PreAP Precalculus Spring Final Exam Review

Name_____ Date_____ Period_____

Calculater Permitted
MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Write each expression in factored form as an algebraic expression of a single trigonometric function.

1) $\sec^4 x + \sec^2 x \tan^2 x - 2 \tan^4 x$ 1) _____

A) $4 \sec^2 x$ B) $\tan^2 x - 1$ C) $3 \sec^2 x - 2$ D) $\sec^2 x + 2$

Use basic identities to simplify the expression.

2) $\dfrac{\cos^2\theta}{\sin^2\theta} + \csc\theta \sin\theta$ 2) _____

A) $\sec^2\theta$ B) 1 C) $\csc^2\theta$ D) $\tan^2\theta$

3) $\sin^2\theta + \tan^2\theta + \cos^2\theta$ 3) _____

A) $\sec^2\theta$ B) $\tan^2\theta$ C) $\cos^3\theta$ D) $\sin\theta$

Rewrite with only sin x and cos x.

4) $\cos 2x + \sin x$ 4) _____

A) $1 + 2 \sin^2 x + \sin x$ B) $1 + 3 \sin^2 x$

C) $1 + 3 \sin x$ D) $1 - 2 \sin^2 x + \sin x$

Eliminate the parameter.

5) $x = t + 4, y = t^2$ 5) _____

A) $y = x^2 - 8x + 16$ B) $y = x^2 + 16$ C) $y = \sqrt{x - 4}$ D) $y = \sqrt{x} + x + 4$

State whether the given measurements determine zero, one, or two triangles.

6) $C = 30°, a = 32, c = 16$ 6) _____

A) Zero B) Two C) One

Convert the angle to decimal degrees and round to the nearest hundredth of a degree.

7) $238°54'38''$ 7) _____

A) 238.97 B) 238.92 C) 238.87 D) 238.91

Solve the problem.

8) A police helicopter is monitoring the speed of two cars on a straight road. The helicopter is at an altitude of 4200 feet directly above the road. At one instant, the angle of elevation from the first car to the helicopter is 20°, and the angle of elevation from the second car to the helicopter is 15°. How far apart are the two cars to the nearest foot? 8) _____

A) 121 feet B) 4135 feet C) 3948 feet D) 403 feet

9) A generator produces an alternating current according to the equation $I = 64 \sin 20\pi t$, where t is time in seconds and I is the current in amperes. What is the smallest time t such that $I = 32$? 9) _____

A) $\dfrac{1}{80}$ sec B) $\dfrac{1}{60}$ sec C) $\dfrac{1}{120}$ sec D) $\dfrac{1}{40}$ sec

10) From a boat on the river below a dam, the angle of elevation to the top of the dam is 23°52'. If the dam is 2314 feet above the level of the river, how far is the boat from the base of the dam (to the nearest foot)? 10) _____

A) 5230 ft B) 5220 ft C) 5210 ft D) 5200 ft

11) For what values of θ ($0 \le \theta < 2\pi$) do maximum r-values occur on the graph of the polar equation $r = 2 \cos 4\theta$? Note that a maximum r-value occurs at a point that is the maximum distance from the pole. 11) _____

A) $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

B) $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$

C) $0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}$

D) $\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$

12) The minute hand of a clock is 9 inches long. What distance does its tip move in 19 minutes? 12) _____

A) $\frac{19}{270}\pi$ in. B) $\frac{57}{10}\pi$ in. C) $\frac{57}{20}\pi$ in. D) $\frac{19}{540}\pi$ in.

13) A boat sails for 2 hours at 35 mph in a direction 91.40°. How far south has it sailed (to the nearest mile)? 13) _____

A) 0 mi B) 2 mi C) -2 mi D) 4 mi

14) The radius of a car wheel is 15 inches. How many revolutions per minute is the wheel making when the car is travelling at 30 mph. Round your answer to the nearest revolution. 14) _____

A) 2101 rpm B) 3318 rpm C) 9 rpm D) 336 rpm

15) A ship travels 98 km on a bearing of 38°, and then travels on a bearing of 128° for 169 km. Find the distance of the end of the trip from the starting point, to the nearest kilometer. 15) _____

A) 267 km B) 77 km C) 60 km D) 195 km

Solve for x in the given interval.

