PreAP Precalculus: Applications of Sinusoids

Example 1: Here is data obtained from the National Weather Service for DFW temperatures in 2009.

Month	Average Temperature in °F
January	44.15
February	48.05
March	56.35
April	65.25
May	73.1
June	81.2
July	85.35
August	84.8
September	77.75
October	67.4
November	55.56
December	47.45

a. Sketch a graph showing how the temperature depends on the time of the year. Also, enter the data in the calculator and create a scatter plot graph.

b. Suppose we wanted to find an equation to fit the data. Since we know that temperatures usually go up and down on a fairly regular basis, a sine curve of the general form $y = A\sin\left(B\left(x - C\right)\right) + D$ is a fairly reasonable starting point. From our previous work with sine curves we know that $y = \sin x$ has a maximum of 1, a minimum of –1 and is centered on the line y = 0. We need to determine the value of the coefficients *A*, *B*, *C*, and *D* to fit the curve to the data. This can be done by hand using the steps that follow.

1. The coefficient *D* tells us the location of the center line. The average of the maximum temperature and the minimum temperature will tell us the value of *D*.
   - What is the maximum temperature of the data set?
   - What is the minimum temperature of the data set?
   - What is the average of the maximum and minimum temperatures?
   - What is the value of the coefficient *D*?

2. The coefficient *A* affects the amplitude. Another way to say this is that A is the distance from the center line to the maximum or minimum on a sine curve.

- What is the value of the coefficient A?

3. The coefficient B affects the period of a sine function. From prior studies we know that the sine function has a period of 2π when the coefficient B is 1, so B is the number of cycles in 2π radians.
   - Since our temperature data has a period of 12 months, what is the value of the coefficient B?

4. Recall that the coefficient C determines the horizontal phase shift of the sine curve. From the graph it is hard to identify the exact value of C, so assume that C = 0 to get started, then adjust as necessary.

5. Enter the equation you have just built into your calculator. Graph the equation to see how well it fits the scatter plot. Then experiment with values of C until you are satisfied that your equation fits the data.
   - What is the value of the coefficient C?

   - What is your equation?

c. Using a TI-83, find a sinusoidal regression equation of the form $y = A\sin\left(B\left(x - C\right)\right) + D$ to fit the data. Record the calculator's equation, based on its values of A, B, C and D.

d. Graph the regression equation you found on the calculator to see how well it fits the scatter plot.
   - Describe how well the equation fits the data.

   - Compare your results with the answers you computed. Are the results identical?

   - Why do you think it might be important to know how to use both techniques?

Example 2: A signal buoy in the Chesapeake Bay bobs up and down with the height of its transmitter (in feet) above sea level modeled by $h = a\sin bt + 5$. During a small squall its height varies from 1 ft to 9 ft and there are 3.5 sec from one 9-ft. height to the next. What are the values of the constants *a* and *b*?

Example 3: The depth of water at the end of a pier varies with the tides throughout the day. Today the high tide occurs at 4:15 a.m. with a depth of 5.2 m. The low tide occurs at 10:27 a.m. with a depth of 2.0 m.

a. Sketch a graph showing how the depth of the water depends on the time since midnight.

b. Find a trigonometric equation that models the depth of the water *t* hours after midnight.

c. Find the depth of the water at noon.

d. A large boat needs at least 3 m of water to moor at the end of the pier. During what time period after noon can it safely moor?

Example 4: The approximate number of hours between sunrise and sunset in Denver varies throughout the year as shown in the chart below.

Date	Day of the year	# hours between sunrise & sunset
January 1	1	9
March 21	80	12
June 21	172	15
December 21	355	9

a. Sketch a graph that shows how the number of hours between sunrise and sunset depends on the day of the year.

b. Give the period, amplitude and an equation of the daylight hours graph.

c. Find the amount of daylight in Denver on July 4.

d. Over the course of a year, during what period of time is the amount of daylight in Denver at least 14 hours?

e. If you were to draw a daylight-hours graph for Seattle, how do you think its amplitude would compare to that of the Denver curve?

