Cal ABC §3.6 p.153(12, 14-32 even, 44, 46, 49, 56, 70, 71, AP1-4) Korpi period $\sqrt{-1}$

(12) Horz axis movement

$s(t) = \sin\left(\frac{3\pi}{2}t\right) + \cos\left(\frac{7\pi}{4}t\right)$

$v(t) = s'(t) = \frac{3\pi}{2}\cos\left(\frac{3\pi}{2}t\right) - \frac{7\pi}{4}\sin\left(\frac{7\pi}{4}t\right)$

(14) $y = (\csc x + \cot x)^{-1}$

$y' = -1(\csc x + \cot x)^{-2} \cdot (-\csc x \cot x - \csc^2 x)$

$= \boxed{\dfrac{\csc x \cot x + \csc^2 x}{(\csc x + \cot x)^2}}$

(16) $y = x^3(2x-5)^4$

$y' = 3x^2(2x-5)^4 + x^3(4)(2x-5)^3$

$= \boxed{3x^2(2x-5)^4 + 4x^3(2x-5)^3}$

or $x^2(2x-5)^3[3(2x-5)+4x]$

$= \boxed{x^2(2x-5)^3[10x-15]}$

(18) $y = 4\sqrt{\sec x + \tan x} = 4(\sec x + \tan x)^{1/2}$

$y' = 2(\sec x + \tan x)^{-1/2}(\sec x \tan x + \sec^2 x)$

$= \boxed{\dfrac{2(\sec x \tan x + \sec^2 x)}{\sqrt{\sec x + \tan x}}}$

(20) $y = \dfrac{x}{\sqrt{1+x^2}} = x(1+x^2)^{-1/2}$

$y' = (1)(1+x^2)^{-1/2} + x\left(-\frac{1}{2}\right)(1+x^2)^{-3/2} \cdot (2x)$

$y' = \dfrac{1}{\sqrt{1+x^2}} - \dfrac{x^2}{(\sqrt{1+x^2})^3}$

or getting a common denom

$= \dfrac{1+x^2-x^2}{(\sqrt{1+x^2})^3} = \boxed{\dfrac{1}{(1+x^2)^{3/2}}}$

(22) $y = (1+\cos(2x))^2$

$y' = 2(1+\cos(2x))' \cdot (-\sin(2x)) \cdot 2$

$\boxed{y' = -4(1+\cos 2x)\sin 2x}$

(24) $y = \sqrt{\tan 5x} = (\tan(5x))^{1/2}$

$y' = \frac{1}{2}(\tan(5x))^{-1/2} \cdot \sec^2(5x) \cdot 5$

$\boxed{y' = \dfrac{5\sec^2 5x}{2\sqrt{\tan 5x}}}$

(26) $r = \overset{f}{(\sec(2\theta))}\overset{g}{(\tan(2\theta))}$

$\dfrac{dr}{d\theta} = \underset{f'}{[(\sec 2\theta \tan 2\theta) \cdot 2]} \cdot \underset{g}{[\tan 2\theta]} + \underset{f}{[\sec 2\theta]} \cdot \underset{g'}{[\sec^2(2\theta) \cdot 2]}$

$= \boxed{2\sec 2\theta \cdot \tan^2 2\theta + 2\sec^3 2\theta}$

or $= \boxed{2\sec 2\theta[\tan^2 2\theta + \sec^2 2\theta]}$

(28) $r = 2\theta\sqrt{\sec\theta} = \overset{f}{(2\theta)} \cdot \overset{g}{(\sec\theta)^{1/2}}$

$\dfrac{dr}{d\theta} = \underset{f' \cdot g}{2((\sec\theta)^{1/2})} + \underset{f \cdot g'}{(2\theta)\left(\frac{1}{2}(\sec\theta)^{-1/2} \cdot \sec\theta\tan\theta\right)}$

$= 2\sqrt{\sec\theta} + \dfrac{\theta\sec\theta\tan\theta}{(\sec\theta)^{1/2}}$

$= \boxed{2\sqrt{\sec\theta} + \theta\sqrt{\sec\theta} \cdot \tan\theta}$

or $\boxed{\sqrt{\sec\theta}[2 + \theta\tan\theta]}$

(30) $y = \cot x$

$y' = -\csc^2 x = -(\csc x)^2$

$y'' = -2(\csc x)' \cdot (-\csc x \cot x)$

$\boxed{y'' = 2\csc^2 x \cot x}$

(32) $y = 9\tan\left(\frac{1}{3}x\right)$

$y' = 9\sec^2\left(\frac{1}{3}x\right) \cdot \frac{1}{3}$

$= 3\sec^2\left(\frac{1}{3}x\right)$

$= 3\left(\sec\left(\frac{1}{3}x\right)\right)^2$

$y'' = 6\left(\sec\left(\frac{1}{3}x\right)\right)' \cdot \sec\left(\frac{1}{3}x\right)\tan\left(\frac{1}{3}x\right) \cdot \left(\frac{1}{3}\right)$