16) $\sec x = -2, \pi \le x \le \frac{3\pi}{2}$ 16) _____

A) $\frac{4\pi}{3}$ B) $\frac{2\pi}{3}$ C) $\frac{7\pi}{6}$ D) $\frac{5\pi}{4}$

Find an exact value.

17) $\sin \frac{-11\pi}{12}$ 17) _____

A) $\frac{\sqrt{2} - \sqrt{6}}{4}$ B) $\frac{\sqrt{6} + \sqrt{2}}{4}$ C) $\frac{\sqrt{6} - \sqrt{2}}{4}$ D) $\frac{-\sqrt{6} - \sqrt{2}}{4}$

Convert the radian measure to degree measure. Use the value of π found on a calculator and round answers to two decimal places.

18) -2.2853 18) _____

A) -131.64° B) -129.94° C) -130.94° D) -130.44°

Suppose that θ is in standard position and the given point is on the terminal side of θ. Give the exact value of the indicated trig function for θ.

19) $(-20, 48)$; find $\sin\theta$. 19) _____

A) $-\frac{12}{13}$ B) $-\frac{5}{13}$ C) $\frac{12}{13}$ D) $\frac{5}{13}$

Decide whether a triangle can be formed with the given side lengths. If so, use Heron's formula to find the area of the triangle.

20) a = 59.3 20) _____
b = 65.4
c = 56.2

A) 1554.77 B) 1484.54
C) No triangle is formed. D) 1498.64

Write an equation for a sine curve that has the given amplitude and period, and which passes through the given point.

21) Amplitude 4, period $\pi/6$, point (1/3, 0) 21) _____

A) $y = 4\sin\left(12x - \frac{\pi}{3}\right)$ B) $y = 4\sin(6x - 2)$

C) $y = 4\sin(12x - 4)$ D) $y = 4\sin\left(\frac{\pi}{6}x - \frac{\pi}{3}\right)$

Solve the equation.

22) Solve $\cot\theta = \sqrt{3}$ for θ, where $0° \le \theta \le 90°$ 22) _____

A) 60° B) 30° C) 75° D) 45°

Convert from degrees to radians. Use the value of π found on a calculator and round answers to four decimal places, as needed.

23) 144° 23) _____

A) $\frac{3\pi}{5}$ B) $\frac{8\pi}{5}$ C) $\frac{4\pi}{5}$ D) $\frac{2\pi}{5}$

Find an algebraic expression equivalent to the given expression.

24) sin (arcsec u) 24) _____

A) $\sqrt{u^2 + 1}$ B) $\frac{u\sqrt{u^2 + 1}}{u2 + 1}$ C) $\sqrt{u^2 - 1}$ D) $\frac{\sqrt{u^2 - 1}}{u}$

Find an equivalent equation in polar coordinates.

25) $x^2 + y^2 - 4x = 0$ 25) _____

A) $r = 4\cos\theta$ B) $r\sin^2\theta = 4\cos\theta$
C) $r = 4\sin\theta$ D) $r\cos^2\theta = 4\sin\theta$

Find the component form and magnitude of the indicated vector.

26) Given that P = (-4, 8) and Q = (-12, 7), find the component form and magnitude of the vector $\overrightarrow{PQ}$. 26) _____

A) $\langle -8, -1\rangle, \sqrt{65}$ B) $\langle 8, 1\rangle, \sqrt{65}$ C) $\langle -16, -1\rangle, \sqrt{257}$ D) $\langle -8, -1\rangle, 65$

Find the rectangular coordinates of the point with the given polar coordinates.

