Name_____ Date_____ Period_____

PreAP PreCalculus End-of-Year Review

Calculator Permitted THIS WILL BE GRADED FOR ACCURACY AND WILL COUNT FOR TWO QUIZ GRADES.

MULTIPLE CHOICE (USE CAPITAL LETTERS)

Two triangles can be formed using the given measurements. Solve both triangles.

1) $B = 32°, b = 27, c = 32$ 1) _____

A) $A = 96.9°, C = 51.1°, a = 14.4; A = 83.1°, C = 128.9°, a = 14.4$

B) $A = 109.1°, C = 38.9°, a = 48.1; A = 6.9°, C = 141.1°, a = 6.1$

C) $A = 96.9°, C = 51.1°, a = 50.6; A = 83.1°, C = 128.9°, a = 50.6$

D) $A = 109.1°, C = 38.9°, a = 15.1; A = 6.9°, C = 141.1°, a = 15.1$

Find the exact value of the real number y.

2) $y = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ 2) _____

A) $-\frac{7\pi}{4}$ B) $\frac{\pi}{3}$ C) $-\frac{\pi}{4}$ D) $\frac{\pi}{4}$

3) $y = \arctan(1)$ 3) _____

A) $\frac{3\pi}{4}$ B) $\frac{\pi}{4}$ C) $\frac{2\pi}{3}$ D) $\frac{\pi}{3}$

Find all solutions to the equation in the interval $[0, 2\pi)$.

4) $\sin 2x = -\sin x$ 4) _____

A) $0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$ B) No solution C) $\frac{\pi}{8}, \frac{9\pi}{8}$ D) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Use basic identities to simplify the expression.

5) $\sin^2\theta + \tan^2\theta + \cos^2\theta$ 5) _____

A) $\tan^2\theta$ B) $\cos^3\theta$ C) $\sin\theta$ D) $\sec^2\theta$

6) $\frac{\cos^2\theta}{\sin^2\theta} + \csc\theta \sin\theta$ 6) _____

A) 1 B) $\csc^2\theta$ C) $\sec^2\theta$ D) $\tan^2\theta$

Find the limit of the function algebraically.

7) $\lim_{x \to 3} \sqrt{x - 9}$ 7) _____

A) 2.44948974 B) -2.4494897 C) Does not exist D) 0

8) $\lim_{x \to 6} \frac{x + 6}{(x - 6)^2}$ 8) _____

A) 6 B) Does not exist C) -6 D) 0

Find an equivalent equation in rectangular coordinates.

9) $r = \frac{5}{1 + \cos\theta}$ 9) _____

A) $y^2 = 10x - 25$ B) $x^2 = 25 - 10y$ C) $y^2 = 25 - 10x$ D) $x^2 = 10y - 25$

10) $r = 10 \sin\theta$ 10) _____

A) $\sqrt{x^2 + y^2} = 10x$ B) $x^2 + y^2 = 10y$ C) $\sqrt{x^2 + y^2} = 10y$ D) $x^2 + y^2 = 10x$

Determine the equation of the line described. Put answer in the slope-intercept form, if possible.

11) Through (5, -3), perpendicular to $-8x - 5y = -25$ 11) _____

A) $y = \frac{8}{5}x - 49$ B) $y = -\frac{5}{8}x + \frac{49}{8}$ C) $y = \frac{5}{8}x$ D) $y = \frac{5}{8}x - \frac{49}{8}$

Find the zeros of the function in the interval $[-2\pi, 2\pi]$.

12) $f(x) = \frac{1}{2}\cos 2x$ 12) _____

A) $\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$ B) $0, \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}$

C) $\pm\pi, \pm 3\pi$ D) $\pm\frac{\pi}{4}, \pm\frac{3\pi}{4}, \pm\frac{5\pi}{4}, \pm\frac{7\pi}{4}$

Find the exact value of the composition.

