresponding points, have reciprocal slopes.
  - If $f(g(x)) = x = g(f(x))$, then $f(x)$ and $g(x)$ are inverses
  - $g(a) = b$ implies $f(b) = a$
  - $g'(a) = \dfrac{1}{f'(b)}$
  - Example: If $f(x) = g^{-1}(x)$ and $f(x) = 2x^2 + 3x - 1$ and if $g(-2) = -1$, find $g'(-2)$.

$f'(x) = 4x + 3$
$f'(-1) = -4 + 3 = -1$
So $g'(-2) = \frac{1}{-1} = -1$

$g: (-2, -1)$
$f: (-1, -2)$

- Finding the slope of a normal line to a function, $f(x)$, at a point $x = a$
  - Normal lines are perpendicular to tangent lines at a point.
  - The normal slope, n, is the opposite, reciprocal of the tangent slope.
  - $n = \dfrac{-1}{f'(a)}$
  - Example: Find the equation of the normal line to the graph of $y = e^{2x}$ at $x = \ln 2$

pt: $y(\ln 2) = e^{2\ln 2} = e^{\ln 2^2} = 4$
pt: $(\ln 2, 4)$

tangent line slope: $y' = 2e^{2x}$
$y'(\ln 2) = 2e^{2\ln 2} = 8 = m$
So $n = -\frac{1}{8}$

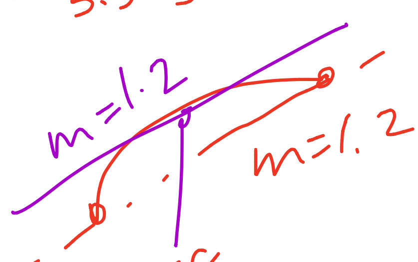
So eq: $y = 4 - \frac{1}{8}(x - \ln 2)$

- Squiggle Alert when the words "approximate" or "estimate" are used in the question
  - Explicitly stated approximations must have an approximation symbol, ≈, rather than an equal sign.
  - This happens with tangent line approximations, numeric methods of integration, linearization, Euler's Method (BC), and Taylor Polynomials (BC).
  - Example: If f is differentiable, and if $f(3) = 2$ and $f(5.5) = 5$, approximate $f'(4.1)$. Show the work that leads to your answer.

$f'(4.1) \approx \dfrac{5-2}{5.5-3} = \dfrac{3}{2.5} = \dfrac{3}{5/2} = \dfrac{6}{5} = 1.2$

$x \in (3, 5.5)$ MVT

open interval



- Average value vs. average rate of change
  - Average value is the average *y*-value for whatever is being measured on the *y*-axis.
  - Average value is "integral over width"
  - Average value $= \dfrac{\int_a^b f(x)\,dx}{b-a} = \dfrac{1}{b-a}\int_a^b f(x)\,dx$
  - Average rate of change is the change in *y* over the change in *x*—the slope of the secant line
  - Average rate of change $= \dfrac{f(b)-f(a)}{b-a}$

- The misuse of equality a.k.a. mathematically prevarification
  - If you use an equal sign, the expressions you are equating better be equal, or you will lose a point.
  - Example: If $f(x)=2x^2-1$, find $f'(1)$
    - WRONG: $f'(x)=4x=4$
    - CORRECT: $f'(x)=4x$, $f'(1)=4(1)=4$
  - Example: To the nearest whole number, approximate $\int_0^2 e^x\,dx$
    - WRONG: $\int_0^2 e^x\,dx = 6.389 = 6$
    - CORRECT: $\int_0^2 e^x\,dx = 6.389 \approx 6$ OR $\int_0^2 e^x\,dx = 6$

- Be sure to include units in all final numeric answers AND any written explanation of this answer.
  - Units can cost you an entire point if you omit them or use the wrong ones.
  - Example: If $r(t)$ is the rate at which water cools in jug in a refrigerator, in $^\circ F/\text{min}$. In the context of the problem, explain the meaning of $r'(5)=-2.10$.

At $t=5$ min, the rate at which water cools is decreasing by 2.10 °F/min per minute.

$v(c)>0, a(c)>0$ $v'(c)>0$

- Speed increasing or decreasing vs. Velocity increasing or decreasing

$v'>0$ $v'<0$

- IVT, EVT, MVT

$x\in(a,b)$ $f'(x)=\dfrac{f(b)-f(a)}{b-a}$

- Geometric formulas to remember

$V=\frac{4}{3}\pi r^3$, $A=4\pi r^2$; $V=\frac{\pi}{3}r^2h$; $V=\pi r^2h$, $A=2\pi r^2+2\pi rh$; $A=\frac{\sqrt{3}}{4}s^2$; $A=\frac{1}{2}(b_1+b_2)h$; $A=\frac{1}{2}\Delta x(y_1+y_2)$

- Justifying relative extrema using the First Derivative test and Second Derivative Test
  - First Derivative Test (at a critical value)

        - "Since $f'(c)=0$ (or $f'(c)=DNE$), and since $f'(x)$ changes from positive to negative at $x=c$, $f(x)$ has a Relative (local) Maximum at $x=c$."
        - "Since $f'(c)=0$ (or $f'(c)=DNE$), and since $f'(x)$ changes from negative to positive at $x=c$, $f(x)$ has a Relative (local) Minimum at $x=c$."
    - Second Derivative Test (at a critical value)
        - "Since $f'(c)=0$ (or $f'(c)=DNE$), and since $f''(c)<0$, $f(x)$ has a Relative (local) Maximum at $x=c$."
        - "Since $f'(c)=0$ (or $f'(c)=DNE$), and since $f''(c)>0$, $f(x)$ has a Relative (local) Minimum at $x=c$."

- Cross-sectional volume magic numbers
    - Squares: Equilateral Triangles: Semi circles:

- Inverse Trig Integral formulas
    - $\int \frac{du}{a^2+u^2}=$ $\int \frac{du}{\sqrt{a^2-u^2}}=$ $\int \frac{du}{u\sqrt{u^2-a^2}}=$

- Convergence Tests
    - *n*th term test for divergence
    - Geometric series *p*-series Direct/Limit Comparison Test
    - Integral Test Ratio Test Alternating Series Test/Error

- Pen or Pencil on the exam???

- 3-decimal accuracy (round or truncate). Store non-exact answers needed for future calculations. NEVER use an approximate answer to calculate a subsequent value.

- Implicit Differentiation—your derivative will have both *x* and *y* in it. If you ever end up with and answer like $\frac{dy}{dx}=\frac{a-b}{c-d}$, realize that this is equivalent to $\frac{dy}{dx}=\frac{b-a}{d-c}$

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* cusplert → x^p, wher $0<p<1$

No pronouns

~~Can't see~~

* indicate numeric methods