Example 5: One particular July 4th in Galveston, high tide occurred at 9:36 a.m. At that time the water at the end of the 61st Street Pier was 2.7 meters deep. Low tide occurred at 3:48 p.m., at which time the water was only 2.1 meters deep. Assume that the depth of the water can be modeled by a trigonometric function whose period is half a lunar day (about 12 hours 24 minutes).

a. Sketch a graph showing how the depth of the water depends on the time since midnight.

b. Find a trigonometric equation that models the depth of the water *t* hours after midnight.

c. At what time on the 4th of July did the first low tide occur?

d. What was the approximate depth of the water at 6 a.m. and at 3 p.m. that day?

e. What was the first time on July 4th when the water was 2.4 meters deep?

HOMEWORK: Application of Sinusoidal Curves

1. Determine the sinusoidal equation of a curve with;
   max = 4
   min = -1
   repeats every 5 units
   the point (3,1.5) is on the curve and "dropping" at that point

2. The pedals of a bicycle are mounted on a bracket whose center is 29 cm above the ground. Each pedal is 16.5 cm from the center of the bracket. Assume that the bicycle is pedaled at 12 cycles per minute. With the starting position of the pedal in a horizontal position at $t = 0$;

   a) Sketch the graph of this sinusoidal function of the height of one pedal for the first three cycles. Label and number the axes appropriately.

   b) Write an equation for this function expressing the height (y) as a function of time (x).

   c) When is the pedal 40 cm above the ground?

   d) How high is the pedal after 23 seconds?

3. An electric water heater cuts in and out on a cyclical basis as it heats the water in a hot tub. The water temperature varies sinusoidally over time. The thermostat turns the heater on when it reaches a low of 30ºC and shuts off when it gets up to 40ºC. It takes 45 minutes for the heater to warm the water the 10ºC, and then it is another 45 minutes as the water cools back down again.

   a) Sketch this relation for exactly 2 hours if the initial temperature was 40ºC. Label and number the axes appropriately.

   b) Write an equation for this function expressing the temperature as a function of time.

   c) How warm is the water 15 minutes after the heater shuts off?

   d) How warm is the water 15 minutes after the heater cuts in?

4. The average daily low for the city of Steinbach reaches –12.6°C on January 10th. The average daily high for the city is 24.2°C on July 11th. *(Assume no leap year.)*

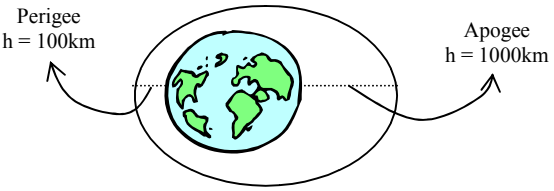
a) The amplitude is half the range of the temperatures. What is this value?
b) The temperatures repeats every year, therefore the period is 365 days.
What is the value of 'b' in the sinusoidal equation?
c) The value of 'c' can be found at the rising point between the max and min.
What is this point? What is the value of 'c' in the sinusoidal equation?
d) The 'y' value at the centerline is equal to the average temperature.
What is this value, and thus the value of 'd'?
e) Write a sinusoidal model for a positive sine graph for the average daily temperatures, where $t = 1$ corresponds to January 1st.
f) According to your model, what will the temperature likely be on March 11?
g) What day of the year will the temperature first likely be the average?
h) For how many days per year is the temperature likely to be at least 20°C?

5. A tsunami (commonly called a tidal wave) is a fast moving ocean wave caused by earthquakes underwater. The water first goes down from its normal level, and then rises an equal distance above its normal level, then returns to its normal level. Suppose the amplitude of a tsunami approaching Honolulu Pier is 10 meters, where the depth of the water is normally 9 meters.

An equation to model the depth of the water during this tsunami is, $y = 10\sin\left[\frac{2\pi}{15}(x - 7.5)\right] + 9$, where y = the water's depth (m) and x = time (min).

