Name__________________________________ Date___________________________ Per____________

Summary: Transformations of Parent Functions

1. You have previously studied three types of transformations: Shifts/Translations, Dilations, and Reflections. For each of the following below, explain the effect of each transformation on the graph of a function $y = f(x)$. Circle the correct word in each box (or scratch out incorrect one) for each transformation. Then give an example of how the equation would look for the three parent functions *f, g, & h* given below. Imagine what the graphs of each of your examples would look like, then verify each of them on your calculator.

$$f(x) = x^2 \qquad g(x) = \sqrt{x} \qquad h(x) = \frac{1}{x}$$

I. For $c > 0$, $y = f(x-c)$: Horizontal shift to the ~~LEFT~~ or RIGHT c units.

For example, for $c = 5$, $f(x-5) = (x-5)^2$, $g(x-5) = \sqrt{x-5}$, $h(x-5) = \dfrac{1}{x-5}$

II. For $c > 0$, $y = f(x+c)$: Horizontal shift to the LEFT or RIGHT c units.

For example, for $c =$ ____

III. For $d > 0$, $y = f(x)+d$: Vertical shift UP or DOWN d units.

For example, for $d =$ ____

IV. For $d > 0$, $y = f(x)-d$: Vertical shift UP or DOWN d units.

For example, for $d =$ ____

V. For $a > 1$, $y = af(x)$: Vertical Dilation STRETCH or COMPRESSION by a factor of a or $1/a$

For example, for $a =$ ____

VI. For $0 < a < 1$, $y = af(x)$: Vertical Dilation STRETCH or COMPRESSION by a factor of a or $1/a$

For example, for $a =$ ____

VII. For $b > 1$, $y = f(bx)$: Horizontal Dilation STRETCH or COMPRESSION by a factor of b or $1/b$

For example, for $b =$ ____

VIII. For $0 < b < 1$, $y = f(bx)$ Horizontal Dilation STRETCH or COMPRESSION by a factor of b or $1/b$

For example, for $b =$ ____

IX. $y = -f(x)$: Negative Dilation, Reflection across the x-axis or y-axis.

For example, ____

X. $y = f(-x)$: Negative Dilation, Reflection across the x-axis or y-axis.

For example, ____ So now that we've done all that using basic function notation, and have listed three specific examples using function notations with different names, what's next? Well the beauty of knowing how these changes in an equation affect its graph, you can generalize it to apply to ANY function and its graph. It even works if you only have the graph of the function, as long as you still understand the basic function notation.

Additionally, we can begin to combine these transformations in a sequence to have a combined affect of a multitude of them. In this case, we always remember our dear aunt sally's rule. . . . to please excuse her. That is, we must do transformations involving multiplication/division first and transformations involving addition/subtraction last. Put differently, we must always do DILATIONS/REFLECTIONS before doing any SHIFTS. Please reread that last sentence.

In general, when faced with any equation, it is imperative, if not extremely helpful to write it in a standardized, systematic way so that we have a better chance of graphing it correctly (provided we already have the individual transformations memorized.) Please reread the last statement in parenthesis again.

Do you remember on the front page how I used the letters *a, b, c,* and *d*? Go ahead and flip to the front to jog your memory. I'll wait …… I'm glad you're back. Using those letters was for a very specific purpose: we use them in our STANDARD TRANSFORMATION FORM of an equation. Here it is:

A new function $h(x)$ that is obtained from transformations of a function $f(x)$ is said to be in **Standard Transformation Form** if it is in the following form:

$$h(x) = af(b(x-c)) + d$$

We sometimes prefer to write this form using capital letter for the transformations. Here's what that would look like:

$$h(x) = Af(B(x-C)) + D$$

Not only does writing equations like this give us a habitual way of graphing transformed functions (without a calculator) but when written like this, you can always simply apply the transformations in the left-to-right sequence!! (Do you know why? Ask your dear Aunt Sally.)

Also note that if *a* and *b* are negative, they may account for TWO transformations, both the dilation, which would technically be $|a|$ and $|b|$, and the negative part of it, technically the (-1) multiplier which would be responsible for the reflections.

Let me show an example of one of my many, many favorite parent function. We'll do a fantastic one that has each type. Here we go.

Awesome Example 1:

Sketch a graph of the following function, then state the domain and range.

$$f(x) = 4 - 3\sqrt{4-2x}$$

Awesome Solution:

Step 1: Identify the parent function. In this case, it is our old friend $f(x) = \sqrt{x}$. When identifying the parent function, we envision its shape, its domain, its range, and any other distinguishable characteristics about it in our head. If not in our head, we can do a quick sketch somewhere off to the side.

