AP Calculus AB
Review 19, No Calculator Permitted on MC

Complete all the following on notebook paper.

_____ 1.

The region in the first quadrant between the x-axis and the graph of $y = 6x - x^2$ is rotated around the y-axis. The volume of the resulting solid of revolution is given by

(A) $\int_0^6 \pi\left(6x - x^2\right)^2 dx$

(B) $\int_0^6 2\pi x\left(6x - x^2\right) dx$

(C) $\int_0^6 \pi x\left(6x - x^2\right)^2 dx$

(D) $\int_0^6 \pi\left(3 + \sqrt{9 - y}\right)^2 dy$

(E) $\int_0^9 \pi\left(3 + \sqrt{9 - y}\right)^2 dy$

_____ 2.

The base of a solid is the region enclosed by the graph of $y = e^{-x}$, the coordinate axes, and the line $x = 3$. If all plane cross sections perpendicular to the x-axis are squares, then its volume is

(A) $\frac{\left(1 - e^{-6}\right)}{2}$ (B) $\frac{1}{2}e^{-6}$ (C) e^{-6} (D) e^{-3} (E) $1 - e^{-3}$

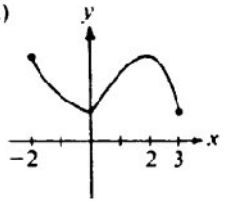
_____ 3.

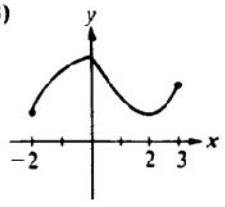
$\int_{-1}^{1} \frac{3}{x^2} dx$ is

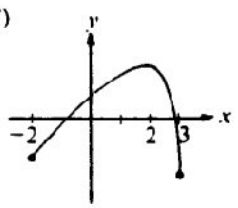
(A) –6 (B) –3 (C) 0 (D) 6 (E) nonexistent

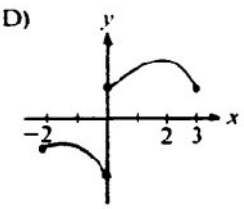
_____ 4.

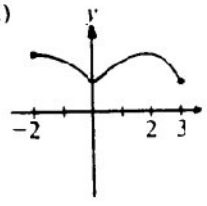
Let f be a function that is continuous on the closed interval $[-2, 3]$ such that $f'(0)$ does not exist, $f'(2) = 0$, and $f''(x) < 0$ for all x except $x = 0$. Which of the following could be the graph of f?

(A) 

(B) 

(C) 

(D) 

(E) 

_____ 5.

If the substitution $u = \frac{x}{2}$ is made, the integral $\int_2^4 \frac{1-\left(\frac{x}{2}\right)^2}{x}\,dx =$

(A) $\int_1^2 \frac{1-u^2}{u}\,du$ (B) $\int_2^4 \frac{1-u^2}{u}\,du$ (C) $\int_1^2 \frac{1-u^2}{2u}\,du$

(D) $\int_1^2 \frac{1-u^2}{4u}\,du$ (E) $\int_2^4 \frac{1-u^2}{2u}\,du$

_____ 6.

At each point (x, y) on a certain curve, the slope of the curve is $3x^2y$. If the curve contains the point $(0,8)$, then its equation is

(A) $y = 8e^{x^3}$ (B) $y = x^3 + 8$ (C) $y = e^{x^3} + 7$

(D) $y = \ln(x+1) + 8$ (E) $y^2 = x^3 + 8$

_____ 7.

What is the domain of the function f given by $f(x) = \frac{\sqrt{x^2-4}}{x-3}$?

(A) $\{x: x \neq 3\}$ (B) $\{x: |x| \leq 2\}$ (C) $\{x: |x| \geq 2\}$

(D) $\{x: |x| \geq 2 \text{ and } x \neq 3\}$ (E) $\{x: x \geq 2 \text{ and } x \neq 3\}$

_____ 8.

A particle with velocity at any time t given by $v(t) = e^t$ moves in a straight line. How far does the particle move from $t = 0$ to $t = 2$?

(A) $e^2 - 1$ (B) $e - 1$ (C) $2e$ (D) e^2 (E) $\frac{e^3}{3}$

_____ 9.

The graph of $y = \frac{-5}{x-2}$ is concave downward for all values of x such that

(A) $x < 0$ (B) $x < 2$ (C) $x < 5$ (D) $x > 0$ (E) $x > 2$

_____ 10.

$$\int \frac{x\,dx}{\sqrt{3x^2+5}} =$$

(A) $\frac{1}{9}\left(3x^2+5\right)^{\frac{3}{2}}+C$ (B) $\frac{1}{4}\left(3x^2+5\right)^{\frac{3}{2}}+C$ (C) $\frac{1}{12}\left(3x^2+5\right)^{\frac{1}{2}}+C$

(D) $\frac{1}{3}\left(3x^2+5\right)^{\frac{1}{2}}+C$ (E) $\frac{3}{2}\left(3x^2+5\right)^{\frac{1}{2}}+C$

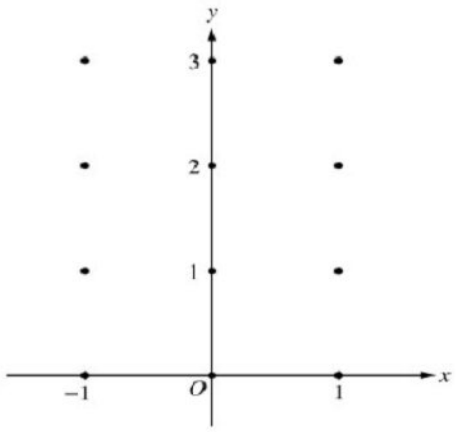
11. 2004-AB6 (No Calculator)

Consider the differential equation $\frac{dy}{dx} = x^2(y-1)$.

(a) On the axes provided, sketch a slope field for the given differential equation at the twelve points indicated. (**Note: Use the axes provided in the pink test booklet.**)

(b) While the slope field in part (a) is drawn at only twelve points, it is defined at every point in the xy-plane. Describe all points in the xy-plane for which the slopes are positive.

(c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$.



12. 2004-AB3 (Calculator Permitted)

A particle moves along the y-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}\left(e^t\right)$.

At time $t = 0$, the particle is at $y = -1$. (Note: $\tan^{-1} x = \arctan x$)

(a) Find the acceleration of the particle at time $t = 2$.

(b) Is the speed of the particle increasing or decreasing at time $t = 2$? Give a reason for your answer.

(c) Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.

(d) Find the position of the particle at time $t = 2$. Is the particle moving toward the origin or away from the origin at time $t = 2$? Justify your answer.