27) $\left(-6, \frac{3}{2}\pi\right)$ 27) _____

A) (-6, 0) B) (6, 0) C) (0, 6) D) (0, -6)

Write the expression as the sine, cosine, or tangent of an angle.

28) $\cos\frac{\pi}{5}\cos\frac{\pi}{7} + \sin\frac{\pi}{5}\sin\frac{\pi}{7}$ 28) _____

A) $\sin\frac{12\pi}{35}$ B) $\cos\frac{12\pi}{35}$ C) $\sin\frac{2\pi}{35}$ D) $\cos\frac{2\pi}{35}$

Find all solutions in the interval $[0, 2\pi)$.

29) $\sin^2\left(\frac{x}{2}\right) = \sin^2 x$ 29) _____

A) $0, \frac{2\pi}{3}, \frac{4\pi}{3}$ B) $0, \pi$ C) $0, \frac{2\pi}{3}$ D) $\frac{2\pi}{3}, \frac{4\pi}{3}$

30) $\sin^2 x - \cos^2 x = 0$ 30) _____

A) $x = \frac{\pi}{4}, \frac{\pi}{6}$ B) $x = \frac{\pi}{4}, \frac{\pi}{3}$

C) $x = \frac{\pi}{4}$ D) $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

Find the unit vector in the direction of the given vector. Write your answer in the indicated form.

31) Let $u = \langle -5, -1\rangle$. Find the unit vector in the direction of u, and write your answer as a linear combination of the standard unit vectors i and j. 31) _____

A) $\frac{-5}{\sqrt{26}}i + \frac{-1}{\sqrt{26}}j$ B) $i + j$ C) $\frac{-5}{26}i + \frac{-1}{26}j$ D) $\frac{-5}{6}i + \frac{-1}{6}j$

Determine two pairs of polar coordinates for the point with $0° \le \theta < 360°$.

32) $(2\sqrt{3}, 6)$ 32) _____

A) $(4\sqrt{3}, 30°), (-6\sqrt{3}, 330°)$ B) $(4\sqrt{3}, 300°), (-4\sqrt{3}, 60°)$

C) $(4\sqrt{3}, 30°), (-4\sqrt{3}, 330°)$ D) $(4\sqrt{3}, 60°), (-4\sqrt{3}, 240°)$

Find the value of the unique real number θ between 0 and 2π that satisfies the given conditions.

33) $\cos\theta = -\frac{\sqrt{2}}{2}$ and $\tan\theta > 0$ 33) _____

A) $\frac{3\pi}{4}$ B) $\frac{\pi}{4}$ C) $\frac{5\pi}{4}$ D) $\frac{4\pi}{3}$

Two triangles can be formed using the given measurements. Solve both triangles.

34) B = 32°, b = 27, c = 32 34) _____

A) A = 109.1°, C = 38.9°, a = 48.1; A = 6.9°, C = 141.1°, a = 6.1

B) A = 96.9°, C = 51.1°, a = 14.4; A = 83.1°, C = 128.9°, a = 14.4

C) A = 109.1°, C = 38.9°, a = 15.1; A = 6.9°, C = 141.1°, a = 15.1

D) A = 96.9°, C = 51.1°, a = 50.6; A = 83.1°, C = 128.9°, a = 50.6

Simplify the expression.

35) $\frac{1}{1 - \cos x} + \frac{1}{1 + \cos x}$ 35) _____

A) $2 \csc^2 x$ B) $\csc^2 x$ C) $2 \csc x$ D) $2 \sec^2 x$

36) $\csc\left(\frac{\pi}{2} - x\right) \cos(-x)$ 36) _____

A) -1 B) 1 C) $\cos^2 x$ D) $-\csc^2 x$

37) $\frac{1 - \sin^2 x}{\sin x - \csc x}$ 37) _____

A) $-\sin x$ B) $\cos^2 x$ C) $-\cos x$ D) $\sin^2 x$

Find an equivalent equation in rectangular coordinates.

