

Operations with Functions:

After completing this worksheet, you should be able to:

1. Find the sum of functions.
2. Find the difference of functions.
3. Find the product of functions.
4. Find the quotient of functions.
5. Find the composition of functions.

In this worksheet we will be working with functions. Note as you are going through this lesson that a lot of the things we are doing we have done before with expressions. Like adding, subtracting, multiplying and dividing. What is new here is we are specifically looking at these same operations with functions this time. I think we are ready to forge ahead.

The following examples will show us how to perform the different operations on functions.

We will use the functions $f(x) = 3x + 1$ and $g(x) = x + 2$ to illustrate the operations

Sum of $f + g$

$$(f + g)(x) = f(x) + g(x)$$

This is a very straight forward process. **When you want the sum of your functions you simply add the two functions together.**



Example 1: If $f(x) = 3x + 1$ and $g(x) = x + 2$ then find $(f + g)(x)$

$$(f + g)(x) =$$

$$(3x + 1) + (x + 2) =$$

$$3x + 1 + x + 2 =$$

$$4x + 3$$

***Add the 2 functions**

***Combine like terms**

Difference of $f - g$

$$(f - g)(x) = f(x) - g(x)$$

Another straight forward idea, **when you want the difference of your functions you simply take the first function minus the second function.**



Example 2: If $f(x) = 3x + 1$ and $g(x) = x + 2$ then find $(f - g)(x)$ and $(f - g)(5)$.

$$(f - g)(x) =$$

$$(3x + 1) - (x + 2) =$$

$$3x + 1 - x - 2 =$$

$$2x - 1$$

***Take the difference of the 2 functions**

***Subtract EVERY term of the 2nd ()**

$$(f - g)(5) =$$

$$2(5) - 1 =$$

$$10 - 1 =$$

$$9$$

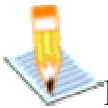
***Plug 5 in for x in the diff. of the 2 functions found above**

Since the difference function had already been found, we didn't have to take the difference of the two functions again. We could just merely plug in 5 into the already found difference function.

Product of fg

$$(fg)(x) = f(x)g(x)$$

Along the same idea as adding and subtracting, **when you want to find the product of your functions you multiply the functions together.**



Example 3: If $f(x) = 3x + 1$ and $g(x) = x + 2$ then find $(fg)(x)$.

$$(fg)(x) =$$

$$(3x + 1)(x + 2) =$$

$$3x^2 + 6x + x + 2 =$$

$$3x^2 + 7x + 2$$

***Take the product of the 2 functions**

***FOIL method to multiply**

Quotient of f/g

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Well, we don't want to leave division of functions out of the loop. It stands to reason that **when you want to find the quotient of your functions you divide the functions.**



Example 4: If $f(x) = 3x + 1$ and $g(x) = x + 2$ then find $\left(\frac{f}{g}\right)(x)$ and $\left(\frac{f}{g}\right)(1)$

$$\left(\frac{f}{g}\right)(x) = \frac{3x+1}{x+2}$$

***Write as a quotient of the 2 functions**
(Notice here that now $x \neq -2$)

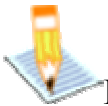
$$\left(\frac{f}{g}\right)(1) = \frac{3(1)+1}{1+2} = \frac{4}{3}$$

***Use the quotient found above to plug 1 in for x**

Composite Function

$$(f \circ g)(x) = f(g(x))$$

Be careful, when you have a composite function, one function is inside of the other. It is not the same as taking the product of those functions.



Example 5: If $f(x) = 3x+1$ and $g(x) = x+2$ then find $(f \circ g)(x)$.

$$\begin{aligned} (f \circ g)(x) &= \\ f(g(x)) &= \\ f(x+2) &= \\ 3(x+2)+1 &= \\ 3x+6+1 &= \\ 3x+7 & \end{aligned}$$

*** g is inside of f**

***Substitute in $x+2$ for g**

***Plug $x+2$ in for x in function f**



Example 6: Let $f(x) = \sqrt{x+1}$, $g(x) = \frac{1}{x}$, and $h(x) = x+3$. Find an equation defining each function and state the domain.