13) $\arctan[\sin(\pi/2)]$ 13) _____

A) $\frac{\pi}{4}$ B) $\frac{\pi}{2}$ C) 0 D) 1

14) $\cos^{-1}\left(\cos\left(-\frac{\pi}{4}\right)\right)$ 14) _____

A) $-\frac{\pi}{4}$ B) $\frac{\pi}{4}$ C) $\frac{5\pi}{4}$ D) $\frac{3\pi}{4}$

Find all solutions in the interval $[0, 2\pi)$.

15) $2\sin^2 x = \sin x$ 15) _____

A) $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{3}, \frac{2\pi}{3}$ B) $x = 0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}$

C) $x = \frac{\pi}{6}, \frac{5\pi}{6}$ D) $x = \frac{\pi}{3}, \frac{2\pi}{3}$

16) $\sin^2\left(\frac{x}{2}\right) = \sin^2 x$ 16) _____

A) $0, \frac{2\pi}{3}$ B) $\frac{2\pi}{3}, \frac{4\pi}{3}$ C) $0, \frac{2\pi}{3}, \frac{4\pi}{3}$ D) $0, \pi$

17) $\tan x + \sec x = 1$ 17) _____

A) No solution B) $x = \frac{\pi}{4}$ C) $x = \frac{5\pi}{4}$ D) $x = 0$

Find the equation of the tangent line to the curve when x has the given value.

18) $f(x) = -5\sqrt{x}$; $x = 9$ 18) _____

A) $y = -\frac{5}{6}x - 10$ B) $y = -\frac{5}{6}x + \frac{15}{2}$ C) $y = \frac{5}{6}x - \frac{15}{2}$ D) $y = -\frac{5}{6}x - \frac{15}{2}$

Find the limit of the function by using direct substitution.

19) $\lim_{x \to 0} (x^2 - 5)$ 19) _____

A) 5 B) 0 C) -5 D) Does not exist

20) $\lim_{x \to 2\pi} \ln(\cos x)$ 20) _____

A) $\ln(2\pi)$ B) 2 C) 1 D) 0

21) $\lim_{x \to 0} \frac{x^3 - 6x + 8}{x - 2}$ 21) _____

A) 4 B) Does not exist C) 0 D) -4

Convert the radian measure to degree measure. Use the value of π found on a calculator and round answers to two decimal places.

22) $\frac{7\pi}{10}$ 22) _____

A) $257.14\pi°$ B) 252° C) 126° D) 154.29°

Give the exact value.

23) $\sec \frac{\pi}{4}$ 23) _____

A) $\sqrt{3}$ B) $\frac{2\sqrt{3}}{3}$ C) $\frac{\sqrt{2}}{2}$ D) $\sqrt{2}$

Find an exact value.

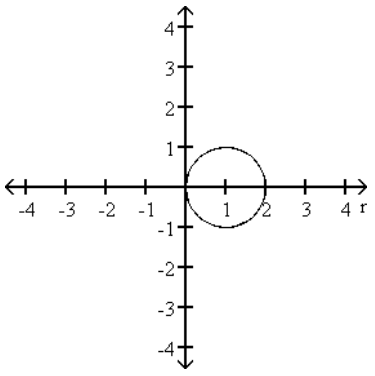
24) $\sin \frac{-11\pi}{12}$ 24) _____

A) $\frac{\sqrt{6} + \sqrt{2}}{4}$ B) $\frac{-\sqrt{6} - \sqrt{2}}{4}$ C) $\frac{\sqrt{2} - \sqrt{6}}{4}$ D) $\frac{\sqrt{6} - \sqrt{2}}{4}$

Use your grapher to determine which of the graphs matches the given polar equation.

