Cal §6.4 p.357 (2, 5, 8, 9, 10, 22, 24, 28, 31, 36, 44, 47, 48, 50, 51, 57) Koyzi, per F1
ABC

Pg 1/2

(2) $\frac{dy}{dx} = -\frac{x}{y}$, $(4,3)$

$\int y\,dy = \int -x\,dx$

$\frac{1}{2}y^2 = -\frac{1}{2}x^2 + C$

$y^2 = -x^2 + 2C$

$y = \sqrt{C - x^2}$

@ $(4,3)$

$3 = \sqrt{C - 4^2}$

$9 = C - 16$

$C = 25$

$\boxed{\text{So } y = \sqrt{25 - x^2} \quad D: [-5, 5] \text{ (semicircle)}}$

(5) $\frac{dy}{dx} = (y+5)(x+2)$, $(0,1)$

$\int \frac{1}{y+5}\,dy = \int x+2\,dx$

$\ln|y+5| = \frac{1}{2}x^2 + 2x + C$

$y+5 = e^{\frac{1}{2}x^2 + 2x + C}$

$y = e^{\frac{1}{2}x + 2x} \cdot e^{C} - 5$

$y = Ce^{\frac{1}{2}x + 2x} - 5$

@ $(0,1)$

$1 = Ce^0 - 5$

$1 = C - 5 \rightarrow C = 6$

$\boxed{\text{So } y = 6e^{\frac{1}{2}x^2 + 2x} - 5 \quad D: \mathbb{R}}$

(8) $\frac{dy}{dx} = e^{x-y}$, $(0,2)$

$\frac{dy}{dx} = \frac{e^x}{e^y}$

$\int e^y\,dy = \int e^x\,dx$

$e^y = e^x + C$

$y = \ln(e^x + C)$

@ $(0,2)$

$2 = \ln(e^0 + C)$

$2 = \ln(1 + C)$

$e^2 = 1 + C \rightarrow C = e^2 - 1$

$\boxed{\text{So } y = \ln(e^x + e^2 - 1) \quad D: \mathbb{R}}$

(9) $\frac{dy}{dx} = -2xy^2$, $(1, \frac{1}{4})$

$\int \frac{1}{y^2}\,dy = \int -2x\,dx$

$-\frac{1}{y} = -x^2 + C$

$y = \frac{1}{x^2 + C}$

@ $(1, \frac{1}{4})$

$\frac{1}{4} = \frac{1}{1+C}$

$4 = 1 + C \rightarrow C = 3$

$\boxed{\text{So } y = \frac{1}{x^2+3} \quad D: \mathbb{R}}$

(10) $\frac{dy}{dx} = \frac{4\sqrt{y}\,\ln x}{x}$, $(e,1)$

$\int y^{-1/2}\,dy = 4\int \frac{\ln x}{x}\,dx$

$2y^{1/2} = 4(\frac{1}{2})(\ln x)^2 + C$

$y^{1/2} = \ln^2 x + C$

$y = ((\ln x)^2 + C)^2$

@ $(e,1)$

$1 = ((\ln e)^2 + C)^2$

$1 = (1 + C)^2$

$1 + C = 1$

$C = 0$

$\boxed{\text{So } y = (\ln^2 x)^2 \quad y = \ln^4 x \quad D: \{x \mid x > 0\}}$

(22) N_p-240, ½ life of 65 min

$\frac{dy}{dt} = -ky$ $\quad (0, 1)$, $(65, \frac{1}{2})$

$\int \frac{1}{y}\,dy = \int -k\,dt$

$\ln|y| = -kt + C$

$y = Ce^{-kt}$ @ $(0,1)$, $C = 1$ so,

$y = e^{-kt}$ @ $(65, \frac{1}{2})$

$\frac{1}{2} = e^{-k(65)}$

$\ln(1/2) = -65k$

$k = \frac{\ln(1/2)}{-65} = \boxed{\frac{\ln 2}{65}}$

$k \approx 0.010663802778\ldots$

(24) Ideal Conditions - yeah!!

(t, P): $(3, 10000)$, $(5, 40000)$

find P @ $t = 0$.

$\boxed{y = Ce^{kt}}$

① : $10000 = Ce^{3k}$

② : $40000 = Ce^{5k}$

② ÷ ① : $4 = e^{2k}$

$\ln 4 = 2k \rightarrow k = \frac{\ln 4}{2}$

$t = 3$: $10000 = Ce^{(3 \cdot \ln 4)/2}$

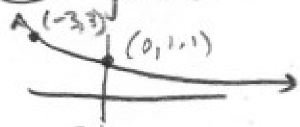
$10000 = Ce^{\ln 4^{3/2}} = C \cdot 4^{3/2}$

