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$n^3 - n$.

ercise 53.

QUICK REVIEW 1.1 *(For help, go to Section A.2.)*

Factor the following expressions completely over the real numbers.

1. $x^2 - 16$

2. $x^2 + 10x + 25$

3. $81y^2 - 4$

4. $3x^3 - 15x^2 + 18x$

5. $16h^4 - 81$

6. $x^2 + 2xh + h^2$

7. $x^2 + 3x - 4$

8. $x^2 - 3x + 4$

9. $2x^2 - 11x + 5$

10. $x^4 + x^2 - 20$

SECTION 1.1 EXERCISES

In Exercises 1–10, match the numerical model to the corresponding graphical model (*a–j*) and algebraic model (*k–t*).

1.

x	3	5	7	9	12	15
y	6	10	14	18	24	30

2.

x	0	1	2	3	4	5
y	2	3	6	11	18	27

3.

x	2	4	6	8	10	12
y	4	10	16	22	28	34

4.

x	5	10	15	20	25	30
y	90	80	70	60	50	40

5.

x	1	2	3	4	5	6
y	39	36	31	24	15	4

6.

x	1	2	3	4	5	6
y	5	7	9	11	13	15

7.

x	5	7	9	11	13	15
y	1	2	3	4	5	6

8.

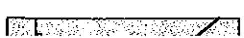
x	4	8	12	14	18	24
y	20	72	156	210	342	600

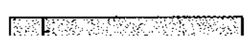
9.

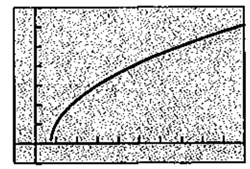
x	3	4	5	6	7	8
y	8	15	24	35	48	63

10.

x	4	7	12	19	28	39
y	1	2	3	4	5	6

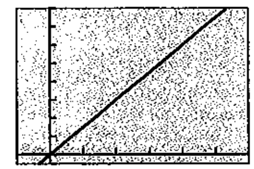






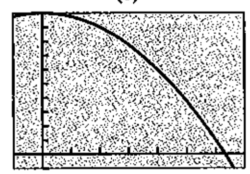
[–4, 40] by [–1, 7]

(c)



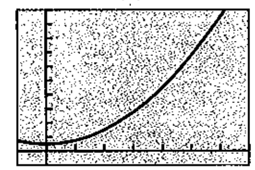
[–3, 18] by [–2, 32]

(d)



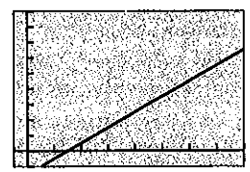
[–1, 7] by [–4, 40]

(e)



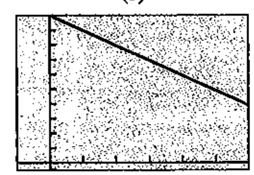
[–1, 7] by [–4, 40]

(f)



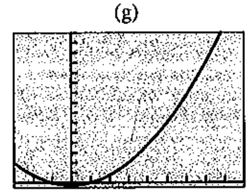
[–1, 16] by [–1, 9]

(g)



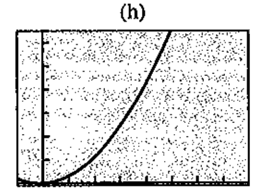
[–5, 30] by [–5, 100]

(h)



[–3, 9] by [–2, 60]

(i)



[–5, 40] by [–10, 650]

(j)

(k) $y = x^2 + x$

(l) $y = 40 - x^2$

Exercises 11–18 refer to the data in Table 1.6 below showing the percentage of the female and male populations in the United States employed in the civilian work force in selected years from 1954 to 2004.

Table 1.6 Employment Statistics

Year	Female	Male
1954	32.3	83.5
1959	35.1	82.3
1964	36.9	80.9
1969	41.1	81.1
1974	42.8	77.9
1979	47.7	76.5
1984	50.1	73.2
1989	54.9	74.5
1994	56.2	72.6
1999	58.5	74.0
2004	57.4	71.9

Source: www.bls.gov

11. (a) According to the numerical model, what has been the trend in females joining the work force since 1954?

(b) In what 5-year interval did the percentage of women who were employed change the most?

12. (a) According to the numerical model, what has been the trend in males joining the work force since 1954?

(b) In what 5-year interval did the percentage of men who were employed change the most?

13. Model the data graphically with two scatter plots on the same graph, one showing the percentage of women employed as a function of time and the other showing the same for men. Measure time in years since 1954.

14. Are the male percentages falling faster than the female percentages are rising, or vice versa?

15. Model the data algebraically with linear equations of the form $y = mx + b$. Write one equation for the women's data and another equation for the men's data. Use the 1954 and 1999 ordered pairs to compute the slopes.

16. If the percentages continue to follow the linear models you found in Exercise 15, what will the employment percentages for women and men be in the year 2009?

17. If the percentages continue to follow the linear models you found in Exercise 15, when will the percentages of women and men in the civilian work force be the same? What percentage will that be?

18. Writing to Learn Explain why the percentages cannot continue indefinitely to follow the linear models that you wrote in Exercise 15.

19. Doing Arithmetic with Lists Enter the data from the "Total" column of Table 1.2 of Example 2 into list L_1 in your calculator. Enter the data from the "Female" column into list L_2. Check a few computations to see that the procedures in (a) and (b) cause the calculator to divide each element of L_2 by the corresponding entry in L_1, multiply it by 100, and store the resulting list of percentages in L_3.

(a) On the home screen, enter the command: $100 \times L_2 / L_1 \rightarrow L_3$.

(b) Go to the top of list L_3 and enter $L_3 = 100(L_2/L_1)$.

20. Comparing Cakes A bakery sells a 9″ by 13″ cake for the same price as an 8″ diameter round cake. If the round cake is twice the height of the rectangular cake, which option gives the most cake for the money?

21. Stepping Stones A garden shop sells 12″ by 12″ square stepping stones for the same price as 13″ round stones. If all of the stepping stones are the same thickness, which option gives the most rock for the money?