$\boxed{y'' = 2\sec^2\left(\frac{1}{3}x\right)\tan\left(\frac{1}{3}x\right)}$

Cal ABC §3.6 cont

44 *BC only Find tan line

$x = \sec t$, $\frac{dx}{dt} = \sec t \tan t$

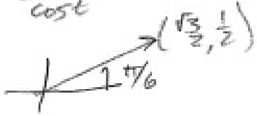
$y = \tan t$, $\frac{dy}{dt} = \sec^2 t$

@ $t = \pi/6$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\sec^2 t}{\sec t \cdot \tan t}$$

$$= \frac{\sec t}{\tan t} = \frac{\frac{1}{\cos t}}{\frac{\sin t}{\cos t}}$$

$$\boxed{\frac{dy}{dx} = \csc t}$$



$$\left.\frac{dy}{dx}\right|_{t=\pi/6} = \csc\frac{\pi}{6} = \boxed{2} = m$$

@ $t = \pi/6$ $(x, y) = (\sec\frac{\pi}{6}, \tan\frac{\pi}{6})$

$= (\frac{2}{\sqrt{3}}, \frac{1}{\sqrt{3}}) = (\frac{2\sqrt{3}}{3}, \frac{\sqrt{3}}{3})$

tan line:

$y - \frac{\sqrt{3}}{3} = 2(x - \frac{2\sqrt{3}}{3})$

or $3y - \sqrt{3} = 6x - 4\sqrt{3}$

$3y = 6x - 3\sqrt{3}$

$\boxed{y = 2x - \sqrt{3}}$

46 *BC only

$x = 2t^2 + 3$, $\frac{dx}{dt} = 4t$

$y = t^4$, $\frac{dy}{dt} = 4t^3$

@ $t = -1$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4t^3}{4t} = t^2$$

$$\left.\frac{dy}{dx}\right|_{t=-1} = (-1)^2 = \boxed{1 = m}$$

@ $t = -1$, $(x, y) = (5, 1)$

tan line

$y - 1 = 1(x - 5)$

$\boxed{y = x - 4}$

49 *BC only

$x = t^2 + t$, $\frac{dx}{dt} = 2t + 1$

$y = \sin t$, $\frac{dy}{dt} = \cos t$

a) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos t}{2t+1}$

b) $\frac{d}{dt}\left[\frac{dy}{dx}\right] = \frac{d}{dt}\left[\frac{\cos t}{2t+1}\right]$ HoDHi

$= \frac{(2t+1)(-\sin t) - \cos t(2)}{(2t+1)^2}$

$= \boxed{\frac{-\sin t(2t+1) - 2\cos t}{(2t+1)^2}}$

c) $\frac{d}{dx}\left[\frac{dy}{dx}\right] = \frac{d(dy/dx)/dt}{dx/dt}$ from b)

$= \frac{(-\sin t(2t+1) - 2\cos t)/(2t+1)^2}{(2t+1)}$ ← dx/dt

$= \boxed{\frac{-\sin t(2t+1) - 2\cos t}{(2t+1)^3}}$

d) $\frac{d^2y}{dx^2} = \frac{d}{dx}\left[\frac{dy}{dx}\right]$ = part c)

70 T or F: $\frac{d}{dx}\sin x = \cos x$

only if x is in Radians (unitless)