- a) $f+g$ b) $f-g$ c) fg d) f/h e) $g \circ f \circ h$

Solutions:

a) $f + g$

$$f + g = \sqrt{x+1} + \frac{1}{x} \quad \text{*Add the 2 functions}$$

Domain:

$$f(x) = \sqrt{x+1}$$

The radicand CANNOT be negative. In other words it has to be greater than or equal to zero:

$$x+1 \geq 0 \quad \text{*Set radicand of } x+1 \text{ greater than or equal to } 0 \text{ and solve}$$

$$x \geq -1$$

$$g(x) = \frac{1}{x}$$

The denominator CANNOT equal zero:

$$x \neq 0 \quad \text{*The den. } x \text{ CANNOT equal zero}$$

Putting these two sets together we get the domain:

$$[-1, 0) \cup (1, \infty) \text{ or } \{x | x \geq -1, x \neq 0\}$$

b) $f - g$

$$f - g = \sqrt{x+1} - \frac{1}{x} \quad \text{*Subtract the 2 functions}$$

Domain:

The domain is the same as the example above, by the same argument:

$$[-1, 0) \cup (1, \infty) \text{ or } \{x | x \geq -1, x \neq 0\}$$

c) fg

$$f \cdot g = \left(\sqrt{x+1}\right)\left(\frac{1}{x}\right) = \frac{\sqrt{x+1}}{x} \quad \text{*Multiply the 2 functions}$$

Domain:

The domain is the same as the two examples above, by the same argument:

$$[-1, 0) \cup (1, \infty) \text{ or } \{x | x \geq -1, x \neq 0\}$$

$$d) f/h = \frac{f}{h}$$

$$\frac{f}{h} = \frac{(\sqrt{x+1})}{x+3}$$

***Divide the 2 functions**

Domain:

$$f(x) = \sqrt{x+1}$$

The radicand CANNOT be negative. In other words it has to be greater than or equal to zero:

$$x+1 \geq 0 \quad \text{*Set radicand of } x+1 \text{ greater than or equal to } 0 \text{ and solve}$$

$$x \geq -1$$

$$h(x) = x+3$$

The denominator $h(x)$ CANNOT equal zero:

$$x \neq -3 \quad \text{*The den. } h(x) = x+3 \text{ CANNOT equal zero}$$

Putting these two sets together we get the domain:

$$[-1, \infty) \text{ or } \{x | x \geq -1\}$$

$$e) g \circ f \circ h$$

$$g \circ f \circ h$$

$$g(f(h(x))) =$$

***h is inside of f is inside of g**

$$g(f(x+3)) =$$

***Substitute in $x+3$ for h**

$$g(\sqrt{x+3+1}) =$$

***Substitute in $\text{sqrt}(x+3+1)$ for $f(x+3)$**

$$g(\sqrt{x+4}) =$$

$$\frac{1}{\sqrt{x+4}}$$

***Plug $\text{sqrt}(x+4)$ in for x in function g**

Domain:

The radicand CANNOT be negative AND the denominator CANNOT equal zero. In other words, the radicand has to be greater than zero:

$$x+4 > 0 \quad \text{*Set radicand of } x+4 \text{ greater than } 0 \text{ and solve}$$

$$x > -4$$

The domain would be:

$$(-4, \infty) \text{ or } \{x | x > -4\}$$

We can also do composition in reverse, essentially DE-composing a function.



Example 7: Let $f(x) = \sqrt{x+3}$ and $g(x) = \frac{1}{x}$. Write $h(x) = \frac{1}{\sqrt{x+3}}$ as a composition function using f and g .

When you are writing a composition function keep in mind that one function is inside of the other. You just have to figure out which function is the inside function and which is the outside function.