22. Free Fall of a Smoke Bomb At the Oshkosh, WI, air show, Jake Trouper drops a smoke bomb to signal the official beginning of the show. Ignoring air resistance, an object in free fall will fall d feet in t seconds, where d and t are related by the algebraic model $d = 16t^2$.

(a) How long will it take the bomb to fall 180 feet?

(b) If the smoke bomb is in free fall for 12.5 seconds after it is dropped, how high was the airplane when the smoke bomb was dropped?

23. Physics Equipment A physics student obtains the following data involving a ball rolling down an inclined plane, where t is the elapsed time in seconds and y is the distance traveled in inches.

t	0	1	2	3	4	5
y	0	1.2	4.8	10.8	19.2	30

Find an algebraic model that fits the data.

24. U.S. Air Travel The number of revenue passengers enplaned in the U.S. over the 14-year period from 1991 to 2004 is shown in Table 1.7.

Table 1.7 U.S. Air Travel

Year	Passengers (millions)	Year	Passengers (millions)
1991	452.3	1998	612.9
1992	475.1	1999	636.0
1993	488.5	2000	666.1
1994	528.8	2001	622.1
1995	547.8	2002	612.9
1996	581.2	2003	646.3
1997	594.7	2004	697.8

Source: www.airlines.org

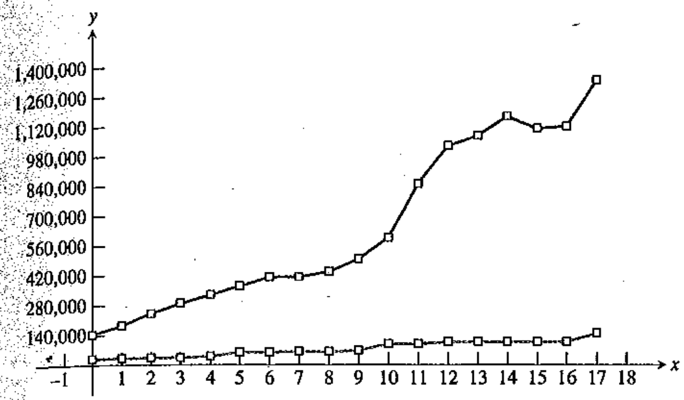
(a) Graph a scatter plot of the data. Let x be the number of years since 1991.

(b) From 1991 to 2000 the data seem to follow a linear model. Use the 1991 and 2000 points to find an equation of the line and superimpose the line on the scatter plot.

(c) According to the linear model, in what year did the number of passengers seem destined to reach 900 million?

(d) What happened to disrupt the linear model?

Exercises 25–28 refer to the graph below, which shows the *minimum* salaries in major league baseball over a recent 18-year period and the *average* salaries in major league baseball over the same period. Salaries are measured in dollars and time is measured after the starting year (year 0).



Source: Major League Baseball Players Association.

25. Which line is which, and how do you know?

26. After Peter Ueberroth's resignation as baseball commissioner in 1988 and his successor's untimely death in 1989, the team owners broke free of previous restrictions and began an era of competitive spending on player salaries. Identify where the 1990 salaries appear in the graph and explain how you can spot them.

27. The owners attempted to halt the uncontrolled spending by proposing a salary cap, which prompted a players' strike in 1994. The strike caused the 1995 season to be shortened and left many fans angry. Identify where the 1995 salaries appear in the graph and explain how you can spot them.

28. Writing to Learn Analyze the general patterns in the graphical model and give your thoughts about what the long-term implications might be for

(a) the players;

(b) the team owners;

(c) the baseball fans.

In Exercises 29–38, solve the equation algebraically and graphically.

29. $v^2 - 5 = 8 - 2v^2$

33. $x(2x - 5) = 12$ **34.** $x(2x - 1) = 10$

35. $x(x + 7) = 14$

36. $x^2 - 3x + 4 = 2x^2 - 7x - 8$

37. $x + 1 - 2\sqrt{x + 4} = 0$ **38.** $\sqrt{x} + x = 1$

In Exercises 39–46, solve the equation graphically by converting it to an equivalent equation with 0 on the right-hand side and then finding the x-intercepts.

39. $2x - 5 = \sqrt{x + 4}$ **40.** $|3x - 2| = 2\sqrt{x + 8}$

41. $|2x - 5| = 4 - |x - 3|$ **42.** $\sqrt{x + 6} = 6 - 2\sqrt{5 - x}$

43. $2x - 3 = x^3 - 5$ **44.** $x + 1 = x^3 - 2x - 5$

45. $(x + 1)^{-1} = x^{-1} + x$ **46.** $x^2 = |x|$

47. Swan Auto Rental charges \$32 per day plus \$0.18 per mile for an automobile rental.

(a) Elaine rented a car for one day and she drove 83 miles. How much did she pay?

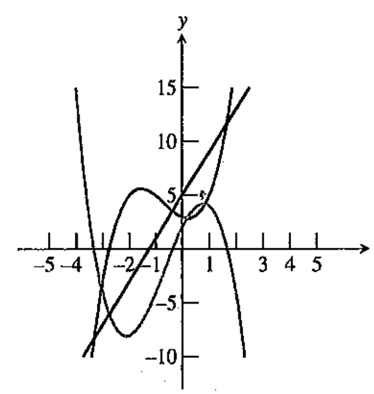
(b) Ramon paid \$69.80 to rent a car for one day. How far did he drive?

48. Connecting Graphs and Equations The curves on the graph below are the graphs of the three curves given by

$y_1 = 4x + 5$

$y_2 = x^3 + 2x^2 - x + 3$

$y_3 = -x^3 - 2x^2 + 5x + 2.$



(a) Write an equation that can be solved to find the points of intersection of the graphs of y_1 and y_2.

(b) Write an equation that can be solved to find the x-intercepts of the graph of y_3.

(c) Writing to Learn How does the graphical model reflect the

49. Exploring Grapher Failure Let $y = (x^{200})^{1/200}$.

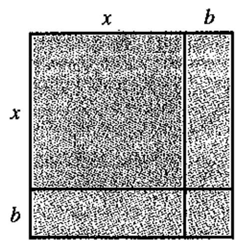
(a) Explain algebraically why $y = x$ for all $x \geq 0$.