$C = 10000/(4^{3/2}) = 10000/8 = \boxed{1250 \text{ bacteria}}$ originally

on to pg. 2

Cal §6.4 cont

(28) $y = Ce^{kt}$



$3 = Ce^{-3k}$

$1.1 = Ce^{(0)(k)} \to C = 1.1$

so $3 = 1.1e^{-3k}$

$\frac{30}{11} = e^{-3k}$

$\ln(30/11) = -3k$

$k = -\frac{1}{3}\ln(30/11)$

$\boxed{\text{so } y = 1.1e^{-1/3\ln(30/11)t} \approx y = 1.1e^{-0.334434t}}$

(31) $T - T_s = (T_0 - T_s)e^{-kt}$

90→60 in 10 min, $T_s = 20$

(0,90), (10,60)

$T - 20 = (90-20)e^{-kt}$

$T = 70e^{-kt} + 20$ @ (10,60)

$60 = 70e^{-10k} + 20$

$\ln\frac{4}{7} = -10k \to \boxed{k = \frac{1}{10}\ln(7/4)}$

$T = 70e^{-\frac{1}{10}\ln(7/4)t} + 20$

a) for $T = 35°$,

$\ln(\frac{3}{14}) = -\frac{1}{10}\ln(\frac{7}{4})t$

$t = -\frac{10\ln(3/14)}{\ln(7/4)}$ min ≈

$t = 27.527$ min, so it will take an additional $27.527 - 10$ ≈ $\boxed{17.527 \text{ min}}$

b) Now $T_s = -15$; $T - -15 = (90 - -15)e^{-kt}$

$T + 15 = 105e^{-kt}$

$T = 105e^{-kt} - 15$

Assuming the same k.

$T = 105e^{-\frac{1}{10}\ln(7/4)t} - 15$

$35 = 105e^{-\frac{1}{10}\ln(7/4)t} - 15$

$\ln(\frac{10}{21}) = -\frac{1}{10}\ln(\frac{7}{4})t$

$t = \frac{-10\ln(10/21)}{\ln(7/4)} \approx \boxed{13.258 \text{ min}}$

(36) a) Carbon-14 ($\frac{1}{2}$ life ≈ 5700)

$\boxed{A = A_0 = (\frac{1}{2})^{t/5700}}$

$.17A_0 = A_0(\frac{1}{2})^{t/5700}$

$\ln(.17) = t\frac{\ln(1/2)}{5700}$

$t \approx \frac{5700(\ln .17)}{\ln(1/2)}$

$t \approx 14571.442$ yrs

so $t - 2000 \approx \boxed{12571 \text{ B.C.}}$

b) $t \approx \frac{5700(\ln .18)}{\ln(1/2)} \approx 14101.408$

$\approx \boxed{12101 \text{ B.C.}}$

c) $t \approx \frac{5700(\ln .16)}{\ln(1/2)} \approx 15069.980$

$\approx \boxed{13070 \text{ B.C.}}$

(44) Napier Space

a) $y = Pe^{rt}$, w/ r=1

$y = Pe^t$

at this rate, you increase your $ by 2.718 (almost triple your $) each year! Sign me up.

b) when will $y = 3P$?

$3P = Pe^t$

$3 = e^t \to t = \ln 3 \approx 1.099$ yrs

$\boxed{\text{or} \approx 1\text{yr } 1\text{month } 6\text{ days}}$

c) for t=1

$y = Pe$; you can earn exactly 2.718281828 46... times your investment in one year

(47) T or F: if $\frac{dy}{dx} = ky$ then $y = e^{kx} + C$ False

it $= e^{kx+c} = e^{kx}\cdot e^c = Ce^{kx}$

(48) T or F: $\frac{dy}{dt} = 2y \Rightarrow y = C\cdot 3^{kt}$?

$\int\frac{1}{y}dy = \int 2dt \to \ln y = 2t + c$

$y = Ce^{2t} = C3^{\log_3 e^{2t}} = Ce^{t/\log_3 e^2}$

True $k = \log_3 e^2$

(50) $y = Ce^{kt}$; (0,100) (199,1)

$y = 100e^{kt}$; $1 = 100e^{199k} \to \ln(\frac{1}{100}) = 199k$

$k = \ln(1/100)/199$, so $y = 100e^{\ln(1/100)/199}$

when will $y = 50$?

$50 = 100e^{\frac{1}{199}\ln(1/100)t}$

$\ln\frac{1}{2} = \frac{1}{199}\ln(1/100)\cdot t$

$t \approx \frac{199\ln(1/2)}{\ln(1/100)} \approx \boxed{29.952 \text{ yrs}}$

(51) $\frac{dy}{dt}$ directly prop to y (find all exp eqs) (II and III only)

(57) Figure it out on your own. Use your table

$\lim_{n\to\infty}(1 + \frac{1}{n})^n = e$

y1: (1+1/x)^x

table: n=1
n=100
n=1000
n=10000
n=100000
⋮

–Kopi