PCPAP Review 5.1 - 5.4 Key (Proofs may vary in solutions)

① $\sin x(\tan x\cos x - \cot x\cos x) \stackrel{?}{=} 1 - 2\cos^2 x$

$\sin x\left(\frac{\sin x}{\cos x}\cdot\cos x - \frac{\cos x}{\sin x}\cdot\cos x\right)$

$\sin x\left(\sin x - \frac{\cos^2 x}{\sin x}\right)$

$\sin^2 x - \cos^2 x$

$(1-\cos^2 x) - \cos^2 x$

$1 - 2\cos^2 x$

Q.E.D

② $\cos x\csc x\tan x \stackrel{?}{=} 1$

$\cos x\left(\frac{1}{\sin x}\right)\left(\frac{\sin x}{\cos x}\right)$

$\frac{\sin x\cos x}{\sin x\cos x}$

1

QED

③ $\frac{\cot^2 x}{\csc x - 1} \stackrel{?}{=} \frac{1+\sin x}{\sin x}$

$\frac{\csc^2 x - 1}{\csc x - 1}$

$\frac{\cancel{(\csc x - 1)}(\csc x + 1)}{\cancel{(\csc x - 1)}}$

$\csc x + 1$

$\frac{1}{\sin x} + \frac{\sin x}{\sin x}$

$\frac{1+\sin x}{\sin x}$

QED

④ $\sec^4 x - \tan^4 x \stackrel{?}{=} \sec^2 x + \tan^2 x$

$(\sec^2 x - \tan^2 x)(\sec^2 x + \tan^2 x)$

$1\cdot(\sec^2 x + \tan^2 x)$

$\sec^2 x + \tan^2 x$

QED

⑤ $\frac{\sin x + \cos x}{\sin x - \cos x} \stackrel{?}{=} \frac{1 + 2\sin x\cos x}{2\sin^2 x - 1}$

$\left(\frac{\sin x + \cos x}{\sin x + \cos x}\right)\frac{\sin x + \cos x}{\sin x - \cos x}$

$\frac{\sin^2 x + 2\sin x\cos x + \cos^2 x}{\sin^2 x - \cos^2 x}$

$\frac{1 + 2\sin x\cos x}{\sin^2 x - (1 - \sin^2 x)}$

$\frac{1 + 2\sin x\cos x}{2\sin^2 x - 1}$

QED

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$\sin^2 x - \cos^2 x$

$(1-\cos^2 x) - \cos^2 x$

$1 - 2\cos^2 x$

Q.E.D

② $\cos x\csc x\tan x \stackrel{?}{=} 1$

$\cos x\left(\frac{1}{\sin x}\right)\left(\frac{\sin x}{\cos x}\right)$

$\frac{\sin x\cos x}{\sin x\cos x}$

1

QED

③ $\frac{\cot^2 x}{\csc x - 1} \stackrel{?}{=} \frac{1+\sin x}{\sin x}$

$\frac{\csc^2 x - 1}{\csc x - 1}$

$\frac{\cancel{(\csc x - 1)}(\csc x + 1)}{\cancel{(\csc x - 1)}}$

$\csc x + 1$

$\frac{1}{\sin x} + \frac{\sin x}{\sin x}$

$\frac{1+\sin x}{\sin x}$

QED

④ $\sec^4 x - \tan^4 x \stackrel{?}{=} \sec^2 x + \tan^2 x$

$(\sec^2 x - \tan^2 x)(\sec^2 x + \tan^2 x)$

$1\cdot(\sec^2 x + \tan^2 x)$

$\sec^2 x + \tan^2 x$

QED

⑤ $\frac{\sin x + \cos x}{\sin x - \cos x} \stackrel{?}{=} \frac{1 + 2\sin x\cos x}{2\sin^2 x - 1}$

$\left(\frac{\sin x + \cos x}{\sin x + \cos x}\right)\frac{\sin x + \cos x}{\sin x - \cos x}$