(b) Graph the equation $y = (x^{200})^{1/200}$ in the window $[0, 1]$ by $[0, 1]$.

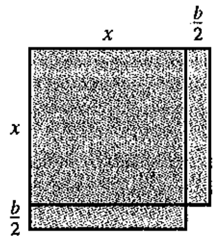
(c) Is the graph different from the graph of $y = x$?

(d) Can you explain why the grapher failed?

50. Connecting Algebra and Geometry Explain how the algebraic equation $(x + b)^2 = x^2 + 2bx + b^2$ models the areas of the regions in the geometric figure shown below on the left:



(Ex. 50)



(Ex. 52)

51. Exploring Hidden Behavior Solving graphically, find all real solutions to the following equations. Watch out for hidden behavior.

(a) $y = 10x^3 + 7.5x^2 - 54.85x + 37.95$

(b) $y = x^3 + x^2 - 4.99x + 3.03$

52. Connecting Algebra and Geometry The geometric figure shown on the right above is a large square with a small square missing.

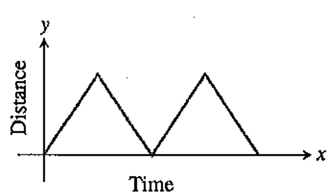
(a) Find the area of the figure.

(b) What area must be added to complete the large square?

(c) Explain how the algebraic formula for completing the square models the completing of the square in (b).

53. Proving a Theorem Prove that if n is a positive integer, then $n^2 + 2n$ is either odd or a multiple of 4. Compare your proof with those of your classmates.

54. Writing to Learn The graph below shows the distance from home against time for a jogger. Using information from the graph, write a paragraph describing the jogger's workout.



Standardized Test Questions

55. True or False A product of real numbers is zero if and only if every factor in the product is zero. Justify your answer.

56. True or False An algebraic model can always be used to make accurate predictions.

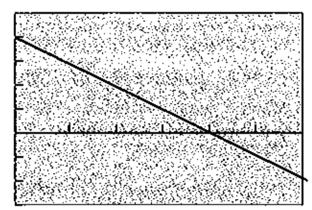
In Exercises 57–60, you may use a graphing calculator to decide which algebraic model corresponds to the given graphical or numerical model.

(A) $y = 2x + 3$ **(B)** $y = x^2 + 5$

(C) $y = 12 - 3x$ **(D)** $y = 4x + 3$

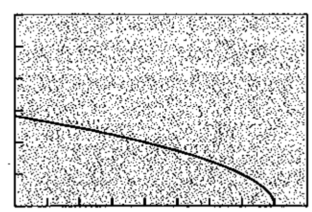
(E) $y = \sqrt{8 - x}$

57. Multiple Choice



[0, 6] by [−9, 15]

58. Multiple Choice



[0, 9] by [0, 6]

59. Multiple Choice

x	1	2	3	4	5	6
y	6	9	14	21	30	41

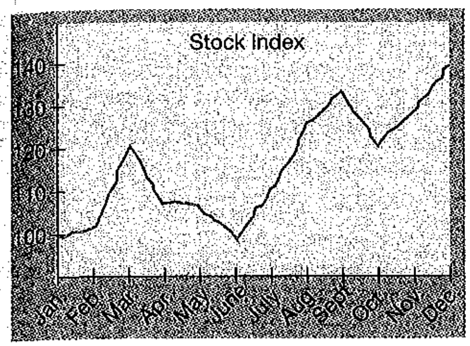
60. Multiple Choice

x	0	2	4	6	8	10
y	3	7	11	15	19	23

Explorations

61. Analyzing the Market Both Ahmad and LaToya watch the stock market throughout the year for stocks that make significant jumps from one month to another. When they spot one, each buys 100 shares. Ahmad's rule is to sell the stock if it fails to perform well for three months in a row. LaToya's rule is to sell in December if the stock has failed to perform well since its purchase.

The graph below shows the monthly performance in dollars (Jan–Dec) of a stock that both Ahmad and LaToya have been watching.



(a) Both Ahmad and LaToya bought the stock early in the year. In which month?

(b) At approximately what price did they buy the stock?

(c) When did Ahmad sell the stock?

(d) How much did Ahmad lose on the stock?

(e) **Writing to Learn** Explain why LaToya's strategy was better than Ahmad's for this particular stock in this particular year.

(f) Sketch a 12-month graph of a stock's performance that would favor Ahmad's strategy over LaToya's.

62. Group Activity Creating Hidden Behavior You can create your own graphs with hidden behavior. Working in groups of two or three, try this exploration.

(a) Graph the equation $y = (x + 2)(x^2 - 4x + 4)$ in the window $[-4, 4]$ by $[-10, 10]$.

(b) Confirm algebraically that this function has zeros only at $x = -2$ and $x = 2$.

(c) Graph the equation $y = (x + 2)(x^2 - 4x + 4.01)$ in the window $[-4, 4]$ by $[-10, 10]$

(d) Confirm algebraically that this function has only one zero, at $x = -2$. (Use the discriminant.)

(e) **Graph the equation** $(x + 2)(x^2 - 4x + 3.99)$ in the window $[-4, 4]$ by $[-10, 10]$.

(f) Confirm algebraically that this function has three zeros. (Use the discriminant.)

Extending the Ideas

63. The Proliferation of Cell Phones Table 1.8 shows the number of cellular phone subscribers in the U.S. and their average monthly bill in the years from 1998 to 2004.

Table 1.8 Cellular Phone Subscribers

Year	Subscribers (millions)	Average Local Monthly Bill ($)
1998	69.2	39.43
1999	86.0	41.24
2000	109.5	45.27
2001	128.4	47.37
2002	140.8	48.40
2003	158.7	49.91
2004	180.4	50.64

Source: Cellular Telecommunication & Internet Association.

(a) Graph the scatter plots of the number of subscribers and the average local monthly bill as functions of time, letting time t = the number of years after 1990.

(b) One of the scatter plots clearly suggests a linear model in the form $y = mx + b$. Use the points at $t = 8$ and $t = 14$ to find a linear model.

(c) Superimpose the graph of the linear model onto the scatter plot. Does the fit appear to be good?

