2008 AB/BC Cal Exam

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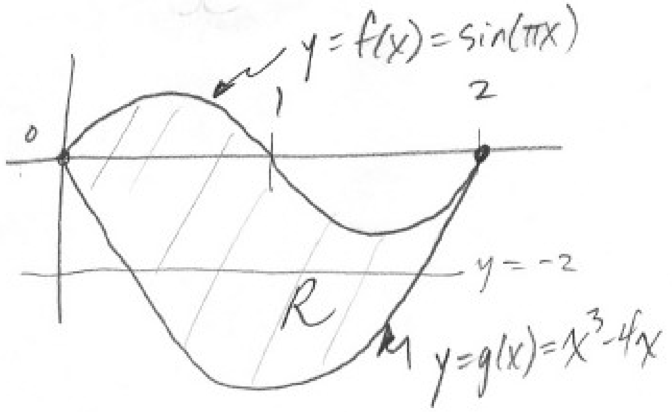

a) $A = \int_0^2 [f(x) - g(x)]\,dx$

$\approx \boxed{4}$

b) poi: $x^3 - 4x = -2$

$x = a = 0.539\ldots$

$x = b = 1.675\ldots$

$\boxed{A = \int_a^b (-2 - g(x))\,dx}$

c) $V = \int_0^2 (f(x) - g(x))^2\,dx$

$\approx \boxed{9.978}$

d) $V = \int_0^2 \left[(f(x) - g(x)) \cdot h(x)\right] dx$

$\approx \boxed{8.369 \text{ or } 8.370}$

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2 a) $L'(5.5) \approx \dfrac{L(7) - L(4)}{7 - 4} = \dfrac{150 - 126}{3} = \boxed{8 \text{ people/hr}}$

b) Avg value $= \dfrac{\int_0^4 L(t)\,dt}{4} = \dfrac{\frac{1}{2}(120+156)(1) + \frac{1}{2}(156+176)(2) + \frac{1}{2}(176+126)(1)}{4}$

$\approx \dfrac{621}{4} = \boxed{155.25 \text{ people}}$

c) Since L' and L are differentiable and $L(t)$ changes from inc to dec or dec to inc at least 3 times, $L'(t) = 0$ $\boxed{\text{at least 3 times}}$ on $0 \le t \le 9$.

d) Tickets $= \int_0^3 r(t)\,dt \approx 972.784$ or about $\boxed{973 \text{ tickets}}$

-Koypi

P1/3 AB

P1/4 BC

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(3) a) $V=\pi r^2 h$

$\frac{dV}{dt} = 2\pi r h \frac{dr}{dt} + \pi r^2 \frac{dh}{dt}$

$\frac{dV}{dt} = 2000$, $r = 100\text{cm}$, $h = \frac{1}{2}\text{cm}$, $\frac{dr}{dt} = 2.5\text{cm/min}$, $\frac{dh}{dt} = ?$

$2000 = 2\pi(100)(\tfrac{1}{2})(2.5) + \pi(100^2)\frac{dh}{dt}$

$\boxed{\frac{dh}{dt} \approx 0.3866 \text{ cm/min}}$

b) $V'(t) = 2000 - 400\sqrt{t}$, cv when $V'(t)=0$

$2000 = 400\sqrt{t}$

$\sqrt{t} = 5$

$t = 25\text{min}$

First deriv test

x	16	25	36
$V'(t)$	+		—

* Volume is a max @ $t=25$ min since $V'(t)$ changes from pos to neg at $t=25$ minutes

c) $\boxed{V(t) = 60{,}000 + \int_0^t (2000 - 400\sqrt{x})\,dx}$

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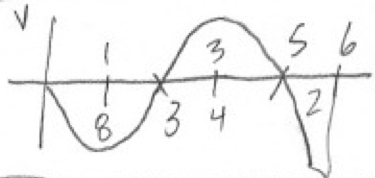
(4)



a) Particle is furthest to left at the minimum position $S(t)$. The occurs at endpts or critical values of $S(t)$:

$S(0) = -2$
$S(6) = -9$
$S(3) = -10$
$S(5) = -7$

*This occurs at $t=3$ with a value of -10

b) Since the Intergal function is continuous, the accumulated areas pass through -8 at least 3 times

c) on $2<t<3$, since $v(t)<0$ and $v'(t)=a(t)>0$, speed is decreasing.

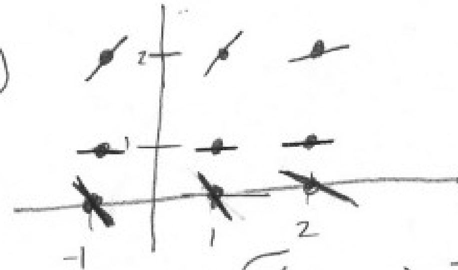
d) Acceleration is negative when $v'(t)$ (slopes of the graph) is negative. This occurs on the open intervals $\boxed{(0,1)\cup(4,6)}$

Korpi (p2/3)

2008 AB exam

(5) $\frac{dy}{dx} = \frac{y-1}{x^2}, x \neq 0$

a)



b) $\int \frac{1}{y-1} dy = \int x^{-2} dx$

$\ln|y-1| = -\frac{1}{x} + C$

* for $f(2)=0$

$\ln 1 = -\frac{1}{2} + C$

$C = \frac{1}{2}$

* so $\ln|y-1| = -\frac{1}{x} + \frac{1}{2}$

$y - 1 = e^{-1/x + \frac{1}{2}}$

$\boxed{y = e^{\left(-1/x + \frac{1}{2}\right)} + 1}$

or $y = \left(e^{1/2}\right) e^{-1/x} + 1$

or $y = \frac{\sqrt{e}}{e^{1/x}} + 1$

c) $\lim_{x \to \infty} \left[e^{\left(-1/x + \frac{1}{2}\right)} + 1 \right] = \boxed{e^{1/2} + 1 = \sqrt{e} + 1}$

(6) $f(x) = \frac{\ln x}{x}, x > 0, \quad f'(x) = \frac{1 - \ln x}{x^2}$

a) $f(e^2) = \frac{2}{e^2}, \quad f'(e^2) = \frac{-1}{e^4}$

Tan line: $\boxed{y - \frac{2}{e^2} = -\frac{1}{e^4}\left(x - e^2\right)}$

b) $f' = 0$ when $1 - \ln x = 0$, $x = e$

$f'(e) = \frac{0}{e} = 0$

$f''(x) = \frac{x^2\left(-\frac{1}{x}\right) - (1 - \ln x)(2x)}{x^2} = \frac{-x - 2x + 2x \ln x}{x^2}$

$f''(x) = \frac{x(-3 + 2\ln x)}{x^2} = \frac{2\ln x - 3}{x}$

$f''(e) = \frac{2-3}{e} < 0$, so f has a Relative Max @ $x = e$ (by 2nd Deriv Test)

c) Point of inflection of f:

$f'' = \frac{2\ln x - 3}{x} = 0$

when $\ln x = \frac{3}{2}$, $x = e^{3/2}$

x	1	$e^{3/2}$	e^2
$f''(x)$	$-$		$+$

Since $f'' = 0$ and f'' changes signs @ $x = e^{3/2}$, it is the point of inflection of f.

d) $\lim_{x \to 0^+} \frac{\ln x}{x} =$

$= \lim_{x \to 0^+} \left(\frac{1}{x}\right)(\ln x) = (\infty)(-\infty) = \boxed{-\infty}$

Kozin p. 3/3