

Name \_\_\_\_\_ Date \_\_\_\_\_ Period \_\_\_\_\_

**Worksheet 3.5—Rational Functions**

Show all work on a separate sheet of paper. All answers must be given as **simplified, exact answers!** No Calculators are permitted unless specified otherwise.

**Multiple Choice**

1. Let  $f(x) = -\frac{2x}{x^2 + 3x}$ . For what values of  $x$  does the graph of  $f(x)$  have a vertical asymptote?  
 (A)  $x = 0$     (B)  $x = 0, x = 3$     (C)  $x = 3$     (D)  $x = -3$     (E)  $x = 0, x = -3$
2. Let  $f(x) = -\frac{2x^2}{x^2 + 3x - 4}$ . Which of the following is an equation of an asymptote of  $f(x)$ ?  
 (A)  $y = 2$     (B)  $x = 1$     (C)  $x = 4$     (D)  $x = -2$     (E)  $y = -4$
3. Let  $f(x) = \frac{x^2}{x+5}$ . Which of the following statements is true about the graph of  $f$ ?  
 (A) There is no VA    (B) There is an HA but no VA    (C) There is an SA but no VA  
 (D) There is a VA and an SA    (E) There is a VA and an HA
4. What is the degree of the end-behavior model of  $f(x) = \frac{x^8 + 1}{x^4 + 1}$ ?  
 (A) 0    (B) 1    (C) 2    (D) 3    (E) 4
5. The equation of the end-behavior model of  $f(x) = \frac{2x^3 - x + 6}{x + 2}$  is given by  
 (A)  $y = 2x^2 - 7$     (B)  $y = 2x^2 - 1$     (C)  $y = 2x^2 + 4x + 7$     (D)  $y = 2x^2 - 4x + 7$     (E)  $y = 2x^2 - 4x - 7$

**Short Answer**

6. Find the  $x$ - and  $y$ - intercepts of the following functions  
 (a)  $t(x) = \frac{x^2 - x - 2}{x - 6}$     (b)  $r(x) = \frac{x^3 - 9x}{x^3}$
7. Find all vertical and horizontal asymptotes (if any).  
 (a)  $k(x) = \frac{6x - 2}{x^2 + 5x - 6}$     (b)  $j(x) = \frac{3x^2}{5 + 2x + x^2}$     (c)  $careful(x) = \frac{2x + x^3}{x - 1}$
8. Analyze the following functions. As in the notes, find the domain, discontinuities, intercepts, and end-behavior. Sketch a graph. Find the equations of all HA's, VA's, and SA's. Give the coordinate of any hole. Find the range after you graph it.  
 (a)  $f(x) = \frac{4x^2 + 4x - 24}{2x^2 + 4x - 16}$     (b)  $h(x) = \frac{x - 3}{x^2 + 3x}$     (c)  $q(x) = \frac{2x^3 - 6x^2 - 14x}{x^2 + 3x}$     (d)  $t(x) = \frac{(x^2 - x - 2)(x - 3)}{x^2 - 4x + 3}$

9. Write an equation of a function,  $f(x)$ , with a VA at  $x = -1$ , a hole at  $x = 3$ , and  $x$ -intercept at  $x = -3$ , and an HA at  $y = 1$ . Once you have the equation, find  $\lim_{x \rightarrow 3} f(x)$ .
10. Write an equation of a function  $d(x)$  with a  $y$ -intercept of  $(0, -2)$ , a VA at  $x = 1$ , an SA at  $y = 2x + 7$ , and a hole at  $x = 2$ . As  $x \rightarrow \infty$ , what do the slopes of the graph of  $d(x)$  approach?
11. Analyze and sketch  $h(x) = \frac{x^5 - 1}{x + 2}$ . Show all asymptotes, including end-behavior asymptotes.
12. (Calculator permitted) A drug is administered to a patient, and the concentration of the drug in the bloodstream is monitored. At time  $t \geq 0$  (in hours since giving the drug), the concentration (in mg/L) is given by

$$c(t) = \frac{5t}{t^2 + 1}$$

Graph the function with your graphing calculator in a reasonable window.

- What is a reasonable X and Y window? Justify.
- What is the highest concentration of drug that is reaching in the patient's bloodstream? How do you know this?
- What happens to the drug concentration after a long period of time? What are the mathematical implications of this if the person lives for many, many, many years after the injection?
- What is the concentration after 5 hours?
- How long does it take for the concentration to drop below 0.3 mg/L?