(d) Why does a linear model seem inappropriate for the other scatter plot? Can you think of a function that might fit the data better?

(e) In 1995 there were 33.8 million subscribers with an average local monthly bill of $51.00. Add these points to the scatter plots.

(f) **Writing to Learn** The 1995 points do not seem to fit the models used to represent the 1998–2004 data. Give a possible explanation for this.

64. Group Activity (Continuation of Exercise 63) Discuss the economic forces suggested by the two models in Exercise 63 and speculate about the future by analyzing the graphs.

SECTION 1.2 EXERCISES

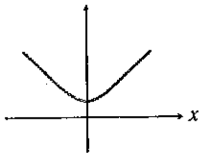
In Exercises 1–4, determine whether the formula determines y as a function of x. If not, explain why not.

1. $y = \sqrt{x - 4}$ **2.** $y = x^2 \pm 3$

3. $x = 2y^2$ **4.** $x = 12 - y$

In Exercises 5–8, use the vertical line test to determine whether the curve is the graph of a function.

5.



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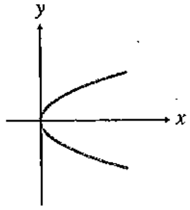

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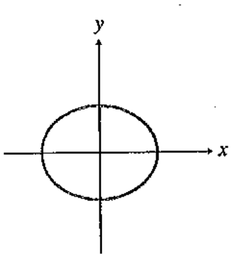

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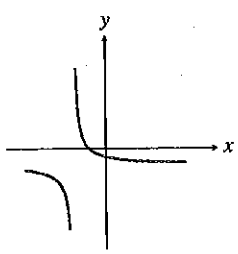


In Exercises 9–16, find the domain of the function algebraically and support your answer graphically.

9. $f(x) = x^2 + 4$ **10.** $h(x) = \dfrac{5}{x - 3}$

11. $f(x) = \dfrac{3x - 1}{(x + 3)(x - 1)}$ **12.** $f(x) = \dfrac{1}{x} + \dfrac{5}{x - 3}$

13. $g(x) = \dfrac{x}{x^2 - 5x}$ **14.** $h(x) = \dfrac{\sqrt{4 - x^2}}{x - 3}$

15. $h(x) = \dfrac{\sqrt{4 - x}}{(x + 1)(x^2 + 1)}$ **16.** $f(x) = \sqrt{x^4 - 16x^2}$

In Exercises 17–20, find the range of the function.

17. $f(x) = 10 - x^2$ **18.** $g(x) = 5 + \sqrt{4 - x}$

19. $f(x) = \dfrac{x^2}{1 - x^2}$ **20.** $g(x) = \dfrac{3 + x^2}{4 - x^2}$

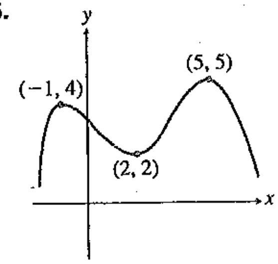
In Exercises 21–24, graph the function and tell whether or not it has a point of discontinuity at $x = 0$. If there is a discontinuity, tell whether it is removable or nonremovable.

21. $g(x) = \dfrac{3}{x}$ **22.** $h(x) = \dfrac{x^3 + x}{x}$

23. $f(x) = \dfrac{|x|}{x}$ **24.** $g(x) = \dfrac{x}{x - 2}$

In Exercises 25–28, state whether each labeled point identifies a local minimum, a local maximum, or neither. Identify intervals on which the function is decreasing and increasing.

25.



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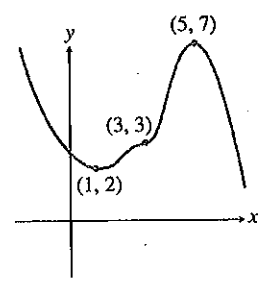

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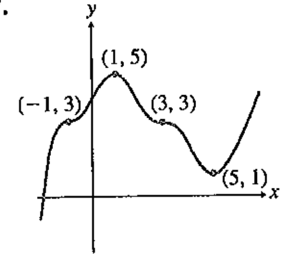

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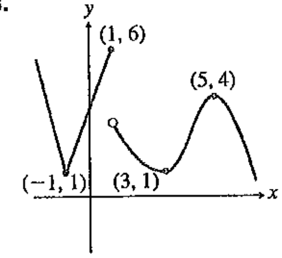


In Exercises 29–34, graph the function and identify intervals on which the function is increasing, decreasing, or constant.

29 $f(x) = |x + 2| - 1$

30. $f(x) = |x + 1| + |x - 1| - 3$

31. $g(x) = |x + 2| + |x - 1| - 2$

32. $h(x) = 0.5(x + 2)^2 - 1$

33. $g(x) = 3 - (x - 1)^2$

34. $f(x) = x^3 - x^2 - 2x$

In Exercises 35–40, determine whether the function is bounded above, bounded below, or bounded on its domain.

35. $y = 32$ **36.** $y = 2 - x^2$

37. $y = 2^x$ **38.** $y = 2^{-x}$

39. $y = \sqrt{1 - x^2}$ **40.** $y = x - x^3$

In Exercises 41–46, use a grapher to find all local maxima and minima and the values of x where they occur. Give values rounded to two decimal places.

41. $f(x) = 4 - x + x^2$ **42.** $g(x) = x^3 - 4x + 1$

43. $h(x) = -x^3 + 2x - 3$ **44.** $f(x) = (x + 3)(x - 1)^2$

45. $h(x) = x^2\sqrt{x + 4}$ **46.** $g(x) = x|2x + 5|$

In Exercises 47–54, state whether the function is odd, even, or neither. Support graphically and confirm algebraically.

47. $f(x) = 2x^4$ **48.** $g(x) = x^3$

49. $f(x) = \sqrt{x^2 + 2}$ **50.** $g(x) = \dfrac{3}{1 + x^2}$

51. $f(x) = -x^2 + 0.03x + 5$ **52.** $f(x) = x^3 + 0.04x^2 + 3$

53. $g(x) = 2x^3 - 3x$ **54.** $h(x) = \dfrac{1}{x}$

In Exercises 55–62, use a method of your choice to find all horizontal and vertical asymptotes of the function.

