

Name _____ Date _____ Period _____

Worksheet 4.4—Properties of Logs

Show all work on a separate sheet of paper. All answers must be given as either simplified, exact answers. No calculator is permitted unless otherwise stated.

Multiple Choice

- $\log 12 =$
 (A) $3 \log 4$ (B) $\log 3 + \log 4$ (C) $4 \log 3$ (D) $\log 3 \cdot \log 4$ (E) $2 \log 6$
- $\log_9 64 =$
 (A) $5 \log_3 2$ (B) $(\log_3 8)^2$ (C) $\frac{\ln 64}{\ln 9}$ (D) $2 \log_9 32$ (E) $\frac{\log 64}{9}$
- $2^{-1} \cdot (-3 \ln 2 - 1) =$
 (A) $-\frac{1}{2} \ln(8e)$ (B) $-\ln(8e)$ (C) $-\frac{3}{2} \ln 2$ (D) $-\frac{1}{2}$ (E) $\frac{1}{8}$
- $\log_{1/2} x^2 =$
 (A) $-2 \log_2 x$ (B) $2 \log_2 x$ (C) $-0.5 \log_2 x$ (D) $0.5 \log_2 x$ (E) $-2 \log_2 |x|$
- $\ln x^5 =$
 (A) $\frac{5 \log_7 x}{\log_7 e}$ (B) $\frac{2 \log x^3}{\log e}$ (C) $\frac{x \log_{1/2} 5}{\log_{1/2} e}$ (D) $3 \ln x^2$ (E) $\ln x^2 \cdot \ln x^3$

Short Answer

6. Evaluate each of the following expressions using the properties of logs (and no calculator).

(a) $\log_3 \sqrt[3]{81}$ (b) $\log 4 + \log 25$ (c) $\log_2 6 - \log_2 15 + \log_2 20$ (d) $\ln(\ln e^{e^{200}})$

7. Use the properties of logs to expand the following expressions.

(a) $\log_5 \sqrt[4]{x^3(x^2+1)}$ (b) $\log_6 \sqrt{\frac{5x^2y^3}{x^2+y^3}}$ (c) $\log \sqrt{x\sqrt{y\sqrt{z}}}$ (d) $\ln \left(\frac{7x^4 \sqrt{x^4-7}}{e^2 (x-5)^2 \sqrt[3]{2-6x^2}} \right)$

8. Use the properties of logs to condense the following expressions.

(a) $4 \ln x - \frac{1}{3} \ln(x^2+1) + 2 \ln(x-1)$ (b) $\frac{1}{3} \ln(2x+1) + \frac{1}{2} [\log(x-4) - \log(x^4-x^2-1)]$

(c) $\log(x^2-1) - \ln(x-1)$ (use the change of base formula on this one first)

9. If $\log_7 x = A \log_{2/3} x$, use the change of base formula to find the value of A ,

10. Simplify the following to a single log expression of the form $\log_b a$: $(\log_7 3)(\log_2 5)(\log_5 7)$
11. Use the properties of logs to prove that $-\ln(x - \sqrt{x^2 - 1}) = \ln(x + \sqrt{x^2 - 1})$. You may want to eventually multiply by a clever form of one.
12. Let $A = \ln 3$ and $B = \ln 5$, write each of the following in terms of A and/or B .
 (a) $\ln 15$ (b) $\ln 27$ (c) $\ln 75$ (d) $\ln 45$ (e) $\log_5 \sqrt{27}$
13. (Calculator Permitted) Solve the following equations graphically on your calculator. Be sure to report three decimals in your answers.
 (a) $\ln x > \sqrt[3]{x}$ (b) $1.2^x \leq \log_{1.2} x$

14. (Calculator Permitted) The “Occupy Wall Street” movement in 2011 is a protest against the unequal distribution of wealth in the United States. Vilfredo Pareto (1848-1923) observed that most of the wealth of any country is owned by a few members of the population. Pareto’s Principle is given by



$$\log P = \log c - k \log W$$

Where W is the wealth level (how much money a person has) and P is the number of people in the population having that much money.

- (a) Solve the equation for P .
- (b) Assume $k = 2.1$, $c = 8000$, and W is measured in millions of dollars. Use part (a) to find the number of people who have \$2 million or more. How many people have \$10 million or more?
- (c) If the population of the US is considered to be 312 million people, what percentage of the US population has \$10 million or more?