38) $r = \frac{5}{1 + \cos \theta}$ 38) _____

A) $y^2 = 25 - 10x$ B) $x^2 = 25 - 10y$ C) $y^2 = 10x - 25$ D) $x^2 = 10y - 25$

Use the fundamental identities to find the value of the trigonometric function.

39) Find $\csc \theta$ if $\cot \theta = -\sqrt{15}$ and $\cos \theta < 0$. 39) _____

A) -4 B) $-\frac{1}{4}$ C) $\frac{1}{4}$ D) 4

Find all solutions to the equation in the interval $[0, 2\pi)$.

40) $\sin 2x = -\sin x$ 40) _____

A) $0, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}$ B) $\frac{\pi}{8}, \frac{9\pi}{8}$ C) $\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ D) No solution

Assume that θ is an acute angle in a right triangle satisfying the given conditions. Evaluate the indicated trigonometric function.

41) $\sin \theta = \frac{8}{9}$; $\cot \theta$ 41) _____

A) $\frac{\sqrt{17}}{8}$ B) $\frac{9}{\sqrt{17}}$ C) $\frac{8}{\sqrt{17}}$ D) $\frac{\sqrt{17}}{9}$

Determine whether the given function is positive or negative for values of t in the specified quadrant.

42) Quadrant II, cot t 42) _____

A) Negative B) Positive

Evaluate without using a calculator.

43) $\sec \beta$, if $\sin \beta = -\frac{7}{10}$ and $\tan \theta > 0$ 43) _____

A) $\frac{\sqrt{10}}{7}$ B) $-\frac{\sqrt{51}}{10}$ C) $-\frac{7\sqrt{51}}{51}$ D) $-\frac{10\sqrt{51}}{51}$

Give the exact value.

44) $\sec \frac{\pi}{4}$ 44) _____

A) $\frac{\sqrt{2}}{2}$ B) $\frac{2\sqrt{3}}{3}$ C) $\sqrt{3}$ D) $\sqrt{2}$

Find the measures of two angles, one positive and one negative, that are coterminal with the given angle.

45) $\frac{9\pi}{5}$ 45) _____

A) $\frac{14\pi}{5}; -\frac{\pi}{5}$ B) $\frac{14\pi}{5}; -\frac{14\pi}{5}$ C) $\frac{\pi}{5}; -\frac{19\pi}{5}$ D) $\frac{19\pi}{5}; -\frac{\pi}{5}$

Find the area. Round your answer to the nearest hundredth if necessary.

46) Find the area of the triangle with the following measurements: 46) _____
B = 104°, a = 12 cm, c = 22 cm

A) 31.93 cm^2 B) 256.16 cm^2 C) 132 cm^2 D) 128.08 cm^2

Describe the transformations required to obtain the graph of the function f(x) from the graph of the function g(x).

47) $f(x) = \cos \frac{x}{5}$; $g(x) = \cos x$ 47) _____

A) Horizontal shrink by a factor of $\frac{1}{5}$ B) Horizontal stretch by a factor of 5

C) Vertical stretch by a factor of 5 D) Vertical shrink by a factor of $\frac{1}{5}$

Use a calculator to find the approximate value of the expression. Express your answer in radians and round to three decimal places.

48) $\csc^{-1}(1.4104)$ 48) _____

A) -3.930 B) 0.788 C) 0.866 D) 3.930

Use the arc length formula and the given information to find the indicated quantity.

49) r = 14 ft, $\theta = 39°$; find s 49) _____

A) $\frac{91}{60}\pi$ ft B) 1092 ft C) 546 ft D) $\frac{91}{30}\pi$ ft

Find the exact value by using a half-angle identity.

50) $\cos\left(-\frac{\pi}{8}\right)$ 50) _____

A) $\frac{1}{2}\sqrt{1+\sqrt{2}}$ B) $\frac{1}{2}\sqrt{2-\sqrt{2}}$ C) $\frac{1}{2}\sqrt{1-\sqrt{2}}$ D) $\frac{1}{2}\sqrt{2+\sqrt{2}}$