55. $f(x) = \dfrac{x}{x - 1}$ **56.** $q(x) = \dfrac{x - 1}{x}$

57. $g(x) = \dfrac{x + 2}{3 - x}$ **58.** $q(x) = 1.5^x$

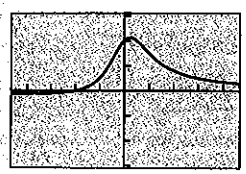
59. $f(x) = \dfrac{x^2 + 2}{x^2 - 1}$ **60.** $p(x) = \dfrac{4}{x^2 + 1}$

61. $g(x) = \dfrac{4x - 4}{x^3 - 8}$ **62.** $h(x) = \dfrac{2x - 4}{x^2 - 4}$

In Exercises 63–66, match the function with the corresponding graph by considering end behavior and asymptotes. All graphs are shown in the same viewing window.

63. $y = \dfrac{x + 2}{2x + 1}$ **64.** $y = \dfrac{x^2 + 2}{2x + 1}$

65. $y = \dfrac{x + 2}{2x^2 + 1}$ **66.** $y = \dfrac{x^3 + 2}{2x^2 + 1}$



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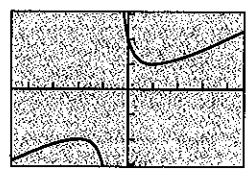

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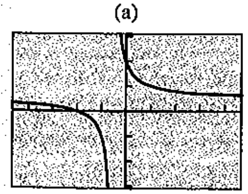

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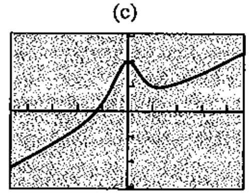

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(d)

67. Can a graph cross its own asymptote? The Greek roots of the word "asymptote" mean "not meeting," since graphs tend to approach, but not meet, their asymptotes. Which of the following functions have graphs that *do* intersect their horizontal asymptotes?

68. Can a graph have two horizontal asymptotes? Although most graphs have at most one horizontal asymptote, it is possible for a graph to have more than one. Which of the following functions have graphs with more than one horizontal asymptote?

(a) $f(x) = \dfrac{|x^3 + 1|}{8 - x^3}$ **(b)** $g(x) = \dfrac{|x - 1|}{x^2 - 4}$

(c) $h(x) = \dfrac{x}{\sqrt{x^2 - 4}}$

69. Can a graph intersect its own vertical asymptote? Graph the function $f(x) = \dfrac{x - |x|}{x^2} + 1$.

(a) The graph of this function does not intersect its vertical asymptote. Explain why it does not.

(b) Show how you can add a single point to the graph of f and get a graph that *does* intersect its vertical asymptote.

(c) Is the graph in (b) the graph of a function?

70. Writing to Learn Explain why a graph cannot have more than two horizontal asymptotes.

Standardized Test Questions

71. True or False The graph of function f is defined as the set of all points $(x, f(x))$ where x is in the domain of f. Justify your answer.

72. True or False A relation that is symmetric with respect to the x-axis cannot be a function. Justify your answer.

In Exercises 73–76, answer the question without using a calculator.

73. Multiple Choice Which function is continuous?

(A) Number of children enrolled in a particular school as a function of time

(B) Outdoor temperature as a function of time

(C) Cost of U.S. postage as a function of the weight of the letter

(D) Price of a stock as a function of time

(E) Number of soft drinks sold at a ballpark as a function of outdoor temperature

74. Multiple Choice Which function is *not* continuous?

(A) Your altitude as a function of time while flying from Reno to Dallas

(B) Time of travel from Miami to Pensacola as a function of driving speed

75. Decreasing Function Which function is decreasing?

(A) Outdoor temperature as a function of time

(B) The Dow Jones Industrial Average as a function of time

(C) Air pressure in the Earth's atmosphere as a function of altitude

(D) World population since 1900 as a function of time

(E) Water pressure in the ocean as a function of depth

76. Increasing or Decreasing Which function cannot be classified as either increasing or decreasing?

(A) Weight of a lead brick as a function of volume

(B) Height of a ball that has been tossed upward as a function of time

(C) Time of travel from Buffalo to Syracuse as a function of driving speed

(D) Area of a square as a function of side length

(E) Height of a swinging pendulum as a function of time

Explorations

77. Bounded Functions As promised in Example 7 of this section, we will give you a chance to prove algebraically that $p(x) = x/(1 + x^2)$ is bounded.

(a) Graph the function and find the smallest integer k that appears to be an upper bound.

(b) Verify that $x/(1 + x^2) < k$ by proving the equivalent inequality $kx^2 - x + k > 0$. (Use the quadratic formula to show that the quadratic has no real zeros.)

(c) From the graph, find the greatest integer k that appears to be a lower bound.

(d) Verify that $x/(1 + x^2) > k$ by proving the equivalent inequality $kx^2 - x + k < 0$.

78. Baylor School Grade Point Averages Baylor School uses a sliding scale to convert the percentage grades on its transcripts to grade point averages (GPAs). Table 1.9 shows the GPA equivalents for selected grades:

Table 1.9 Converting Grades

Grade (x)	GPA (y)
60	0.00
65	1.00
70	2.05
75	2.57
80	3.00
85	3.36
90	3.69
95	4.00
100	4.28

Source: Baylor School College Counselor.

(a) Considering GPA (y) as a function of percentage grade (x), is it increasing, decreasing, constant, or none of these?

(b) Make a table showing the *change* (Δy) in GPA as you move down the list. (See Exploration 1.)

(c) Make a table showing the change in Δy as you move down the list. (This is $\Delta\Delta y$.) Considering the *change* (Δy) in GPA as a function of percentage grade (x), is it increasing, decreasing, constant, or none of these?

(d) In general, what can you say about the shape of the graph if y is an increasing function of x and Δy is a decreasing function of x?

(e) Sketch the graph of a function y of x such that y is a decreasing function of x and Δy is an increasing function of x.

