BC Review 09 No Calculator
Do all work on separate notebook paper

_____ 1.

What are all values of x for which the function f defined by $f(x) = x^3 + 3x^2 - 9x + 7$ is increasing?

(A) $-3 < x < 1$
(B) $-1 < x < 1$
(C) $x < -3$ or $x > 1$
(D) $x < -1$ or $x > 3$
(E) All real numbers

_____ 2.

In the xy-plane, the graph of the parametric equations $x = 5t + 2$ and $y = 3t$, for $-3 \le t \le 3$, is a line segment with slope

(A) $\frac{3}{5}$ (B) $\frac{5}{3}$ (C) 3 (D) 5 (E) 13

_____ 3.

The slope of the line tangent to the curve $y^2 + (xy+1)^3 = 0$ at $(2, -1)$ is

(A) $-\frac{3}{2}$ (B) $-\frac{3}{4}$ (C) 0 (D) $\frac{3}{4}$ (E) $\frac{3}{2}$

_____ 4.

$\int \frac{1}{x^2 - 6x + 8}\,dx =$

(A) $\frac{1}{2}\ln\left|\frac{x-4}{x-2}\right| + C$

(B) $\frac{1}{2}\ln\left|\frac{x-2}{x-4}\right| + C$

(C) $\frac{1}{2}\ln\left|(x-2)(x-4)\right| + C$

(D) $\frac{1}{2}\ln\left|(x-4)(x+2)\right| + C$

(E) $\ln\left|(x-2)(x-4)\right| + C$

_____ 5.

If f and g are twice differentiable and if $h(x) = f\left(g(x)\right)$, then $h''(x) =$

(A) $f''\left(g(x)\right)\left[g'(x)\right]^2 + f'\left(g(x)\right)g''(x)$

(B) $f''\left(g(x)\right)g'(x) + f'\left(g(x)\right)g''(x)$

(C) $f''\left(g(x)\right)\left[g'(x)\right]^2$

(D) $f''\left(g(x)\right)g''(x)$

(E) $f''\left(g(x)\right)$

_____ 6.

$\int_1^e \left(\frac{x^2-1}{x}\right)dx =$

(A) $e - \frac{1}{e}$ (B) $e^2 - e$ (C) $\frac{e^2}{2} - e + \frac{1}{2}$ (D) $e^2 - 2$ (E) $\frac{e^2}{2} - \frac{3}{2}$

_____ 7.

If $\frac{dy}{dx} = \sin x \cos^2 x$ and if $y = 0$ when $x = \frac{\pi}{2}$, what is the value of y when $x = 0$?

(A) -1 (B) $-\frac{1}{3}$ (C) 0 (D) $\frac{1}{3}$ (E) 1

_____ 8.

A particle moves on a plane curve so that at any time $t > 0$ its x-coordinate is $t^3 - t$ and its y-coordinate is $(2t-1)^3$. The acceleration vector of the particle at $t = 1$ is

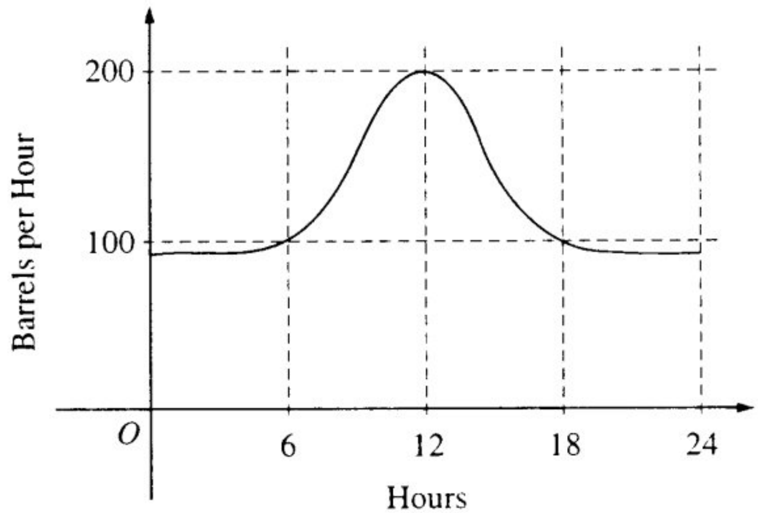
(A) $(0,1)$ (B) $(2,3)$ (C) $(2,6)$ (D) $(6,12)$ (E) $(6,24)$

____ 9.

If f is a linear function and $0 < a < b$, then $\int_a^b f''(x)\,dx =$

(A) 0 (B) 1 (C) $\frac{ab}{2}$ (D) $b - a$ (E) $\frac{b^2 - a^2}{2}$

_____ 10.

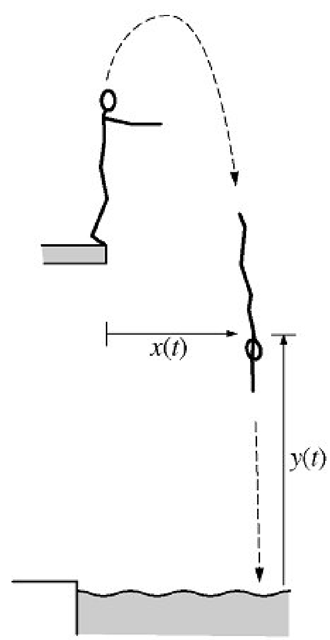


The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

(A) 500 (B) 600 (C) 2,400 (D) 3,000 (E) 4,800

Free Response

11. 2009-BC3 (Calculator Permitted)



Note: Figure not drawn to scale.

A diver leaps from the edge of a diving platform into a pool below. The figure above shows the initial position of the diver and her position at a later time. At time t seconds after she leaps, the horizontal distance from the front edge of the platform to the diver's shoulders is given by $x(t)$, and the vertical distance from the water surface to her shoulders is given by $y(t)$, where $x(t)$ and $y(t)$ are measured in meters. Suppose that the diver's shoulders are 11.4 meters above the water when she makes her leap and that

$$\frac{dx}{dt} = 0.8 \text{ and } \frac{dy}{dt} = 3.6 - 9.8t,$$

for $0 \le t \le A$, where A is the time that the diver's shoulders enter the water.

(a) Find the maximum vertical distance from the water surface to the diver's shoulders.

(b) Find A, the time that the diver's shoulders enter the water.

(c) Find the total distance traveled by the diver's shoulders from the time she leaps from the platform until the time her shoulders enter the water.

(d) Find the angle θ, $0 < \theta < \frac{\pi}{2}$, between the path of the diver and the water at the instant the diver's shoulders enter the water.

12. 2009-BC4 (No Calculator)

Consider the differential equation $\frac{dy}{dx} = 6x^2 - x^2y$. Let $y = f(x)$ be a particular solution to this differential equation with the initial condition $f(-1) = 2$.

(a) Use Euler's method with two steps of equal size, starting at $x = -1$, to approximate $f(0)$. Show the work that leads to your answer.

(b) At the point $(-1, 2)$, the value of $\frac{d^2y}{dx^2}$ is -12. Find the second-degree Taylor polynomial for f about $x = -1$.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(-1) = 2$.