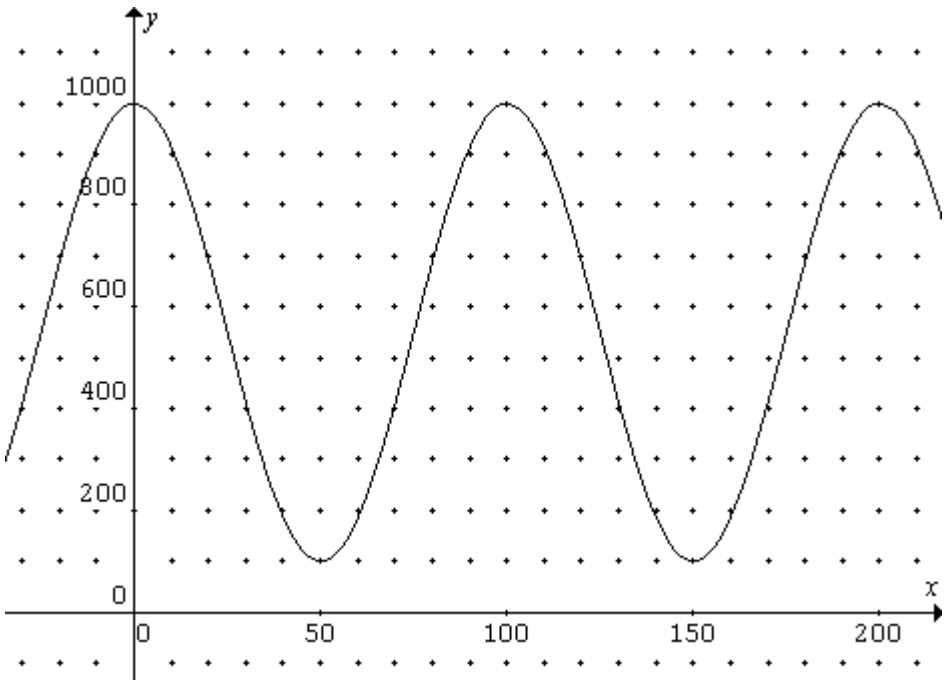
a) Create a graph showing the depth of water over from 10 minutes before the tsunami hits to 10 minutes after it passes.
b) What is the period of this function? What does this number represent?
c) What does the *x*-axis represent in this model?
d) Is there a time period when the pier is drained of water? Explain.
e) A sunken treasure rests 5 meters below the normal surface of the water. For how long will it be exposed before the crest of the tsunami crushes everything in its path?
f) The wavelength of a wave is the distance from one crest of a wave to the next crest. Our tsunami travels at 1200 km/hr. What is its wavelength?

6. A spacecraft is in an elliptical orbit around Earth as shown in the diagram.



At time x = 0 hours, it's at its apogee *(highest point)* y = 1000 km above the Earth's surface. Fifty minutes later, it's at its perigee *(lowest point)* y = 100 km.

The sketch below shows this sinusoidal relationship.



a) In order to transmit data to Earth, it must be within 700 km of the Earth's surface. For how many consecutive minutes *(correct to 2 decimal places)* will the spacecraft be able to transmit? Sketch this info on the diagram.

b) The spacecraft is vulnerable to attack from ground missiles for 20 minutes while it is traveling through its lowest orbit. What is the maximum height *(correct to 3 decimal places)* these missiles can reach? Sketch this info on the diagram.

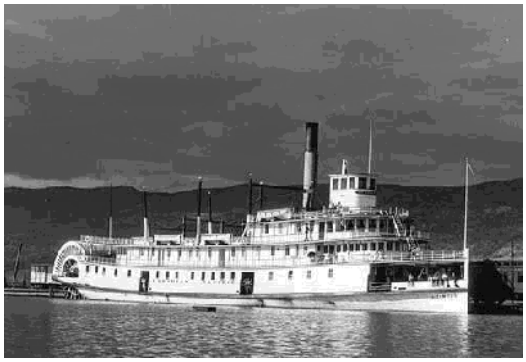
7. The average temperature in Buenos Aires for 2003 is shown below.
   Using your TI-83 determine the sinusoidal equation for the data.
   a) Determine the expected temperature on June 9th.
   b) For how many consecutive days was the temperature no more than 12°C?

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Temp	23	22	20	16	13	10	10	12	14	16	18	22

8. The paddlewheel of an old riverboat is 10 meters in diameter.
   It spins on its axis that is 2 meters above the water's surface.
   It spins at a rate of 15 revolutions per minute.