$\frac{\sin^2 x + 2\sin x\cos x + \cos^2 x}{\sin^2 x - \cos^2 x}$

$\frac{1 + 2\sin x\cos x}{\sin^2 x - (1 - \sin^2 x)}$

$\frac{1 + 2\sin x\cos x}{2\sin^2 x - 1}$

QED

Review §5.1–5.4 PCPAP

(6) $\sin^3 x \cos^2 x \stackrel{?}{=} \sin x\left(\cos^2 x - \cos^4 x\right)$

$\sin x \cdot \cos^2 x\left(1 - \cos^2 x\right)$

$\sin x \cdot \cos^2 x\left(\sin^2 x\right)$

$\sin^3 x \cdot \cos^2 x$

QED

(7) $\dfrac{\cos x}{\sec x - 1} - \dfrac{\cos x}{\sec x + 1} = \dfrac{2\cos x}{\tan^2 x}$

$\dfrac{\cos x(\sec x + 1) - \cos x(\sec x - 1)}{(\sec x - 1)(\sec x + 1)}$

$\dfrac{1 + \cos x - 1 + \cos x}{\sec^2 x - 1}$

$\dfrac{2\cos x}{\tan^2 x}$

QED

(8) $\cos\left(x + \frac{\pi}{6}\right) \stackrel{?}{=} \frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$

$\cos x \cos\frac{\pi}{6} - \sin x \sin\frac{\pi}{6}$

$\frac{\sqrt{3}}{2}\cos x - \frac{1}{2}\sin x$

QED.

(9) $\cos 4x + \cos 2x \stackrel{?}{=} 2 - 2\sin^2 2x - 2\sin^2 x$

$\cos(2(2x)) + \cos^2 x - \sin^2 x$

$(\cos^2 2x) - \sin^2 2x + (\cos^2 x) - \sin^2 x$

$(1 - \sin^2 2x) - \sin^2 2x + (1 - \sin^2 x) - \sin^2 x$

$2 - 2\sin^2 2x - 2\sin^2 x$

QED

(10) $\cos(x - y) - \cos(x + y) \stackrel{?}{=} 2\sin x \sin y$

$\cos x \cos y + \sin x \sin y - (\cos x \cos y - \sin x \sin y)$

$\cancel{\cos x \cos y} + \underline{\sin x \sin y} - \cancel{\cos x \cos y} + \underline{\sin x \sin y}$

$2\sin x \sin y$

QED

(11) $\cos 4u \stackrel{?}{=} \cos^2 2u - \sin^2 2u$

$\cos(2(2u))$

$\cos^2 2u - \sin^2 2u$

QED

(12) $\cos 3x \stackrel{?}{=} \cos^3 x - 3\sin^2 x \cos x$

$\cos(x + 2x)$

$\cos x \cos 2x - \sin x \sin 2x$

$\cos x\left(\cos^2 x - \sin^2 x\right) - \sin x\left(2\sin x \cos x\right)$

$\cos^3 x - \cos x \sin^2 x - 2\sin^2 x \cos x$

$\cos^3 x - 3\sin^2 x \cos x$

QED

Review §5.1–5.4 ACPAP

(13) Solve: $\cos^2 x + 2\cos x + 1 = 0$

$(\cos x + 1)^2 = 0$

$\cos x + 1 = 0$

$\cos x = -1$

$\boxed{x = \pi}$

(14) Solve $x \in [0, 2\pi)$

$2\sin^2 x = \sin x$

$2\sin^2 x - \sin x = 0$

$\sin x(2\sin x - 1) = 0$

$\sin x = 0$ or $\sin x = \frac{1}{2}$

$\boxed{x = 0, \pi}$ $\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$

(15) Solve

$\cos x = \sin x$

* from unit circle

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right), \left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\boxed{x = \frac{\pi}{4}, \frac{5\pi}{4}}$

