

Name _____ Date _____ Period _____

Worksheet 2.5—Building Functions from other Functions

Show all work on a separate sheet of paper. Give simplified, exact values for all answers. **No Calculator is Permitted unless specifically stated.**

I. Multiple Choice

1. If the point $(3, 4)$ lies on the graph of an invertible function f , then which of the following points lies on the graph of its inverse?

(A) $(4, 3)$ (B) $(3, -4)$ (C) $\left(3, \frac{1}{4}\right)$ (D) $(-3, 4)$ (E) None of these

2. The inverse of the function $f(x) = 7x + 8$ will be

(A) $g(x) = \frac{x-8}{7}$ (B) $g(x) = \frac{1}{7x+8}$ (C) $g(x) = \frac{8}{x-7}$ (D) $g(x) = -7x - 8$ (E) $g(x) = -\frac{1}{7}x + 8$

3. If $f(x) = \sqrt{x}$ and $g(x) = x^2$, then $(gf)(x) =$

(A) $\frac{\sqrt{x}}{x}$ (B) $|x|$ (C) $x^{5/2}$ (D) x (E) $\frac{x}{\sqrt{x}}$

4. If $f(x) = \sqrt{x}$ and $g(x) = x^2$, then $(g \circ f)(x) =$

(A) $\frac{\sqrt{x}}{x}$ (B) $|x|$ (C) $x^{5/2}$ (D) x (E) $\frac{x}{\sqrt{x}}$

5. If $f(x) = \sqrt{x}$ and $g(x) = x^2$, then $(f \circ g)(x) =$

(A) $\frac{\sqrt{x}}{x}$ (B) $|x|$ (C) $x^{5/2}$ (D) x (E) $\frac{x}{\sqrt{x}}$

6. Suppose f and g are functions with domain of all real numbers. Which of the following is NOT necessarily true?

(A) $(f + g)(x) = (g + f)(x)$ (B) $(fg)(x) = (gf)(x)$ (C) $f(g(x)) = g(f(x))$
 (D) $(f - g)(x) = -(g - f)(x)$ (E) $(f \circ g)(x) = f(g(x))$

7. If $f(x) = x - 7$ and $g(x) = \sqrt{4 - x}$, what is the domain of $\frac{f}{g}$?

(A) $(-\infty, 4)$ (B) $(-\infty, 4]$ (C) $(4, \infty)$ (D) $[4, \infty)$ (E) $(4, 7) \cup (7, \infty)$

8. If $f(x) = x^2 + 1$, then $(f \circ f)(x) =$

(A) $2x^2 + 2$ (B) $2x^2 + 1$ (C) $x^4 + 1$ (D) $x^4 + 2x^2 + 1$ (E) $x^4 + 2x^2 + 2$

9. Which of the following relations is equivalent to $y = |x|$?

(A) $y = x$ (B) $y = \sqrt{x^2}$ (C) $y^3 = x^3$ (D) $y = (\sqrt{x})^2$ (E) $x = |y|$

10. Let $h(x) = \frac{4x+5}{2x-7}$ and $f(x) = x+6$. If $h(x) = (g \circ f)(x)$, then $g(x)$ is ??

(A) $\frac{4x+1}{2x-13}$ (B) $\frac{4x-1}{2x+13}$ (C) $\frac{4x}{2x} - \frac{5}{7}$ (D) $\frac{4x-19}{2x-5}$ (E) None of these

II. Short Answer

11. If $f(x) = \sqrt{x+3}$ and $g(x) = \sqrt{x-4}$, find formulas for $h =: \frac{f}{g}, \frac{g}{f}, f+g, f \circ g$, and $g \circ f$. Give the domain of each.

12. For each of the following, find $f(g(x))$ and $g(f(x))$. Find the domain of each and decide if f and g are inverses. Give an explanation for your answers.

(a) $f(x) = \frac{1}{x-1}, g(x) = \sqrt{x}$ (b) $f(x) = \frac{1}{x+1}, g(x) = \frac{1}{x-1}$

13. Decompose each of the following functions h into two functions f and g such that $h(x) = f(g(x))$. Find two, different, non-trivial decompositions.

(a) $h(x) = \sqrt{x^2 - 5x}$ (b) $h(x) = \frac{3}{x^3 - 5x + 6}$ (c) $h(x) = \sqrt{x + e^{\sqrt{x}}}$

14. Assume f is a one-to-one function.

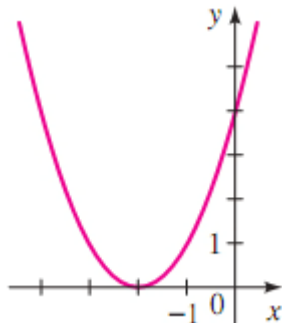
(a) If $f(2) = 9$, find $f^{-1}(9)$ (b) If $f^{-1}(-3) = 1$, find $f(1)$ (c) if $f(x) = 5 - 2x$, find $f^{-1}(-3)$

15. Find the inverse, $g(x)$, of the following functions, then compose the functions to verify.

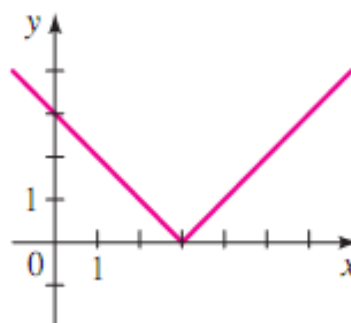
(a) $f(x) = (2 - x^3)^5$ (b) $f(x) = \frac{2-7x}{3x-1}$

16. The following functions are not one-to-one. Restrict each's domain so that the resulting function IS one-to-one. Write an equation for each graph (assume no dilations), then find the equation of the inverse function under the restricted domain.

(a)

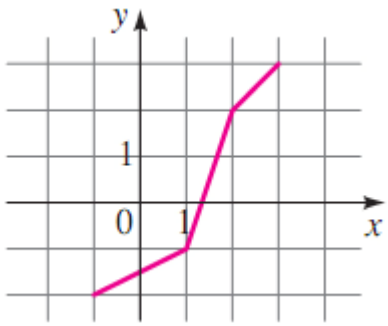


(b)

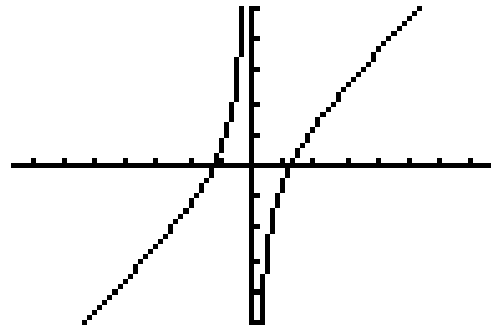


17. Use the graph of each function, f , to sketch the graph of f^{-1} . Assume the scales are square.

(a)



(b)



18. Korpicello's Pizza charges a base price of \$5 for a large pizza, plus \$2 for each topping.

- Write an equation for the total cost, C , of a large pizza with n toppings.
- Find the equation for $C^{-1}(n)$, the inverse function of $C(n)$.
- What is practical interpretation (or what is the usefulness) of $C^{-1}(n)$?
- What are **your** favorite toppings? If you only had \$10 to spend, how many, and which, toppings would you/could you get?