79. Group Activity Sketch (freehand) a graph of a function f with domain all real numbers that satisfies all of the following conditions:

(a) f is continuous for all x;

(b) f is increasing on $(-\infty, 0]$ and on $[3, 5]$;

(c) f is decreasing on $[0, 3]$ and on $[5, \infty)$;

(d) $f(0) = f(5) = 2$;

(e) $f(3) = 0$.

80. Group Activity Sketch (freehand) a graph of a function f with domain all real numbers that satisfies all of the following conditions:

(a) f is decreasing on $(-\infty, 0)$ and decreasing on $(0, \infty)$;

(b) f has a nonremovable point of discontinuity at $x = 0$;

(c) f has a horizontal asymptote at $y = 1$;

(d) $f(0) = 0$;

(e) f has a vertical asymptote at $x = 0$.

81. Group Activity Sketch (freehand) a graph of a function f with domain all real numbers that satisfies all of the following conditions:

(a) f is continuous for all x;

(b) f is an even function;

(c) f is increasing on $[0, 2]$ and decreasing on $[2, \infty)$;

(d) $f(2) = 3$.

82. Group Activity Get together with your classmates in groups of two or three. Sketch a graph of a function, but do not show it to the other members of your group. Using the language of functions (as in Exercises 79–81), describe your function as completely as you can. Exchange descriptions with the others in your group and see if you can reproduce each other's graphs.

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Extending the Ideas

83. A function that is bounded above has an infinite number of upper bounds, but there is always a *least upper bound,* i.e., an upper bound that is less than all the others. This least upper bound may or may not be in the range of f. For each of the following functions, find the least upper bound and tell whether or not it is in the range of the function.

(a) $f(x) = 2 - 0.8x^2$

(b) $g(x) = \dfrac{3x^2}{3 + x^2}$

(c) $h(x) = \dfrac{1 - x}{x^2}$

(d) $p(x) = 2 \sin (x)$

(e) $q(x) = \dfrac{4x}{x^2 + 2x + 1}$

84. Writing to Learn A continuous function f has domain all real numbers. If $f(-1) = 5$ and $f(1) = -5$, explain why f must have at least one zero in the interval $[-1, 1]$. (This generalizes to a property of continuous functions known as the Intermediate Value Theorem.)

85. Proving a Theorem Prove that the graph of every odd function with domain all real numbers must pass through the origin.

86. Finding the Range Graph the function $f(x) = \dfrac{3x^2 - 1}{2x^2 + 1}$ in the window $[-6, 6]$ by $[-2, 2]$.

(a) What is the apparent horizontal asymptote of the graph?

(b) Based on your graph, determine the apparent range of f.

(c) Show algebraically that $-1 \leq \dfrac{3x^2 - 1}{2x^2 + 1} < 1.5$ for all x, thus confirming your conjecture in part (b).

QUICK REVIEW 1.3 *(For help, go to Sections P.1, P.2, 3.1, and 3.3.)*

In Exercises 1–10, evaluate the expression without using a calculator.

1. $|-59.34|$
2. $|5 - \pi|$
3. $|\pi - 7|$
4. $\sqrt{(-3)^2}$
5. $\ln(1)$
6. e^0
7. $(\sqrt[3]{3})^3$
8. $\sqrt[3]{(-15)^3}$
9. $\sqrt[3]{-8^2}$
10. $|1 - \pi| - \pi$

SECTION 1.3 EXERCISES

In Exercises 1–12, each graph is a slight variation on the graph of one of the twelve basic functions described in this section. Match the graph to one of the twelve functions (a)–(l) and then support your answer by checking the graph on your calculator. (All graphs are shown in the window $[-4.7, 4.7]$ by $[-3.1, 3.1]$.)

(a) $y = -\sin x$ **(b)** $y = \cos x + 1$ **(c)** $y = e^x - 2$
(d) $y = (x + 2)^3$ **(e)** $y = x^3 + 1$ **(f)** $y = (x - 1)^2$
(g) $y = |x| - 2$ **(h)** $y = -1/x$ **(i)** $y = -x$
(j) $y = -\sqrt{x}$ **(k)** $y = \text{int}(x + 1)$ **(l)** $y = 2 - 4/(1+e^{-x})$

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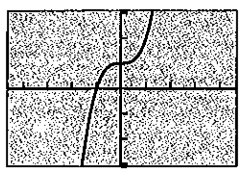

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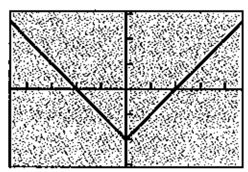

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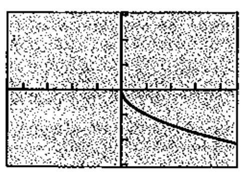

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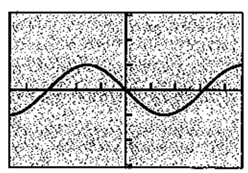

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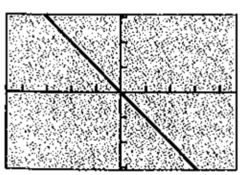

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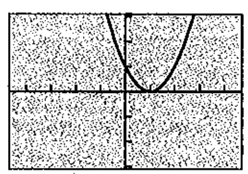

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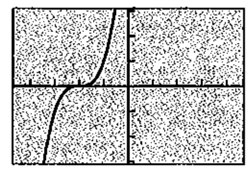

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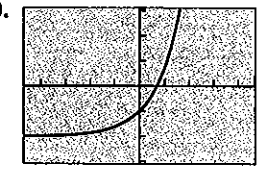

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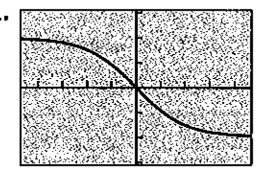

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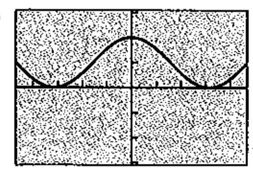

In Exercises 13–18, identify which of Exercises 1–12 display functions that fit the description given.

13. The function whose domain excludes zero.

14. The function whose domain consists of all nonnegative real numbers.

15. The two functions that have at least one point of discontinuity.

16. The function that is not a *continuous function.*

17. The six functions that are bounded below.

18. The four functions that are bounded above.

In Exercises 19–28, identify which of the twelve basic functions fit the description given.