Note that if you put g inside of f you would get:

$$\begin{aligned} f \circ g &= \\ f(g(x)) &= \\ f\left(\frac{1}{x}\right) &= \\ \sqrt{\frac{1}{x} + 3} & \end{aligned} \quad \text{*Put } g \text{ inside of } f$$

Note that if you put f inside of g you would get:

$$\begin{aligned} g \circ f &= \\ g(f(x)) &= \\ g(\sqrt{x+3}) &= \\ \frac{1}{\sqrt{x+3}} & \end{aligned} \quad \text{*Put } f \text{ inside of } g$$

Hey, this looks familiar! Our answer is $h(x) = (g \circ f)(x) = g(f(x))$.

Practice Problems:

These are practice problems to help bring you to the next level. It will allow you to check and see if you have an understanding of these types of problems. **Math works just like anything else, if you want to get good at it, then you need to practice it. Even the best athletes and musicians had help along the way and lots of practice, practice, practice, to get good at their sport or instrument.** In fact there is no such thing as too much practice.

For problems 1a – 1j, Let $f(x) = x^2 + 3$ and $g(x) = 3x - 1$. Find the following

1a. $(f + g)(x)$

1b. $(f + g)(3)$

1c. $(f - g)(x)$

1d. $(fg)(x)$

1e. $(fg)(-1)$

1f. $\left(\frac{f}{g}\right)(x)$

1g. $(f \circ g)(x)$

1h. $(f \circ g)(1)$

1i. $(g \circ f)(x)$

1j. $(g \circ g)(x)$

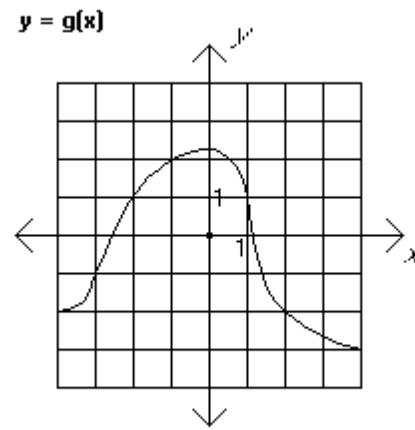
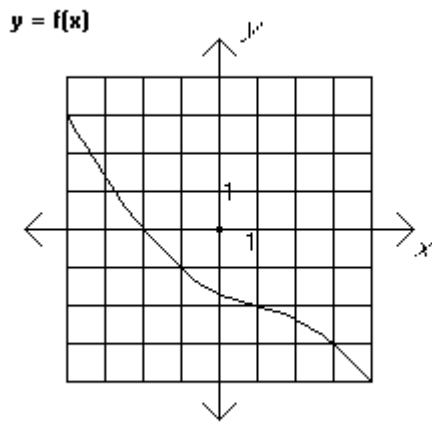
2. Let $f(x) = |x - 5|$ and $g(x) = \frac{1}{x}$. Write the given function $h(x)$ as a composition function using f and g .

2a. $h(x) = \left| \frac{1}{x} - 5 \right|$

2b. $h(x) = \frac{1}{|x - 5|}$

*2c. $h(x) = x$

3. Let $f(x)$ and $g(x)$ be given by the following graphs:



a. Find $f(1)$

b. Find $g(-3)$

c. Find $(f + g)(1)$

d. Find $(f - g)(-1)$

e. Find $(fg)(4)$

f. Find $(f/g)(4)$

g. Find $(f \circ g)(2)$

h. Find $(g \circ f)(4)$

i. Find $(g \circ g)(-3)$

4. For each of the following pairs of functions, find $f \circ g$, $g \circ f$, $f \circ f$, and $g \circ g$ and state the resulting domain for each. You do not need to simplify each new function once you create it.

a. $f(x) = 4x^3 - 6x + 5$, $g(x) = x^2 + 2x$

b. $f(x) = \sqrt{x+2}$, $g(x) = x^2 - 6$

c. $f(x) = \frac{x-1}{x-4}$, $g(x) = \frac{5}{x}$