Step 2: We write the function in standard transformation form. Don't ever assume you will be given such an easy function that didn't require SOME type of rewriting. For this example, we want that +4 in front to go the very back. We get $f(x) = -3\sqrt{4-2x} + 4$. Inside the function, which in this case is under the radical, we not only want the *x* listed before and constant, but additionally, if there IS a horizontal dilation AND a horizontal shift, that is a *b* AND *c* value, we want to factor out the *b* from BOTH the *x*-term and the constant. This is SOOOOO important. Here it is in several steps, proceeding from above. Follow along please:

$$f(x) = -3\sqrt{4-2x} + 4$$

$$f(x) = -3\sqrt{-2x+4} + 4$$

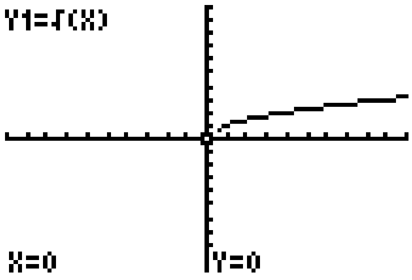
$$f(x) = -3\sqrt{-2(x-2)} + 4$$

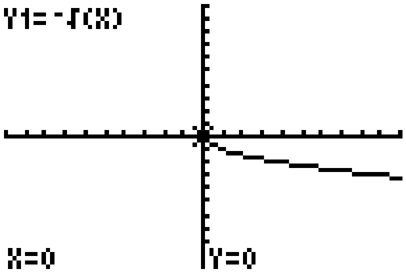
Step 4: Time to go through the problem and mentally tell yourself what each transformation is doing to our parent function. You can proceed from left to right. Here's what goes on in MY head.

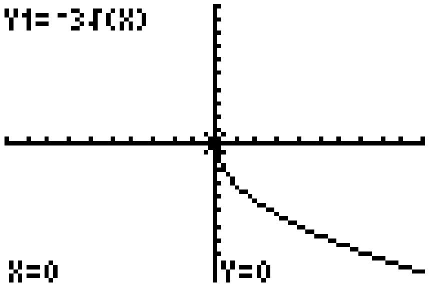
$$f(x) = -3\sqrt{-2(x-2)} + 4$$

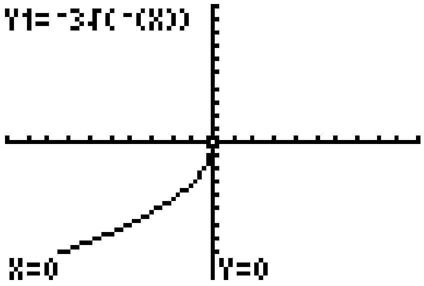
"Gosh I love math . . . . where was I . . . . .The parent function is reflected across the x-axis, THEN vertically stretched by a factor of 3, THEN reflected across the y-axis, THEN it is horizontally compressed by a factor of 2, THEN the graph is shifted to the right 2 units, and FINALLY, the graph is shifted up 4 units., yeah, that ought to do it. Anything that happens INSIDE the function is always OPPOSITE of what it actually appears to be, hence 'compress' and 'right', right? Right!! I DO love math."

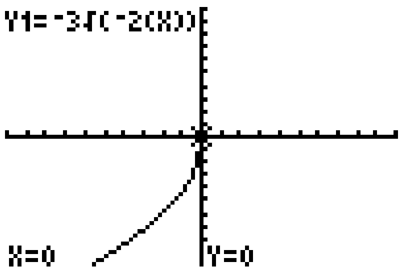
Step 5: Start graphing the new function as a sequence. You can physically graph one transformation at a time using a faint or dotted line, then apply subsequent transformations to the previous graphs until you ultimately arrive at the final graph . Whenever you have either a vertical and/or horizontal dilation, you don't necessarily have to get the stretches or compressions right on. In fact, if you don't even label the *y*-axis, you just need to show a graph as being RELATIVELY steeper or shallower than the previous. Here's how to do this one. I used the graphing calculator, but this type of process is easily done in the style of sketching without one.

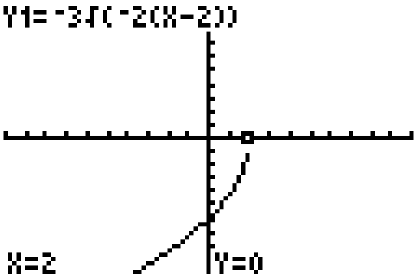




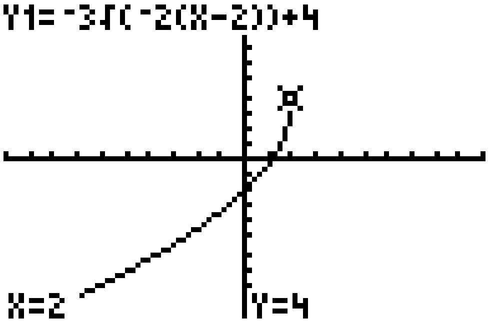








Doing the final transformation, we get the final graph of $f(x)$.