19. The four functions that are odd.

20. The six functions that are increasing on their entire domains.

21. The three functions that are decreasing on the interval $(-\infty, 0)$.

25. The four functions that do *not* have end behavior $\lim_{x \to +\infty} f(x) = +\infty$.

26. The three functions with end behavior $\lim_{x \to -\infty} f(x) = -\infty$.

27. The four functions whose graphs look the same when turned upside-down and flipped about the y-axis.

28. The two functions whose graphs are identical except for a horizontal shift.

In Exercises 29–34, use your graphing calculator to produce a graph of the function. Then determine the domain and range of the function by looking at its graph.

29. $f(x) = x^2 - 5$ **30.** $g(x) = |x - 4|$

31. $h(x) = \ln(x + 6)$ **32.** $k(x) = 1/x + 3$

33. $s(x) = \text{int}(x/2)$ **34.** $p(x) = (x + 3)^2$

In Exercises 35–42, graph the function. Then answer the following questions:

(a) On what interval, if any, is the function increasing? Decreasing?

(b) Is the function odd, even, or neither?

(c) Give the function's extrema, if any.

(d) How does the graph relate to a graph of one of the twelve basic functions?

35. $r(x) = \sqrt{x - 10}$ **36.** $f(x) = \sin(x) + 5$

37. $f(x) = 3/(1+e^{-x})$ **38.** $q(x) = e^x + 2$

39. $h(x) = |x| - 10$ **40.** $g(x) = 4\cos(x)$

41. $s(x) = |x-2|$ **42.** $f(x) = 5 - \text{abs}(x)$

43. Find the horizontal asymptotes for the graph shown in Exercise 11.

44. Find the horizontal asymptotes for the graph of $f(x)$ in Exercise 37.

In Exercises 45–52, sketch the graph of the piecewise-defined function. (Try doing it without a calculator.) In each case, give any points of discontinuity.

45. $f(x) = \begin{cases} x & \text{if } x \le 0 \\ x^2 & \text{if } x > 0 \end{cases}$ **46.** $g(x) = \begin{cases} x^3 & \text{if } x \le 0 \\ e^x & \text{if } x > 0 \end{cases}$

47. $h(x) = \begin{cases} |x| & \text{if } x < 0 \\ \sin x & \text{if } x \ge 0 \end{cases}$ **48.** $w(x) = \begin{cases} 1/x & \text{if } x < 0 \\ \sqrt{x} & \text{if } x \ge 0 \end{cases}$

49. $f(x) = \begin{cases} \cos x & \text{if } x \le 0 \\ e^x & \text{if } x > 0 \end{cases}$ **50.** $g(x) = \begin{cases} |x| & \text{if } x < 0 \\ x^2 & \text{if } x \ge 0 \end{cases}$

51. $f(x) = \begin{cases} -3 - x & \text{if } x \le 0 \\ 1 & \text{if } 0 < x < 1 \\ x^2 & \text{if } x \ge 1 \end{cases}$

52. $f(x) = \begin{cases} x^2 & \text{if } x < -1 \\ |x| & \text{if } -1 \le x < 1 \\ \text{int}(x) & \text{if } x \ge 1 \end{cases}$

53. Writing to Learn The function $f(x) = \sqrt{x^2}$ is one of our twelve basic functions written in another form.

(a) Graph the function and identify which basic function it is.

(b) Explain algebraically why the two functions are equal.

54. Uncovering Hidden Behavior The function $g(x) = \sqrt{x^2 + 0.0001} - 0.01$ is *not* one of our twelve basic functions written in another form.

(a) Graph the function and identify which basic function it appears to be.

(b) Verify numerically that it is not the basic function that it appears to be.

55. Writing to Learn The function $f(x) = \ln(e^x)$ is one of our twelve basic functions written in another form.

(a) Graph the function and identify which basic function it is.

(b) Explain how the equivalence of the two functions in (a) shows that the natural logarithm function is *not* bounded above (even though it *appears* to be bounded above in Figure 1.42).

56. Writing to Learn Let $f(x)$ be the function that gives the cost, in cents, to mail a letter that weighs x ounces. As of June 2002, the cost is 37 cents for a letter that weighs up to one ounce, plus 23 cents for each additional ounce or portion thereof.

(a) Sketch a graph of $f(x)$.

(b) How is this function similar to the greatest integer function? How is it different?

57. Analyzing a Function Set your calculator to DOT mode and graph the greatest integer fünction, $y = \text{int}(x)$, in the window $[-4.7, 4.7]$ by $[-3.1, 3.1]$. Then complete the following analysis.

BASIC FUNCTION
The Greatest Integer Function

$f(x) = \text{int } x$

Domain:
Range:
Continuity:
Increasing/decreasing behavior:
Symmetry:
Boundedness:
Local extrema:
Horizontal asymptotes:
Vertical asymptotes:
End behavior:

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In Exerc question

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Standardized Test Questions

58. True or False The greatest integer function has an inverse function. Justify your answer.

59. True or False The logistic function has two horizontal asymptotes. Justify your answer.

In Exercises 60–63, you may use a graphing calculator to answer the question.

60. Multiple Choice Which function has range {all real numbers}?

(A) $f(x) = 4 + \ln x$

(B) $f(x) = 3 - 1/x$

(C) $f(x) = 5/(1 + e^{-x})$

(D) $f(x) = \text{int}\,(x - 2)$

(E) $f(x) = 4\cos x$

61. Multiple Choice Which function is bounded both above and below?

(A) $f(x) = x^2 - 4$

(B) $f(x) = (x - 3)^3$

(C) $f(x) = 3e^x$

(D) $f(x) = 3 + 1/(1 + e^{-x})$

(E) $f(x) = 4 - |x|$

62. Multiple Choice Which of the following is the same as the restricted-domain function $f(x) = \text{int}\,(x),\ 0 \le x < 2$?

