CALCULUS BC
LAST AP REVIEW

1. The population $P(t)$ of fish in a lake satisfies the logistic differential equation

$$\frac{dP}{dt} = 3P - \frac{P^2}{6000}.$$

(a) If $P(0) = 4000$, what is $\lim_{t\to\infty} P(t)$? Is the solution curve increasing or decreasing? Justify your answer.

(b) If $P(0) = 10{,}000$, what is $\lim_{t\to\infty} P(t)$? Is the solution curve increasing or decreasing? Justify your answer.

(c) If $P(0) = 20{,}000$, what is $\lim_{t\to\infty} P(t)$? Is the solution curve increasing or decreasing? Justify your answer.

(d) If $P(0) = 4000$, what is the population when it is growing the fastest? Where is the solution curve concave up? Concave down? Justify your answer.

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2. $\int x\sin(2x)\,dx =$

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3. $\int \frac{dx}{x^2 - 6x + 8} =$

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4. Write an integral expression which gives the area of the region inside the polar curve $r = 4\cos\theta$ and outside $r = 2$.

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5. Given $\frac{dy}{dx} = \frac{xy}{2}$. Let $f(x)$ be the particular solution to the given differential equation with initial condition $f(0) = 3$. Use Euler's method starting at $x = 0$, with a step size of 0.1, to approximate $f(0.2)$.

6. Given $\frac{dx}{dt}=\cos\left(t^3\right)$ and $\frac{dy}{dt}=3\sin\left(t^2\right)$ for $0\le t\le 3$. At time $t=2$, the object is at position (4, 5).

(a) Find the speed of the object at time $t=2$.

(b) Find the total distance traveled by the object over the time interval $0\le t\le 1$.

(c) Find the position of the object at time $t=3$.

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7. Given $f(x)=\frac{1}{3}+\frac{2x}{9}+\frac{3x^2}{27}+\ldots+\frac{(n+1)}{3^{n+1}}x^n+\ldots$

(a) Find $\lim_{x\to 0}\frac{f(x)-\frac{1}{3}}{x}$.

(b) Write the first three nonzero terms and the general term for the infinite series that represents $\int_0^1 f(x)\,dx$.

(c) Find the sum of the series found in part (b).

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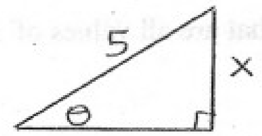
8. If $y=xy+x^2+1$, then when $x=-1$, $\frac{dy}{dx}=?$

9. If θ increases at a constant rate of 3 rad/min, at what rate is x increasing in units/min when $x = 3$ units?



10. Write an integral equation which gives the length of the path described by the parametric equations $x = \cos^3 t$ and $y = \sin^3 t$ for $0 \le t \le \frac{\pi}{2}$.

11. If f is a function such that $f'(x) = \sin(x^2)$, then the coefficient of x^7 in the Taylor series for $f(x)$ about $x = 0$ is ?

12. If $\frac{dy}{dx} = y\sec^2 x$ and $y = 5$ when $x = 0$, then $y = ?$

13. $\lim_{h \to 0} \frac{\cos\left(\frac{\pi}{2} + h\right) - \cos\left(\frac{\pi}{2}\right)}{h} =$

14. The coefficient of x^3 in the Taylor series for e^{2x} about $x = 0$ is ?

15. What are all values of x for which the series $\sum_{n=1}^{\infty}\frac{(x-2)^n}{n3^n}$ converge?

16. Which of the following converge?

I. $\sum_{n=1}^{\infty}\frac{n}{n+2}$ II. $\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{n}$ III. $\sum_{n=1}^{\infty}\frac{1}{n}$

(A) None (B) II only (C) III only (D) I and II only (E) I and III only

17. The function f has derivatives of all orders for all real numbers x. Assume that $f(2)=5$, $f'(2)=-3$, $f''(2)=4$, $f'''(2)=-1$, and $\left|f^{(4)}(x)\right|\le 3$ for all x in [2, 2.2].

(a) Write the third-degree Taylor polynomial for f about $x=2$.

(b) Use your answer to (a) to approximate $f(2.15)$. Give your answer correct to five decimal places.

(c) Use the Lagrange error bound on the approximation of $f(2.15)$ to explain why $f(2.15)\neq 4.7$.

18. The Taylor series about $x=5$ for a certain function f converges to $f(x)$ for all x in its interval of convergence. The nth derivative of f at $x=5$ is given by

$$f^{(n)}(5)=\frac{(-1)^n n!}{2^n(n+2)} \text{ and } f(5)=\frac{1}{2}.$$

Show that the sixth-degree Taylor polynomial for f about $x=5$ approximates $f(6)$ with an error less than $\frac{1}{1000}$.