25) $r = 1 + 2\cos\theta$ 25) _____

A)



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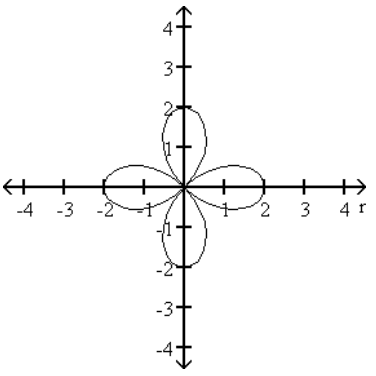

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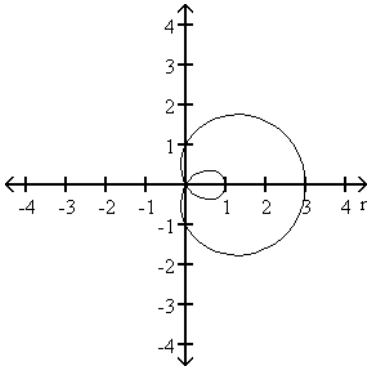

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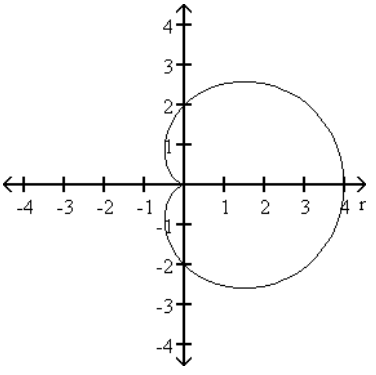
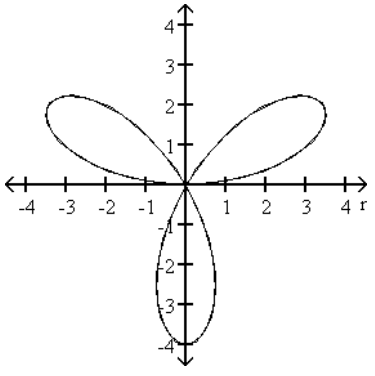


26) $r = 4\cos 3\theta$ 26) _____

A)



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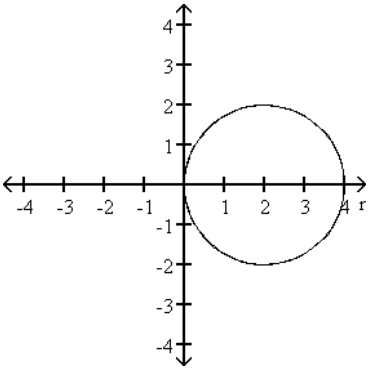

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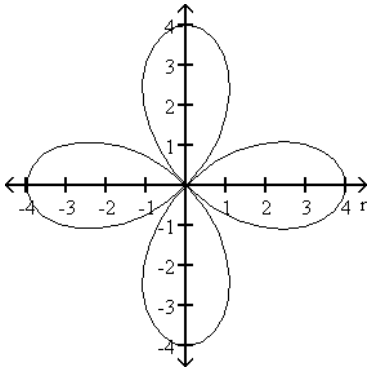
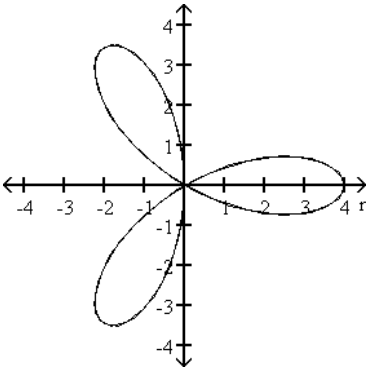


D)



Solve.

27) A ranger in fire tower A spots a fire at a direction of 40°. A ranger in fire tower B, which is 28 miles directly east of tower A, spots the same fire at a direction of 116°. How far from tower A is the fire? 27) _____

A) 26 miles B) 22 miles C) 24 miles D) 33 miles

Find the measures of two angles, one positive and one negative, that are coterminal with the given angle.

28) $\frac{9\pi}{5}$ 28) _____

A) $\frac{14\pi}{5}; -\frac{\pi}{5}$ B) $\frac{19\pi}{5}; -\frac{\pi}{5}$ C) $\frac{\pi}{5}; -\frac{19\pi}{5}$ D) $\frac{14\pi}{5}; -\frac{14\pi}{5}$

Simplify the expression.