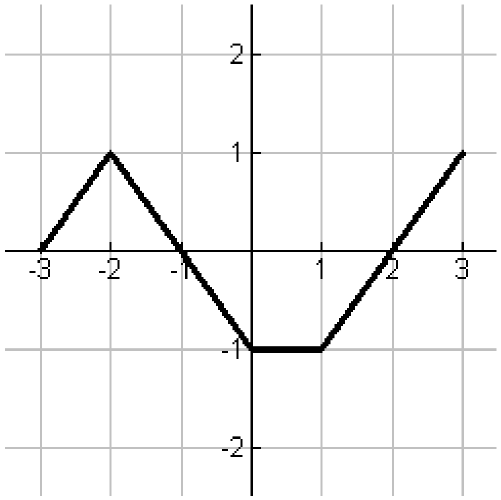


Notice that the point $(0,0)$ on the original graph is a point of interest, since it is the starting point for our sketching the graph. This point ended up moving to right 2 and up 4 to $(2,4)$ because of the two shift transformations. You can actually increase your efficiency in sketching these types of functions if you can visualize the final "shape" of the graph, determined by the dilations/reflections, then find the new locations of these "points of interest," then draw the final shape from that new point. This takes a bit a practice, so practice a lot.

Also be aware that the above transformations will have the same effect on **ANY** given graph of a function.

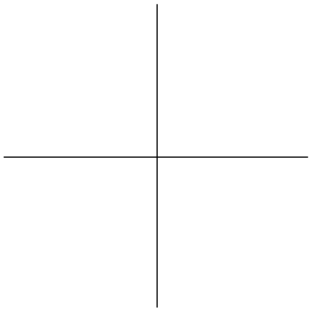
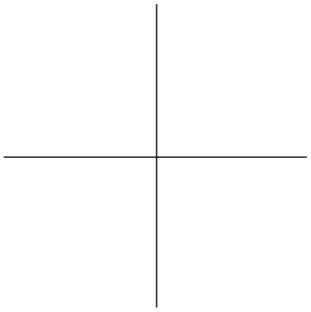
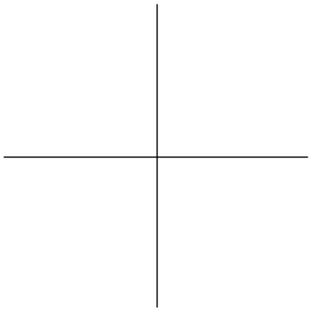
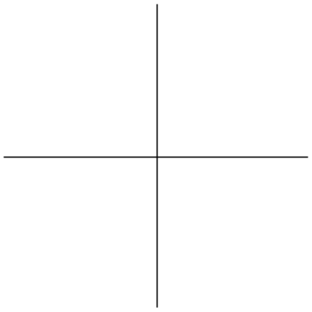
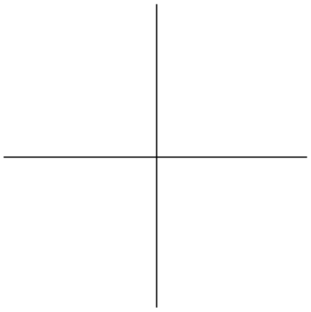
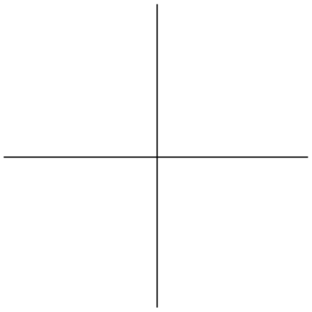
In this example, you are given the graph of a function, but not its equation. You can still sketch transformations of the graph by interpreting the transformations from the general function form, and applying them to the graph. In some cases, it might be easier to following specific "points of interest," like *x*- or *y*- intercepts, relative extrema, etc. of the graph to see where they go, then come back and connect the dots between the points of interest you choose.

2. Now try the following problems, given only a graph of $f(x)$ on the domain $-3 \le x \le 3$.



Use the above graph as the parent function, $f(x)$. On the grids below, sketch what the new graph will look like under the indicated transformations. Be sure to indicated where your original points on the graph end up, and have your axes labeled. Remember to rewrite the functions in standard transformation form first, if necessary.

a. $f_1(x) = f(-x)$	b. $f_2(x) = -f(x)$	c. $f_3(x) = f(x) - 1$
d. $f_4(x) = f(x-1)$	e. $f_5(x) = f(2x)$	f. $f_6(x) = 1 - f(x)$

g. $f_7(x) = f(2-x)$ 	h. $f_8(x) = \frac{1}{2} f\left(\frac{x}{2}\right)$ 	i. $f_9(x) = \lvert f(x) \rvert$ 
j. $f_{10}(x) = f(\lvert x \rvert)$ 	k. $f_{11}(x) = \lvert f(\lvert x \rvert) \rvert$ 	l. $f_{12}(x) = f(-\lvert x \rvert) - 1$ 

Problems *i* through *l* involve absolute values. We haven't discussed these yet, but see if you can, nonetheless, figure out the results. As a hint, I would recommend plugging in individual *x*-values, $x = -3, -2, -1, 0, -1, -2, -3$ and "evaluate" them in the function notation. You can obtain the function values you get from the original graph. Example: for $|f(-x)| + 1$, plugging in $x = -1$, we get:

$$\left|f(-(-1))\right| + 1 = |f(1)| + 1 = |-1| + 1 = 1 + 1 = 2$$

So we plot the point $(-1, 2)$

Plot your new 7 points like that and connect the dots. We'll discuss these transformations next time.

Have fun! ☺

--Mr. Korpi