Worksheet: Finding complex roots of polynomials
KEY

1. Possible rational roots: $x = \pm1, \pm2, \pm3, \pm4, \pm6, \pm12$. Roots: $x = -3, -2, 1, 2$
2. Possible rational roots: $x = \pm1, \pm5, \pm\frac{5}{2}, \pm\frac{1}{2}$. Roots: $x = 1, -\frac{1}{2} \pm \frac{3}{2}i$
3. Roots: $x = -\frac{2}{3}, \frac{5}{2}, \pm\sqrt{3}i$
4. a. $f(x) = Ax(x+15)(x+10)^2(x+5)(x-10)^2(x-15),\ A > 0$

   b. $f(x) = \frac{1}{4{,}073{,}472}x(x+15)(x+10)^2(x+5)(x-10)^2(x-15)$
5. c. $f(x) = A(x+5)^2(x-5)^3, A > 0$

   d. $f(x) = \frac{6}{625}(x+5)^2(x-5)^3$
6. $f(x) = -\frac{1}{900}(x+4)^2(x+1)^3(x-4)^2$
7. $f(x) = -\frac{1}{5600}(x+5)^2(x+1)(x-4)^3(x-7)$
8. $f(x) = 15(x+2)(x-1)\left(x-\frac{4}{3}\right)\left(x+\frac{2}{5}\right)$
9. Roots: $x = -3 \pm \sqrt{11}, \pm\sqrt{5}$
10. Roots: $x = -1, 2 \pm i, \frac{3 \pm \sqrt{5}}{2}$
11. Possible rational roots: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24, \pm\frac{1}{2}, \pm\frac{3}{2}$. Roots: $x = -1, -\frac{3}{2}, 4, \frac{-3 \pm \sqrt{17}}{2}$
12. $f(x) = 30(x+4)\left(x+\frac{4}{5}\right)\left(x-\frac{1}{3}\right)(x-5)^2$
13. Remainder $= (-1)^{36} + 4(-1)^{27} + 7 = 4$
14. Quotient $= x^2 - 5x + 10$, Remainder $= -32$
15. $P(x) = x^4 - 2x^2 - 1$
16. a. $P(x) = A(x+2)(x^2-2x-2)(x^2+25), A \neq 0$

    b. $P(x) = \frac{1}{4}(x+2)(x^2-2x-2)(x^2+25)$
17. a. False, imaginary roots occur in conjugate pairs
    b. False, with only 5 reals, that leaves 3 imaginaries, which cannot happen.
    c. True, since irrationals and imaginaries occur in pairs, one must be rational
    d. False, and even degree function must have an odd number of relative extrema
    e. False, for example $f(x) = x^2 + x$ is even degree but does not have *y*-axis symmetry
    f. True
    g. True
    h. True
    i. False, it's not awesome, it's ***TOTALLY*** awesome.

18. a. $f(x)=Ax^2\left(x^2-8x+25\right), A\neq 0$

b. $f(x)=A\left(x^2-14x+54\right)^2\left(x^2-5\right)^2, A\neq 0$

19. $k=26$

20. Roots: $x=\pm 2i, 1\pm\sqrt{5}, -4, 3$