29) $\frac{1}{\csc x - \cot x} + \frac{1}{\csc x + \cot x}$ 29) _____

A) 2 cot x B) $\csc^2 x$ C) csc x D) 2 csc x

30) $\cos\left(\frac{\pi}{2} - x\right) \csc(-x)$ 30) _____

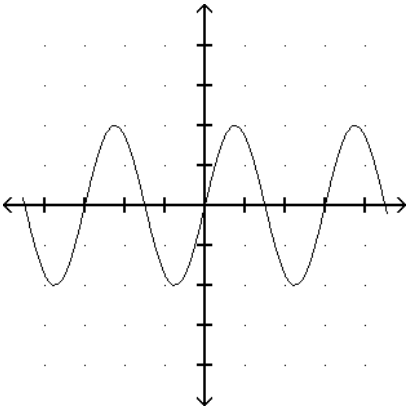
A) $-\sin^2 x$ B) 1 C) -1 D) -cot x

31) $\frac{1 - \sin^2 x}{\sin x - \csc x}$ 31) _____

A) -cos x B) $\cos^2 x$ C) $\sin^2 x$ D) -sin x

Find the period of the function.

32) y = 2 cos 6x 32) _____



A) 2π B) $\frac{\pi}{6}$ C) $\frac{\pi}{3}$ D) 2

State whether the given measurements determine zero, one, or two triangles.

33) A = 66°, a = 26, b = 28 33) _____

A) Zero B) Two C) One

Find an equivalent equation in polar coordinates.

34) $x^2 + y^2 - 4x = 0$ 34) _____

A) $r \sin^2\theta = 4 \cos\theta$ B) $r = 4 \sin\theta$

C) $r \cos^2\theta = 4 \sin\theta$ D) $r = 4 \cos\theta$

Solve the problem.

35) Suppose that the average monthly low temperatures for a small town are shown in the table. 35) _____

Month	1	2	3	4	5	6	7	8	9	10	11	12
Temperature (°F)	19	27	38	45	57	62	65	58	51	41	33	25

Model this data using $f(x) = a \sin(b(x - c)) + d$.

A) $f(x) = 23 \sin\left[\frac{\pi}{6}(x - 4)\right] + 42$ B) $f(x) = 42 \sin\left[\frac{\pi}{6}(x - 4)\right] + 23$

C) $f(x) = 23 \sin\left[\frac{\pi}{12}(x - 4)\right] + 42$ D) $f(x) = 23 \sin\left[\frac{\pi}{6}(x - 7)\right] + 42$

36) The position of an object at time t is given by s(t). Find the instantaneous rate of change (velocity) at the indicated value of t. 36) _____

$s(t) = \frac{2}{t - 3}$ at $t = 0$

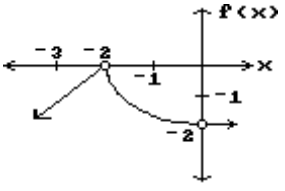
A) $\frac{4}{3}$ B) $-\frac{4}{9}$ C) $-\frac{2}{9}$ D) $-\frac{2}{3}$

37) A kite is currently flying at an altitude of 17 meters above the ground. If the angle of elevation from the ground to the kite is 30°, find the length of the kite string to the nearest meter. 37) _____

A) 20 meters B) 29 meters C) 34 meters D) 8 meters

Use the given graph to determine the limit, if it exists.

38) $\lim_{x \to 0} f(x)$ 38) _____



A) -1 B) 0 C) Does not exist D) -2

Rewrite with only sin x and cos x.

39) $\sin 2x - \cos 2x$ 39) _____

A) $2 \sin^2 x + 2 \sin x \cos x - 1$ B) $2 \sin^2 x - 2 \sin x \cos x + 1$

C) $2 \sin x$ D) $2 \sin^2 x - 2 \sin x \cos x - 1$

Find an algebraic expression equivalent to the given expression.

