

## AP REVIEW 6

Work these on notebook paper. Use your calculator on problems 56 and 60.

56. A particle moves along the  $x$ -axis so that its velocity at time  $t$  is given by

$$v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right).$$

At time  $t = 0$ , the particle is at position  $x = 1$ .

- (a) Find the acceleration of the particle at time  $t = 2$ . Is the speed of the particle increasing at  $t = 2$ ? Why or why not?  
(b) Find all times  $t$  in the open interval  $0 < t < 3$  when the particle changes direction. Justify your answer.  
(c) Find the total distance traveled by the particle from time  $t = 0$  until time  $t = 3$ .  
(d) During the time interval  $0 \leq t \leq 3$ , what is the greatest distance between the particle and the origin? Show the work that leads to your answer.

57. Let  $f$  and  $g$  be differentiable functions with the following properties:

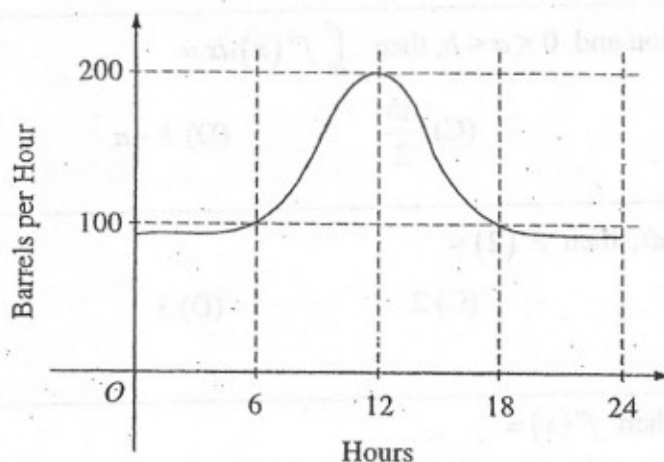
(i)  $g(x) > 0$  for all  $x$

(ii)  $f(0) = 1$

If  $h(x) = f(x)g(x)$  and  $h'(x) = f(x)g'(x)$ , then  $f(x) =$

- (A)  $f'(x)$       (B)  $g(x)$       (C)  $e^x$       (D) 0      (E) 1

58.



The flow of oil, in barrels per hour, through a pipeline on July 9 is given by the graph shown above. Of the following, which best approximates the total number of barrels of oil that passed through the pipeline that day?

- (A) 500      (B) 600      (C) 2,400      (D) 3,000      (E) 4,800

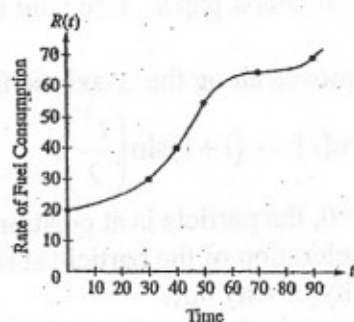
59. What is the instantaneous rate of change at  $x = 2$  of the function  $f$  given

by  $f(x) = \frac{x^2 - 2}{x - 1}$ ?

- (A) -2      (B)  $\frac{1}{6}$       (C)  $\frac{1}{2}$       (D) 2      (E) 6

60.

The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function  $R$  of time  $t$ . The graph of  $R$  and a table of selected values of  $R(t)$ , for the time interval  $0 \leq t \leq 90$  minutes, are shown above.



| $t$<br>(minutes) | $R(t)$<br>(gallons per minute) |
|------------------|--------------------------------|
| 0                | 20                             |
| 30               | 30                             |
| 40               | 40                             |
| 50               | 55                             |
| 70               | 65                             |
| 90               | 70                             |

- (a) Use data from the table to find an approximation for  $R'(45)$ . Show the computations that lead to your answer. Indicate units of measure.
- (b) The rate of fuel consumption is increasing fastest at time  $t = 45$  minutes. What is the value of  $R''(45)$ ? Explain your reasoning.
- (c) Approximate the value of  $\int_0^{90} R(t) dt$  using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of  $\int_0^{90} R(t) dt$ ? Explain your reasoning.
- (d) For  $0 < b \leq 90$  minutes, explain the meaning of  $\int_0^b R(t) dt$  in terms of fuel consumption for the plane. Explain the meaning of  $\frac{1}{b} \int_0^b R(t) dt$  in terms of fuel consumption for the plane. Indicate units of measure in both answers.

61. If  $f$  is a linear function and  $0 < a < b$ , then  $\int_a^b f''(x) dx =$

- (A) 0                      (B) 1                      (C)  $\frac{ab}{2}$                       (D)  $b - a$                       (E)  $\frac{b^2 - a^2}{2}$

62. If  $F(x) = \int_0^x \sqrt{t^3 + 1} dt$ , then  $F'(2) =$

- (A) -3                      (B) -2                      (C) 2                      (D) 3                      (E) 18

63. If  $f(x) = \sin(e^{-x})$ , then  $f'(x) =$

- (A)  $-\cos(e^{-x})$                       (B)  $\cos(e^{-x}) + e^{-x}$                       (C)  $\cos(e^{-x}) - e^{-x}$   
 (D)  $e^{-x} \cos(e^{-x})$                       (E)  $-e^{-x} \cos(e^{-x})$

64. If  $f''(x) = x(x+1)(x-2)^2$ , then the graph of  $f$  has inflection points when  $x =$

- (A) -1 only                      (B) 2 only                      (C) -1 and 0 only  
 (D) -1 and 2 only                      (E) -1, 0, and 2 only

65. What are all the values of  $k$  for which  $\int_{-3}^k x^2 dx = 0$ ?

- (A) -3                      (B) 0                      (C) 3                      (D) -3 and 3                      (E) -3, 0, and 3