Name_____ Date_____ Period_____

Worksheet 3.4—Complex Zeros of Polynomial Functions

Show all work on a separate sheet of paper. All answers must be given as **simplified, exact answers**! Calculators are permitted, but only to help you narrow down choices of rational zeros or to find simplified values of leading coefficients, *A*.

Multiple Choice

1. The reciprocal of the number *i* is
   (A) $-i$ (B) -1 (C) 1 (D) i (E) none of these

2. State the **possible** number of imaginary zeros of $g(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$, where *a, b, c, d,* and *e* are real number coefficients.
   (A) 3 or 1 (B) 2, 4, or 0 (C) Exactly 1 (D) Exactly 3 (E) Exactly 4

3. What is the simplified form of $\dfrac{9i^5 - 5i^{1735}}{2i^4}$
   (A) $-7i$ (B) $-2i$ (C) $2i$ (D) $7i$ (E) none of these

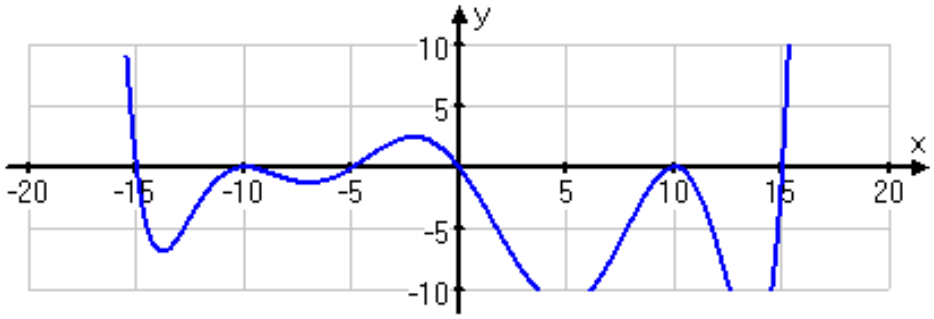
4. $\dfrac{-2-2i}{5-2i} =$ (A) $-\dfrac{2}{5}$ (B) $-\dfrac{14}{29}-\dfrac{14}{29}i$ (C) $-\dfrac{6}{29}-\dfrac{14}{29}i$ (D) $-\dfrac{6}{21}-\dfrac{14}{21}i$ (E) $-\dfrac{14}{21}-\dfrac{14}{21}i$

Short Answer

5. List all possible rational *x*-intercepts of $y = 2x^3 + 3x - 5$, then find all complex roots. Use your calculator to narrow down your rational root possibilities. Show the synthetic division.

6. Use the Rational Root Theorem to find possible rational zeroes of $y = 6x^4 - 11x^3 + 8x^2 - 33x - 30$, then find the all complex roots. Use your calculator to narrow down your rational root possibilities. Show the synthetic division. You do not have to list the rational root possibilities.

7. The following is a graph of a 8th degree polynomial function, $f(x)$, with all real roots.
   (a) Write a general equation of the function.



   (b) If the function satisfies $f(-2) = 3$, find the particular equation of $f(x)$. Show work and use proper notation.

8. Sketch a graph of the following polynomial. Show all *x*-intercepts.
   $y = -{}^{1}/_{5600}(x+5)^2(x+1)(x-4)^3(x-7)$

9. Given that $-i+2$ is a zero of $f(x)=x^5-6x^4+11x^3-x^2-14x+5$, find all complex roots using synthetic division. List your possible rational roots also.

10. Find all the complex zeroes of the following polynomial: $f(x)=2x^5+3x^4-30x^3-57x^2-2x+24$. List all possible rational roots first, then use your calculator to help narrow down the search. Show your synthetic division.

11. Find the remainder when $x^{36}+4x^{27}+7$ is divided by $x+1$.

12. Find $P(x)$ if $P(x)$ divided by $x-1$ has a remainder of -2 and a quotient of x^3+x^2-x-1. Write $P(x)$ in expanded form.

13. A polynomial function has the following complex roots: A polynomial $P(x)$ has the following roots: $-2,\ 1+\sqrt{3},\ 5i$.
    a. Write an equation of the function of lowest possible degree. Remember to expand any factors containing radicals or imaginary units.
    b. If $P(0)=-25$, write the particular equation.

14. Determine a polynomial of lowest degree with real coefficients that has the given roots:
    a. 0(m2), $4+3i$ (b) $7-i\sqrt{5}$ (m2), $\sqrt{5}$ (m2)

15. Determine k so that $f(x)=x^3-11x^2+kx-6$ has $x-3$ as a factor.

16. True or False: if False, explain why or provide a counterexample.
    a. A polynomial of the 5th degree can have only 2 real roots and 3 imaginary roots
    b. A polynomial function of degree 8 can only have 5 real roots.
    c. A polynomial function of degree 7 must have at least one rational root.
    d. A 44^{th} degree polynomial function can have exactly 12 relative extrema.
    e. Every even degree function is even.
    f. Every odd polynomial function is also of odd degree.
    g. An odd degree polynomial has a range of all real numbers.
    h. An even degree polynomial has a domain of all real numbers.
    i. Precalculus is awesome.