

- ① Regular Dodecagon area in a circle with radius 12 (work with 1 $\Delta$ )

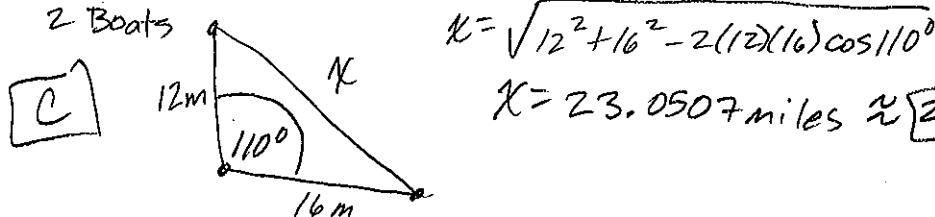
**B** Central angle =  $\frac{360^\circ}{12} = 30^\circ$        Area =  $\frac{1}{2}(12)(12)\sin 30^\circ = \frac{144}{4} = \frac{72}{2}$

Total Area =  $12\left(\frac{72}{2}\right) = 6(72) = 432$

- ② Area of  $\Delta$ . Sides = 7, 8, 9      Semiperimeter =  $s = \frac{7+8+9}{2} = 12$

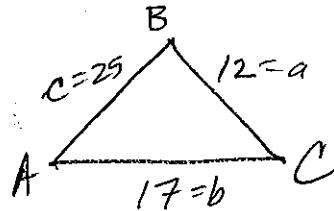
**B** Area =  $\sqrt{12(12-7)(12-8)(12-9)} = \sqrt{12(5)(4)(3)} = \sqrt{720} = \sqrt{144 \times 5}$   
 $= 12\sqrt{5}$

③

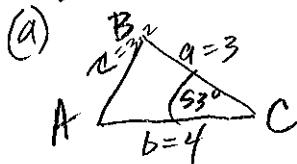


- ④  $\Delta$  with sides 12, 17, 25; smallest angle

**E**  $12^2 = 25^2 + 17^2 - 2(25)(17)\cos A$   
 $A = \cos^{-1}\left[\frac{12^2 - 25^2 - 17^2}{-2 \times 25 \times 17}\right], A = 25.057^\circ \approx 25^\circ$



- ⑤  $\triangle ABC$



$c^2 = 3^2 + 4^2 - 2(3)(4)\cos 53^\circ$   
 $c = 3.249\dots \rightarrow C$

$\frac{\sin A}{3} = \frac{\sin 53^\circ}{3.249\dots} * A \text{ is the smaller of angles } A \& B$

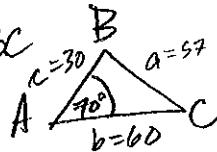
$A = \sin^{-1}\left(\frac{3 \sin 53^\circ}{3.249}\right)$

$A = 47.511^\circ$

$B = 180^\circ - 53^\circ - 47.511^\circ$

$B = 79.488^\circ$

- (b)  $\triangle ABC$



$a^2 = 30^2 + 60^2 - 2(30)(60)\cos 70^\circ$   
 $a = 57.172\dots \rightarrow A$

$\frac{\sin C}{30} = \frac{\sin 70^\circ}{57.172\dots} * C \text{ is smaller of angles } C \& B$

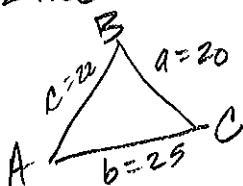
$C = \sin^{-1}\left(\frac{30 \sin 70^\circ}{57.172\dots}\right)$

$C = 29.543^\circ$

$B = 180^\circ - 70^\circ - 29.543^\circ$

$B = 80.456^\circ$

- (c)  $\triangle ABC$



$25^2 = 22^2 + 20^2 - 2(22)(20)\cos B$

$B = \cos^{-1}\left(\frac{25^2 - 22^2 - 20^2}{-2 \times 22 \times 20}\right)$

$B = 72.083^\circ \rightarrow B$

$\frac{\sin A}{20} = \frac{\sin B}{25}$

$A = \sin^{-1}\left(\frac{20 \sin B}{25}\right)$

$A = 49.816B^\circ$

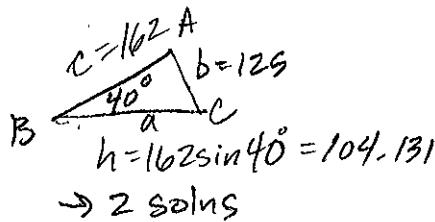
$C = 180^\circ - B - A$

$C = 57.248^\circ$

# Precal Matters WS 6.6 KEY

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⑥ (a)  $b = 125$ ,  $c = 162$ ,  $B = 40^\circ$   
(ambig. case)



