Name_____ Date_____ Period_____

Worksheet 5.7—Inverse Trig Functions

Show all work on a separate sheet of paper. No calculator is permitted unless specified otherwise. Unless otherwise stated, report three decimals and units in all final answers.

Multiple Choice

1. $\text{Cos}^{-1}\left(-\frac{\sqrt{3}}{2}\right) =$

(A) $-\frac{7\pi}{6}$ (B) $\frac{7\pi}{6}$ (C) $-\frac{\pi}{6}$ (D) $-\frac{11\pi}{6}$ (E) $\frac{5\pi}{6}$

2. $\text{Arcsin}\left(-\frac{1}{2}\right) =$

(A) $-\frac{7\pi}{6}$ (B) $\frac{7\pi}{6}$ (C) $-\frac{\pi}{6}$ (D) $-\frac{11\pi}{6}$ (E) $\frac{5\pi}{6}$

3. $\arcsin\left(\sin\pi\right) =$

(A) -2π (B) $-\pi$ (C) π (D) 0 (E) 2π

4. $\sec\left(\arctan x\right) =$

(A) x (B) $\csc x$ (C) $\sqrt{1+x^2}$ (D) $\sqrt{1-x^2}$ (E) $\frac{\sqrt{1-x^2}}{x}$

5. The range of the function $f\left(x\right) = \arcsin x$ is

(A) $\left(-\infty,\infty\right)$ (B) $\left(-1,1\right)$ (C) $\left[-1,1\right]$ (D) $\left[0,\pi\right]$ (E) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

6. The range of the function $f\left(x\right) = \arccos x$ is

(A) $\left(-\infty,\infty\right)$ (B) $\left(-1,1\right)$ (C) $\left[-1,1\right]$ (D) $\left[0,\pi\right]$ (E) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

7. The range of the function $f\left(x\right) = \arctan x$ is

(A) $\left(-\infty,\infty\right)$ (B) $\left[-1,1\right]$ (C) $\left[0,\pi\right]$ (D) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ (E) None of these

8. Find the exact value of each expression, if it is defined. Give your answers in radians in the principal value range of each function.

(a) $\text{Sin}^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ (b) $\text{Arccos}\left(-\frac{\sqrt{3}}{2}\right)$ (c) $\text{Tan}^{-1}\sqrt{3}$ (d) $\text{Arcsin}\sqrt{3}$ (e) $\text{Cos}^{-1}(-1)$

9. Use a calculator to find an approximate value of each expression correct to 5 decimal places, if it is defined. Give your answers in decimal degrees in the interval $\left[0^\circ, 360^\circ\right)$.

(a) $\sin^{-1}(0.13844)$ (b) $\arccos(-0.92761)$ (c) $\tan^{-1}(26.23110)$

10. The following facts to find an approximate value (using a calculator) of each expression correct to 5 decimal places, if it is defined. Give your answers in radians in the interval $[0, 2\pi)$.

$$\csc\theta = \frac{1}{\sin\theta},\ \sec\theta = \frac{1}{\cos\theta},\ \text{and}\ \cot\theta = \frac{1}{\tan\theta}$$

(Hint: rewrite each inverse trig function as a trig function, then express each in terms of their reciprocals, then resolve for θ.)

(a) $\theta = \csc^{-1}(10.13844)$ (b) $\theta = \text{arcsec}(-1.92761)$ (c) $\theta = \cot^{-1}(26.23110)$

11. Find the **exact value** of each expression if it is defined.

(a) $\sin\left(\sin^{-1}\frac{1}{4}\right)$ (b) $\tan(\arctan 3)$ (c) $\cos\left(\cos^{-1}3\right)$ (d) $\cos^{-1}(\cos 3)$

(e) $\arcsin\left(\sin\left(-\frac{\pi}{7}\right)\right)$ (f) $\sin^{-1}\left(\sin\left(\frac{4\pi}{7}\right)\right)$ (g) $\tan^{-1}\left(2\sin\frac{2\pi}{3}\right)$ (h) $\arccos\left(\sqrt{3}\sin\frac{11\pi}{6}\right)$

12. Evaluate each expression by sketching a triangle and finding the missing side of the triangle.

(a) $\sin\left(\arccos\frac{3}{5}\right)$ (b) $\sin\left(\tan^{-1}\frac{12}{5}\right)$ (c) $\csc\left(\cos^{-1}\left(-\frac{7}{25}\right)\right)$

13. Rewrite each expression as an algebraic expression in *x*.

(a) $\cos\left(\sin^{-1}x\right)$ (b) $\sec(\arctan 2x)$

14. Rewrite each of the following into a composition of a tring and inverse trig function. Give two equivalent compostitions for each.

(a) $\frac{\sqrt{1-x^2}}{x}$ (b) $\frac{1}{2}\sqrt{4-9x^2}$

15. Using a graphing calculator, (a) find all the solutions to the following equation correct to three decimal places, then (b) find the exact solution using your knowledge of the unit circle.

$$\text{Arcsin}x - \text{Arccos}x = 0$$

16. (Calculator Permitted) The figures indicate that the higher the orbit of a satellite, the more of the earth the satellite can "see." Let θ, s, and h be as in the figure, and assume the earth is a sphere of radius 3960 miles. **(RADIAN MODE!!)**

    (a) Express the angle θ as a function of h, $\theta(h)$.

    (b) Express the distance s as a function of θ, $s(\theta)$.

    (c) Express the distance s as a function of h, $s(h)$, using your results from parts (a) and (b).

    (d) If the satellite is 100 miles above the earth, what is the distance s that it can see? Show your set up.

    (e) How high does the satellite have to be in order to see both Los Angeles and New York, 2450 miles apart? Show your set up.