40) $\sin(\text{arcsec } u)$ 40) _____

A) $\frac{u\sqrt{u^2 + 1}}{u2 + 1}$ B) $\sqrt{u^2 - 1}$ C) $\sqrt{u^2 + 1}$ D) $\frac{\sqrt{u^2 - 1}}{u}$

Find the derivative of the function at the specified point.

41) $f(x) = -2x^2 + 8x$ at $x = 7$ 41) _____

A) -20 B) -147 C) 28 D) 4

Find the specified quantity.

42) Find the phase shift of $y = -4 - 3 \sin\left(5x - \frac{\pi}{3}\right)$. 42) _____

A) $\frac{\pi}{15}$ B) $\frac{\pi}{9}$ C) $-\frac{\pi}{3}$ D) $-\frac{\pi}{15}$

The given measurements may or may not determine a triangle. If not, then state that no triangle is formed. If a triangle is formed, then use the Law of Sines to solve the triangle, if it is possible, or state that the Law of Sines cannot be used.

43) $C = -3°$, $a = 46$, $c = 22$ 43) _____

A) No triangle is formed.

B) $A = 177.8°$, $B = 5.2°$, $b \approx -15.8$

C) $A = 5.2°$, $B = 177.8°$, $b \approx -15.8$

D) The triangle cannot be solved with the Law of Sines.

Suppose that θ is in standard position and the given point is on the terminal side of θ. Give the exact value of the indicated trig function for θ.

44) (6, 8); find $\cos \theta$. 44) _____

A) $\frac{4}{3}$ B) $\frac{4}{5}$ C) $\frac{3}{4}$ D) $\frac{3}{5}$

Use graphs and tables to find the limit and identify any vertical asymptotes.

45) $\lim\limits_{x \to 2^-} \frac{1}{x - 2}$ 45) _____

A) $-\infty$; $x = -2$ B) 1 ; no vertical asymptotes

C) $-\infty$; $x = 2$ D) ∞ ; $x = -2$

Find the rectangular coordinates of the point with the given polar coordinates.

46) $(\sqrt{3}, \pi/6)$ 46) _____

A) $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ B) $\left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right)$ C) $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ D) $\left(\frac{\sqrt{3}}{2}, \frac{3}{2}\right)$

Solve the triangle.

47) $A = 49°$, $B = 64°$, $a = 8$ 47) _____

A) $C = 23°$, $b \approx 6.7$, $c \approx 8.2$ B) $C = 67°$, $b \approx 9.5$, $c \approx 9.8$

C) $C = 67°$, $b \approx 6.7$, $c \approx 8.2$ D) $C = 67°$, $b \approx 9.5$, $c \approx 8.2$

Write each expression in factored form as an algebraic expression of a single trigonometric function.

48) $1 - 2 \sin^2 x + \sin^4 x$ 48) _____

A) $\cos^4 x$ B) $(1 + \tan^2 x)$ C) $(1 - \sin^2 x)$ D) $\sin^2 x$

Convert from degrees to radians. Use the value of π found on a calculator and round answers to four decimal places, as needed.

49) 570° 49) _____

A) $\frac{19\pi}{3}$ B) $\frac{19\pi}{12}$ C) $\frac{19\pi}{5}$ D) $\frac{19\pi}{6}$

Assume that θ is an acute angle in a right triangle satisfying the given conditions. Evaluate the indicated trigonometric function.

50) $\sin\theta = \frac{8}{9}$; $\cot\theta$ 50) _____

A) $\frac{8}{\sqrt{17}}$ B) $\frac{\sqrt{17}}{9}$ C) $\frac{\sqrt{17}}{8}$ D) $\frac{9}{\sqrt{17}}$

Find the value of the unique real number θ between 0 and 2π that satisfies the given conditions.