Aacute Case

$$\frac{\sin C}{162} = \frac{\sin 40^\circ}{125}$$

$$C = \sin^{-1}\left(\frac{162 \sin 40^\circ}{125}\right)$$

$$C = 56.413^\circ \rightarrow C$$

$$A = 180^\circ - 40^\circ - C$$

$$A = 83.586^\circ \rightarrow A$$

$$\frac{a}{\sin A} = \frac{125}{\sin 40^\circ}$$

$$a = \frac{125 \sin A}{\sin 40^\circ} \quad a = 193.248$$

Obtuse Case

$$C = 180^\circ - 56.413^\circ$$

$$C = 123.586^\circ \rightarrow C$$

$$A = 180^\circ - 40^\circ - C$$

$$A = 106.413^\circ \rightarrow A$$

$$a = \frac{125 \sin A}{\sin 40^\circ}$$

$$a = 54.950$$

(c)  $a = e$ ,  $b = \pi$ ,  $C = e\pi^\circ$   
(SAS)

$$c^2 = e^2 + \pi^2 - 2e\pi \cos(e\pi^\circ)$$

$$C = 0.607 \rightarrow C$$

$$\frac{\sin A}{e} = \frac{\sin(e\pi^\circ)}{0.607}$$

$$A = \sin^{-1}\left(\frac{e \sin(e\pi^\circ)}{0.607}\right)$$

$$A = 41.674^\circ \rightarrow A$$

$$B = 180^\circ - e\pi^\circ - A$$

$$B = 129.785^\circ$$

(b)  $a = 73.5$ ,  $B = 61^\circ$ ,  $C = 83^\circ$  (ASA)

$$A = 180^\circ - 61^\circ - 83^\circ$$

$$A = 36^\circ$$

$$\frac{b}{\sin 61^\circ} = \frac{73.5}{\sin 36^\circ}$$

$$b = \frac{73.5 \sin 61^\circ}{\sin 36^\circ}$$

$$b = 109.367$$

$$\frac{c}{\sin 83^\circ} = \frac{73.5}{\sin 36^\circ}$$

$$c = \frac{73.5 \sin 83^\circ}{\sin 36^\circ}$$

$$c = 124.113$$

(d)  $b = 3$ ,  $c = 4$ ,  $A = 90^\circ$ , SAS

$$a^2 = 3^2 + 4^2 - 2(3)(4)\cos 90^\circ$$

$$a = 5$$

$$\sin C = \frac{4}{5}$$

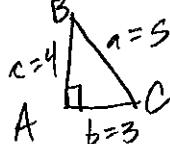
$$C = \sin^{-1}\left(\frac{4}{5}\right)$$

$$C = 53.130^\circ \rightarrow C$$

$$B = 180^\circ - 90^\circ - C$$

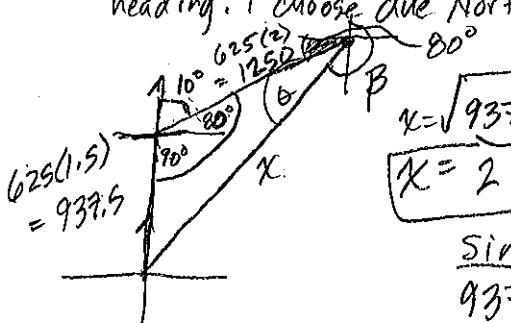
$$B = 36.869^\circ$$

(Right  $\Delta$ )  
(3-4-5 RT $\Delta$ )



(7)

\* we can choose any original heading. I choose due North



$$x = \sqrt{937.5^2 + 1250^2 - 2(937.5)(1250)\cos(170^\circ)}$$

$$x = 2179.346 \text{ miles} \rightarrow x$$

$$\frac{\sin \theta}{937.5} = \frac{\sin 170^\circ}{x}$$

$$\theta = \sin^{-1}\left(\frac{937.5 \sin 170^\circ}{x}\right)$$

$$\theta = 4.2839^\circ \rightarrow A$$

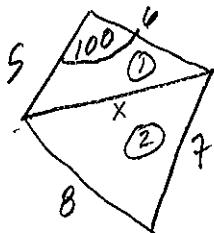
$$\beta = 180^\circ + (90^\circ - 80^\circ - \theta)$$

$$\beta = 185.716^\circ$$

(Relative to her original heading)

(8)

(a)



$$x = \sqrt{5^2 + 6^2 - 2 \times 5 \times 6 \cos 100^\circ}$$

$$x = 8.450 \rightarrow x$$

$$\text{Area of } \Delta ① = \frac{1}{2}(5)(6) \sin 100^\circ$$

$$= 14.772 \rightarrow A$$

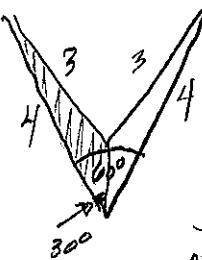
$$\text{Area of } \Delta ② = \frac{\sqrt{c(c-x)(c-x)(c-x)}}{4}$$

$$s = \frac{x+7+8}{2} = 11.725 \rightarrow c$$

$$s = 11.725 \rightarrow c$$

$$\text{Total Area} = A+B = 40.770$$

(b)