if x is in degrees

$x° \cdot \frac{\pi}{180°}$ = rads 's

$\frac{d}{dx}[\sin x] = \frac{d}{dx}\left[\sin(\frac{\pi}{180}x)\right]$

$= \frac{\pi}{180}\cos x$

FALSE

71 T or F: $x = 3\cos t$, $\frac{dx}{dt} = -3\sin t$

$y = -1 + \sin t$, $\frac{dy}{dt} = \cos t$

@ $t = \pi/4$

$\frac{dy}{dx} = \frac{\cos \pi/4}{-3\sin \pi/4} = -\frac{1}{3} = m$

Normal slope $= m_\perp = 3$

FALSE

56

x	f(x)	g(x)	f'(x)	g'(x)
2	8	2	1/3	-3
3	3	-4	2π	5

a) $\frac{d}{dx}[2f(x)] = 2f'(x)$

@ $x = 2$, $2f'(2) = 2(\frac{1}{3}) = \boxed{\frac{2}{3}}$

b) $\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$

@ $x = 3$, $f'(3) + g'(3) = \boxed{2\pi + 5}$

c) $\frac{d}{dx}[f(x) \cdot g(x)] = f'(x)g(x) + f(x) \cdot g'(x)$

@ $x = 3$, $f'(3)g(3) + f(3)g'(3)$

$= (2\pi)(-4) + (3)(5) = \boxed{-8\pi + 15}$

d) $\frac{d}{dx}\left[\frac{f}{g}\right] = \frac{gf' - fg'}{g^2}$, @ $x = 2$,

$\frac{(2)(\frac{1}{3}) - (8)(-3)}{2^2} = \frac{\frac{2}{3} + 24}{4}\left(\frac{3}{3}\right) = \frac{2 + 72}{12}$

$= \frac{74}{12} = \boxed{\frac{37}{6}}$

e) $\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$

@ $x = 2$, $f'(g(2)) \cdot g'(2)$

$= f'(2) \cdot (-3) = (\frac{1}{3})(-3) = \boxed{-1}$

f) $\frac{d}{dx}\left[(f(x))^{1/2}\right] = \frac{1}{2}[f(x)]^{-1/2} \cdot f'(x)$

@ $x = 2$, $\frac{1}{2}[8^{-1/2}][\frac{1}{3}]$

$= \frac{1}{6 \cdot 2\sqrt{2}} = \boxed{\frac{1}{12\sqrt{2}}}$

g) $\frac{d}{dx}\left[(g(x))^{-2}\right] = -2[g(x)]^{-3} \cdot g'(x)$

@ $x = 3$, $-2(-4)^{-3}(5)$

$= \frac{-10}{-64} = \boxed{\frac{5}{32}}$

h) $\frac{d}{dx}\left[(f(x))^2 + (g(x))^2\right]^{1/2}$

$= \frac{1}{2}[f^2(x) + g^2(x)]^{-1/2}(2f(x) \cdot f' + 2g \cdot g')$

@ $x = 2$, $\frac{1}{2}(64 + 4)^{-1/2}(16(\frac{1}{3}) + 4(-3))$

$= \frac{\frac{8}{3} - 6}{\sqrt{68}} = \frac{-10}{3\sqrt{68}} = \frac{-10}{3 \cdot 2\sqrt{17}} = \boxed{\frac{-5}{3\sqrt{17}}}$

Cal ABC §3.6 cont

AP1

$y = \sin^4(3x)$

$y = (\sin(3x))^4$

$y' = 4(\sin(3x))^3 \cdot \cos(3x) \cdot 3$

$y' = 12\sin^3(3x)\cos(3x)$

$\boxed{B}$

AP2

$y = \cos x + \tan x$

$y' = -\sin x + \sec^2 x = -\sin x + (\sec x)^2$

$y'' = -\cos x + 2(\sec x)^1 \cdot (\sec x \tan x)$

$y'' = -\cos x + 2\sec^2 x \tan x$

$\boxed{A}$

⊛BC only

AP3

$x = 3\sin t, \quad \frac{dx}{dt} = 3\cos t$

$y = 2\cos t, \quad \frac{dy}{dt} = -2\sin t$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{-2\sin t}{3\cos t} = -\frac{2}{3}\tan t$

$\boxed{C}$

AP4

$s(t) = -t^2 + t + 2 \quad s(m), t(sec), \quad t \geq 0$

a) Initial position: $\boxed{s(0) = 2 \text{ meters}}$

b) $\boxed{v(t) = s'(t) = -2t + 1}$ m/sec

c) Particle moves Right when $v(t) > 0$

$v(t) = -2t + 1 > 0$

$-2t > -1$

$0 \leq t < \frac{1}{2}$

* So particle moves to the right on the interval $0 \leq t < \frac{1}{2}$

d) $a(t) = v'(t) = s''(t) = \boxed{-2 \text{ m/sec}^2}$

e) Speed when $s(t) = 0$:

$s(t) = 0 = -t^2 + t + 2$

$t^2 - t - 2 = 0$

$(t-2)(t+1) = 0$

$t = 2$ or $\cancel{t = -1}$

$\text{Speed}_{\text{at } t=2} = |v(2)| = |-2(2)+1| = |-4+1|$

$= |-3| = \boxed{3 \text{ m/sec}}$

f) Bonus question: @ $t=2$, the particle is speeding up (speed is increasing) since $v(2) = -3$ (neg) and $a(t) = a(2) = -2$ (neg) ($v(t)$ and $a(t)$ have same sign at $t=2$)

— Kanyi