51) $\cos\theta = -\frac{\sqrt{2}}{2}$ and $\tan\theta > 0$ 51) _____

A) $\frac{3\pi}{4}$ B) $\frac{4\pi}{3}$ C) $\frac{5\pi}{4}$ D) $\frac{\pi}{4}$

Solve the equation.

52) Solve $\tan\theta = \frac{1}{\sqrt{3}}$ for θ, where $0 \le \theta \le \frac{\pi}{2}$. 52) _____

A) $\frac{\pi}{6}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{2}$ D) $\frac{\pi}{3}$

Convert the angle to decimal degrees and round to the nearest hundredth of a degree.

53) 138°48′17″ 53) _____

A) 138.80 B) 138.81 C) 138.76 D) 138.86

Use the arc length formula and the given information to find the indicated quantity.

54) s = 2 m, r = 3 m; find θ 54) _____

A) 6 rad B) $\frac{2}{3}$ rad C) 12 rad D) $\frac{3}{2}$ rad

Use the definition $f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ to find the derivative of the given function at the indicated point.

55) $f(x) = -9/x$, $a = -11$ 55) _____

A) $\frac{121}{9}$ B) $\frac{9}{11}$ C) $\frac{11}{9}$ D) $\frac{9}{121}$

56) $f(x) = 6 - 8x^2$, $a = 2$ 56) _____

A) -32 B) 32 C) 38 D) -26

57) $f(x) = 19 - 20x$, $a = 1$ 57) _____

A) 19 B) -19 C) - 20 D) -1

Solve the problem.

58) Find an equation of the tangent line to the graph of $y = x^2 - x$, at the point (2, 2). 58) _____

A) $y = 3x - 6$ B) $y = 3x - 4$ C) $y = 3x + 4$ D) $y = 3x + 6$

59) Find the equation of the normal line to the curve $y = 3x - 5x^2$ at the point (-3, -54). 59) _____

A) $x - 27y + 975 = 0$ B) $x + 33y + 975 = 0$

C) $x + 33y + 1785 = 0$ D) $x - 27y + 1785 = 0$

Find y', the derivative of y.

60) $y = \frac{1}{2}x^8 - \frac{1}{4}x^4$ 60) _____

A) $4x^8 - x^4$ B) $\frac{1}{2}x^7 - \frac{1}{4}x^3$ C) $4x^7 - x^3$ D) $4x^9 - x^5$

Find the horizontal tangents of the curve.

61) $y = x^4 - 2x^2 + 1$ 61) _____

A) At x = 0, 1 B) At x = 0 C) At x = 1, -1, D) At x = 0, 1, -1

Identify the type of polar graph.

62) $r = 4 \cos 2\theta$ 62) _____

A) lemniscate B) rose with 4 petals

C) rose with 2 petals D) cardioid

63) $r = 2 - 4 \cos \theta$ 63) _____

A) lemniscate B) cardioid

C) limacon with loop D) rose with 4 petals

64) $r = 4 + 4 \sin \theta$ 64) _____

A) lemniscate B) limacon with loop

C) dimpled limacon D) cardiod

65) $r = 7 - 6 \sin \theta$ 65) _____

A) limacon with loop B) cardiod

C) dimpled limacon D) lemniscate

66) $r = \frac{2}{3 \cos \theta - 5 \sin \theta}$ 66) _____

A) lemniscate B) straight line

C) cardioid D) limacon with loop

The rectangular coordinates of a point are given. Express the point in polar coordinates with $r \geq 0$ and $0 \leq \theta < 2\pi$.

67) $\left(\frac{1}{6}, \frac{\sqrt{3}}{6}\right)$ 67) _____

A) $\left(\frac{1}{3}, \frac{\pi}{6}\right)$ B) $\left(\frac{1}{6}, \frac{\pi}{6}\right)$ C) $\left(\frac{1}{3}, \frac{2\pi}{3}\right)$ D) $\left(\frac{1}{3}, \frac{\pi}{3}\right)$