Focus on one triangle

obtuse SSA angle case

$$\frac{\sin 30^\circ}{3} = \frac{\sin x}{4}$$

$$x = \sin^{-1}\left(\frac{4 \sin 30^\circ}{3}\right)$$

$$x = 41.810 \text{ & acute}, 90^\circ$$

$$X = 180 - 41.810 \dots = 138.189^\circ \rightarrow X$$

$$y = 180 - x - 30 = 11.810^\circ \rightarrow Y$$

$$\text{Area} = 2 \left[ \frac{1}{2}(3)(4) \sin y \right]$$

$$= 2.456$$

(9)

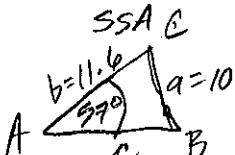
$$s = \frac{112 + 149 + 191}{2} = 226$$

$$\text{Area} = \sqrt{226(226-112)(226-149)(226-191)}$$

$$\text{Area} = 8332.705 \text{ ft}^2$$

$$\text{Value} = \$200(8332.705 \text{ ft}^2)$$

$$= \$2,499,811.63$$

(10)  $A = 57^\circ, b = 11.6, a = 10$ 

$$10^2 = 11.6^2 + c^2 - 2(11.6)c \cos 57^\circ$$

$$1c^2 - (2(11.6)\cos 57^\circ)c + (11.6^2 - 10^2) = 0$$

$$c = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} = 8.631 \rightarrow x$$

$$\text{or } 4.003 \rightarrow y$$

$$\begin{aligned} &\text{Acute case} \\ &c = 8.631 \rightarrow x \\ &\frac{\sin B}{11.6} = \frac{\sin 57^\circ}{10} \end{aligned}$$

$$B = \sin^{-1}\left(\frac{11.6 \sin 57^\circ}{10}\right)$$

$$B = 76.620^\circ \rightarrow B$$

$$C = 180 - 57 - B$$

$$C = 19.620^\circ$$

OBTUSE CASE

$$c = 4.003$$

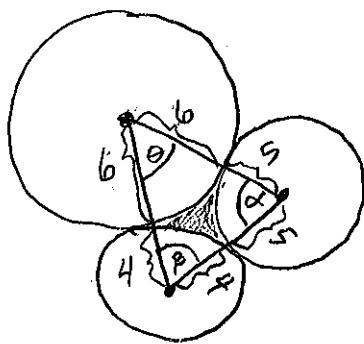
$$B = 180 - 76.620^\circ$$

$$B = 103.379^\circ \rightarrow B$$

$$C = 180 - 57 - B$$

$$C = 46.379^\circ$$

(11)

Triangle Area

$$\Delta = \frac{10+11+9}{2} = 15$$

$$\text{Area} = \sqrt{15(15-10)(15-11)(15-9)}$$

$$\text{Area} = \boxed{42.426} \rightarrow A$$

Area of Sectors

$$A_\theta = \frac{1}{2} r^2 \theta \quad (\theta \text{ in radians})$$

$$\begin{aligned} \text{Area}(\theta) &= \frac{1}{2}(6^2)\theta = \frac{1}{2}(6^2)(0.881..) \\ &= 15.858 \rightarrow E \end{aligned}$$

$$\begin{aligned} \text{Area}(\beta) &= \frac{1}{2}(4^2)\beta = \frac{1}{2}(4^2)(1.230) \\ &= 9.847 \rightarrow F \end{aligned}$$

$$\begin{aligned} \text{Area}(\alpha) &= \frac{1}{2}(5^2)\alpha = \frac{1}{2}(5^2)(1.029) \\ &= 12.870 \rightarrow G \end{aligned}$$

Area of Shaded Region

$$= \text{Area of Triangle} - \text{Sum of Sector Areas}$$

$$= A - (E+F+G)$$

$$= \boxed{3.850 \text{ in}^2}$$

Find Angles  $\theta, \alpha, \beta$ 

$$11^2 = 10^2 + 9^2 - 2 \times 10 \times 9 \cos \beta$$

$$\beta = \cos^{-1} \left[ \frac{11^2 - 10^2 - 9^2}{-180} \right]$$

$$\boxed{\beta = 70.528^\circ} \rightarrow B$$

$$\frac{\sin \theta}{9} = \frac{\sin B}{11}$$

$$\theta = \sin^{-1} \left( \frac{9 \sin B}{11} \right)$$

$$\boxed{\theta = 50.478^\circ} \rightarrow C$$

$$\alpha = 180 - B - C$$

$$\boxed{\alpha = 58.992^\circ} \rightarrow D$$

You must convert these to radians, or find them in radians from the beginning.

\* All angles must be in radians (mult by  $\frac{\pi}{180}$ )

$$\beta = 70.528^\circ \left( \frac{\pi}{180} \right) = \boxed{1.230} \rightarrow B$$

$$\theta = 50.478^\circ \left( \frac{\pi}{180} \right) = \boxed{0.881} \rightarrow C$$

$$\alpha = 58.992^\circ \left( \frac{\pi}{180} \right) = \boxed{1.029} \rightarrow D$$