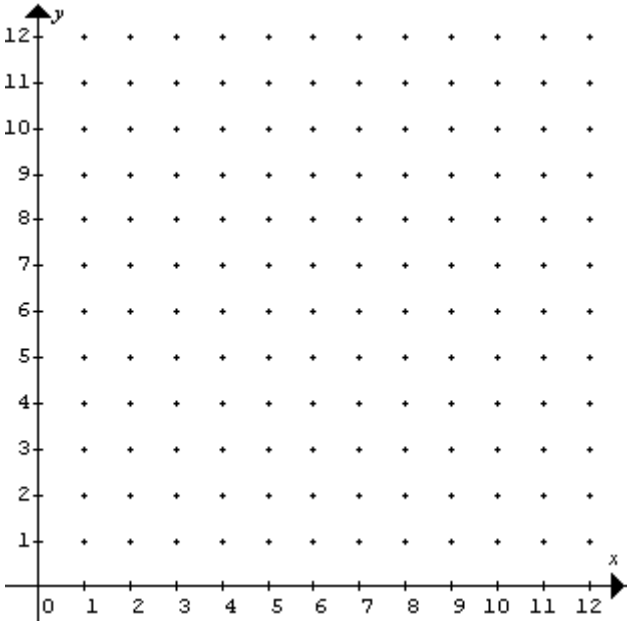
   Rising from the water, one of the paddles picks up a lily pad which sticks to the end of the 5 meter paddle. The height of the lily pad varies sinusoidally with time. Sketch the height of the lily pad from when it was first picked up by the wheel, to when it flew off (see below). Label the axes appropriately.

   The lily pad stayed on the wheel as it went through the water twice, but fell off after 9 seconds. At what height above the water did the lily pad fall off?



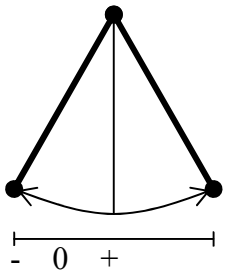
   HINT: Of course you must first find the equation.
   Getting a, b, and d should not be that hard by now. Getting c is still fun, though.

9. The number of people employed by a restaurant is modeled by the function;
$y = 3\sin(0.52x + 1) + 7.2$, where x represents the month and y represents the number of people employed (assume January = month 1).
   a) Sketch a graph for this function for one year.
   b) What is the maximum number of people employed?



10. A pendulum is observed and data is collected as shown below. When the pendulum is to the right of the resting place, a positive distance is recorded. A negative distance indicates the pendulum is to the left of the resting place.

time (sec)	distance (cm)
1.70	1.81
2.98	6.76
3.49	-7.89
4.77	-0.52
5.85	12.67



   a) Write an equation to represent the sinusoidal motion of the pendulum.
   b) What is the position of the pendulum when after 5 seconds?

11. The Great Canadian Bungee Company offers jumps from a 200-ft bridge. A jumper's height above the water for the first 10 seconds is modeled by;

$$y = 98\sin\left(0.63x + 1.5\right) + 102$$

where y is the height in feet and x is the time in seconds.

a) Determine the period and the amplitude.
b) Describe what the period represents in this situation.

12. The following chart gives the average monthly temperature for Morris in ˚C.

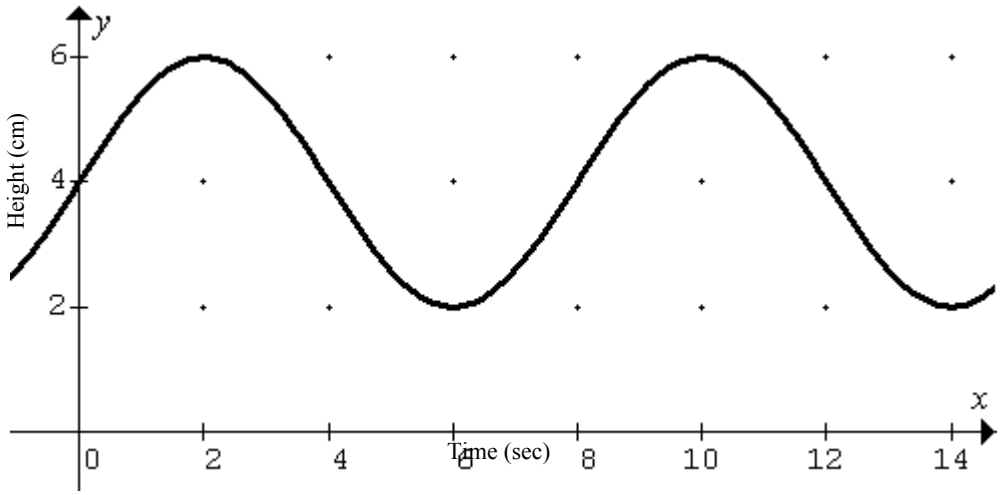
Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
-23.6	-24.3	-19.7	-10.9	-0.4	9.2	15.1	15.9	11.2	2.5	-8	-17.6