(16) Solve $\sec^2 x - 2 = \tan^2 x$

$(\sec^2 x - \tan^2 x) - 2 = 0$

$1 - 2 = 0$

$-1 \neq 0$

$\boxed{\text{No solution}}$

(17) Solve

$\sin(\cos x) = 0$

$\sin^{-1}(\sin(\cos x)) = \sin^{-1} 0$

$\cos x = 0$ or $\cos x = \pi$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$; $x = \cos^{-1}\pi$

$x =$ No Solution

(18) Solve

$\cos x = \sin 2x$

$\cos x - \sin 2x = 0$

$\cos x - 2\sin x\cos x = 0$

$\cos x(1 - 2\sin x) = 0$

$\cos x = 0$ or $\sin x = \frac{1}{2}$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$; $\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}}$

(19) Solve:

$\cos x - \cos 3x = 0$

$\cos x - \cos(x + 2x) = 0$

$\cos x - [\cos x\cos 2x - \sin x\sin 2x] = 0$

$\cos x - \cos x(\cos^2 x - \sin^2 x) + \sin x(2\sin x\cos x) = 0$

$\cos x[1 - \cos^2 x + \sin^2 x + 2\sin^2 x] = 0$

$\cos x[1 - (\cos^2 x) + 3\sin^2 x] = 0$

$\cos x[1 - (1 - \sin^2 x) + 3\sin^2 x] = 0$

$\cos x[4\sin^2 x] = 0$

$\cos x = 0$ or $4\sin^2 x = 0$

$\sin^2 x = 0$

$\sin x = 0$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$ $\boxed{x = 0, \pi}$

PCPAP Review §5.1 - §5.4

20) $\sin\frac{11\pi}{12} = \sin(165^\circ) = \sin(225^\circ - 60^\circ)$

$= \sin 225^\circ \cos 60^\circ - \cos 225^\circ \sin 60^\circ$

$\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) - \left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$-\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$\boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$

21) $\cos\frac{\pi}{12} = \cos(15^\circ) = \cos(45^\circ - 30^\circ)$

$= \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$= \boxed{\frac{\sqrt{6} + \sqrt{2}}{4}}$

22) $\tan\frac{7\pi}{12} = \tan(105^\circ) = \tan(60^\circ + 45^\circ) = \frac{\sin(60^\circ + 45^\circ)}{\cos(60^\circ + 45^\circ)} = \frac{\sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ}{\cos 60^\circ \cos 45^\circ - \sin 60^\circ \sin 45^\circ}$

$= \frac{\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)} = \frac{\frac{\sqrt{6} + \sqrt{2}}{4}}{\frac{\sqrt{2} - \sqrt{6}}{4}} = \frac{\sqrt{2} + \sqrt{6}}{\sqrt{2} - \sqrt{6}}\left(\frac{\sqrt{2} + \sqrt{6}}{\sqrt{2} + \sqrt{6}}\right) = \frac{2 + 2\sqrt{12} + 6}{2 - 6} = \frac{8 + 2 \cdot 2\sqrt{3}}{-4}$

$= \frac{8 + 4\sqrt{3}}{-4} = \frac{4(2 + \sqrt{3})}{-4} = \boxed{-2 - \sqrt{3}}$

23) $\sin\frac{\pi}{5} \cdot \cos\frac{\pi}{11} + \cos\frac{\pi}{5}\sin\frac{\pi}{11}$

$= \sin\left(\frac{\pi}{5} + \frac{\pi}{11}\right) = \sin\left(\frac{11\pi + 5\pi}{55}\right)$

$= \boxed{\sin\left(\frac{16\pi}{55}\right)}$

24) $\cos 7x \cos 2x - \sin 7x \sin 2x$

$\cos(7x + 2x)$

$\boxed{\cos(9x)}$

— done

Test: Part I: Prove 5 of 6 Identities

(calculator ok) Part II: Solve 2 of 3 equations

Part III: Prove 5 of 6 Identities

No Multiple Choice, All Free Respose.
Everything has equal weight.