a) Using January as Month 1, represent the data using a sinusoidal equation using your TI-83
b) According to your equation, what is the maximum temperature in Morris?

13. The centre of a bicycle wheel is 33cm above the ground. A reflector mounted on the spokes is located 17.5cm from the centre of the wheel that is turning at a constant rate of one revolution per second. If y is the height of the reflector in cm, and x represents the time in seconds;

a) What is the amplitude (a) of the reflector?
b) What is the period? What is the value of 'b' in the equation?
c) Assuming that 'c' in the equation is ¾, what is the equation representing this function?
d) Sketch and label this sinusoidal graph for two complete revolutions.
e) How high was the reflector when the bicycle first started moving? Was the reflector initially moving up or down?
f) At what time does the reflector first pass through a height of 25cm above the ground?
Be sure to show the answers for parts e) and f) on your sketch.
g) How would the equation change if the reflector was in a position 2cm further from the center of the wheel?

14. A weight is suspended on a spring above a tabletop. The height (in cm) varies sinusoidally with the time (in seconds). The data is presented in the following graph.



What is the equation of this function?

15. The following data shows how the depth of water at Qualicum Beach, BC, varies during part of the day due to tides.

Time (hours)	1	3	5	9	11.5
Depth (meters)	2.3	4.7	5.3	1.3	0.6

a) Find the equation of the sinusoidal curve for this data using your TI-83
b) Sketch and label this curve, and show on the diagram the length of time the depth of water is less than 1m in the first 12 hours. Explain how you arrived at your answer.

16. Two Ferris wheels are located at a theme park.

The first rotates according to the model $y = 10\sin(0.105x - 1.571) + 11$.

The second rotates according to the model $y = 4\sin(0.262x - 1.571) + 5$.

Each ride costs $3 and lasts for 5 minutes. Describe two differences you can experience on each ride. Justify your reasoning.

17. Brent takes a ride on a Ferris wheel. Once all the passengers are finally loaded, Brent begins the ride halfway from the top at the point B, moving down, as shown in the diagram. He is always 25m from the center of the wheel. It takes Brent 30 seconds to reach the top of the ride. Create a possible equation describing Brent's height on the ride.
(*The water level must be represented by a height of zero meters.*)

b) If Brent traveled 20 times around in 4 minutes, what would be the value of 'b' in the equation?

18. Rachel manages a fishing operation. She passes through a channel where the depth of the water (measured in meters) varies with time and can be modeled by the following function:

$$d(t) = 2.5\sin\left(0.523t\right) + 2.9$$

Where *d* represents the depth of the water in meters and *t* represents the time in hours since midnight. Rachel's boat needs 2 meters of water in order to pass safely through the channel. During what times today could she <u>not</u> travel through the channel? Display this information on a sketch.

Look up http://www.bio-chart.com/ to determine what a biorhythm is before attempting question 19.

19. Two business partners, Mr. Korpi and Mr. Kilford, have consulted biorhythm graphs to determine the ideal and most appropriate time to conduct the grand opening of their new store. Their biorhythm described below show how their energy levels fluctuate during the day.

Mr. Korpi	$E = 1.3\sin(1.3t) + .5$
Mr. Kilford	$E = 1.3\sin(1.87t) + .5$

where the energy level E is measured over time t measured in hours.

a) If $t = 0$ is midnight, at what time of day should they hold their grand opening? Justify your response.

b) Identify the period for both biorhythms. Explain what this means by comparing the two men.

20. Graph your biorhythms using your TI-83 for the next several months.
When can you expect to have all 3 (Korpi, Kilford, and Yours) on a high?