(A) $f(x) = \begin{cases} 0 & \text{if } 0 \le x < 1 \\ 1 & \text{if } x = 1 \\ 2 & \text{if } 1 < x < 2 \end{cases}$

(B) $f(x) = \begin{cases} 0 & \text{if } x = 0 \\ 1 & \text{if } 0 < x \le 1 \\ 2 & \text{if } 1 < x < 2 \end{cases}$

(C) $f(x) = \begin{cases} 0 & \text{if } 0 \le x < 1 \\ 1 & \text{if } 1 \le x < 2 \end{cases}$

(D) $f(x) = \begin{cases} 1 & \text{if } 0 \le x < 1 \\ 2 & \text{if } 1 \le x < 2 \end{cases}$

(E) $f(x) = \begin{cases} x & \text{if } 0 \le x < 1 \\ 1 + x & \text{if } 1 \le x < 2 \end{cases}$

63. Multiple Choice Increasing Functions Which function is increasing on the interval $(-\infty, \infty)$?

(A) $f(x) = \sqrt{3 + x}$

(B) $f(x) = \text{int}\,(x)$

Explorations

64. Which is Bigger? For positive values of x, we wish to compare the values of the basic functions x^2, x, and $\sqrt{x}$.

(a) How would you order them from least to greatest?

(b) Graph the three functions in the viewing window [0, 30] by [0, 20]. Does the graph confirm your response in (a)?

(c) Now graph the three functions in the viewing window [0, 2] by [0, 1.5].

(d) Write a careful response to the question in (a) that accounts for all positive values of x.

65. Odds and Evens There are four odd functions and three even functions in the gallery of twelve basic functions. After multiplying these functions together pairwise in different combinations and exploring the graphs of the products, make a conjecture about the symmetry of:

(a) a product of two odd functions.

(b) a product of two even functions.

(c) a product of an odd function and an even function.

66. Group Activity Assign to each student in the class the name of one of the twelve basic functions, but secretly so that no student knows the "name" of another. (The same function name could be given to several students, but all the functions should be used at least once.) Let each student make a one-sentence self-introduction to the class that reveals something personal "about who I am that really identifies me." The rest of the students then write down their guess as to the function's identity. Hints should be subtle and cleverly anthropomorphic. (For example, the absolute value function saying "I have a very sharp smile" is subtle and clever, while "I am absolutely valuable" is not very subtle at all.)

67. Pepperoni Pizzas For a statistics project, a student counted the number of pepperoni slices on pizzas of various sizes at a local pizzeria, compiling the following table:

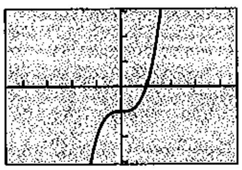
Table 1.10

Type of Pizza	Radius	Pepperoni count
Personal	4"	12
Medium	6"	27
Large	7"	37
Extra Large	8"	48

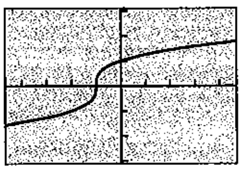
(d) Some pizza places have charts showing their kitchen staff how much of each topping should be put on each size of pizza. Do you think this pizzeria uses such a chart? Explain.

Extending the Ideas

68. Inverse Functions Two functions are said to be *inverses* of each other if the graph of one can be obtained from the graph of the other by reflecting it across the line $y = x$.
For example, the functions with the graphs shown below are inverses of each other:



[–4.7, 4.7] by [–3.1, 3.1]
(a)



[–4.7, 4.7] by [–3.1, 3.1]
(b)

(a) Two of the twelve basic functions in this section are inverses of each other. Which are they?

(b) Two of the twelve basic functions in this section are their own inverses. Which are they?

(c) If you restrict the domain of one of the twelve basic functions to $[0, \infty)$, it becomes the inverse of another one. Which are they?

69. Identifying a Function by Its Properties

(a) Seven of the twelve basic functions have the property that $f(0) = 0$. Which five do not?

(b) Only one of the twelve basic functions has the property that $f(x + y) = f(x) + f(y)$ for all x and y in its domain. Which one is it?

(c) One of the twelve basic functions has the property that $f(x + y) = f(x)f(y)$ for all x and y in its domain. Which one is it?

(d) One of the twelve basic functions has the property that $f(xy) = f(x) + f(y)$ for all x and y in its domain. Which one is it?

(e) Four of the twelve basic functions have the property that $f(x) + f(-x) = 0$ for all x in their domains. Which four are they?

QUICK REVIEW 1.4 *(For help, go to Sections P.1, 1.2, and 1.3.)*

In Exercises 1–10, find the domain of the function and express it in interval notation.

1. $f(x) = \dfrac{x-2}{x+3}$ **2.** $g(x) = \ln(x-1)$

3. $f(t) = \sqrt{5-t}$ **4.** $g(x) = \dfrac{3}{\sqrt{2x-1}}$

5. $f(x) = \sqrt{\ln(x)}$ **6.** $h(x) = \sqrt{1-x^2}$

7. $f(t) = \dfrac{t+5}{t^2+1}$ **8.** $g(t) = \ln(|t|)$

9. $f(x) = \dfrac{1}{\sqrt{1-x^2}}$ **10.** $g(x) = 2$

SECTION 1.4 EXERCISES

In Exercises 1–4, find formulas for the functions $f+g$, $f-g$, and fg. Give the domain of each.

1. $f(x) = 2x-1$; $g(x) = x^2$ **2.** $f(x) = (x-1)^2$; $g(x) = 3-x$

3. $f(x) = \sqrt{x}$; $g(x) = \sin x$ **4.** $f(x) = \sqrt{x+5}$; $g(x) = |x+3|$

In Exercises 5–8, find formulas for f/g and g/f. Give the domain of each.

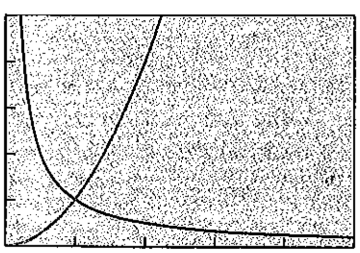
5. $f(x) = \sqrt{x+3}$; $g(x) = x^2$

6. $f(x) = \sqrt{x-2}$; $g(x) = \sqrt{x+4}$

7. $f(x) = x^2$; $g(x) = \sqrt{1-x^2}$

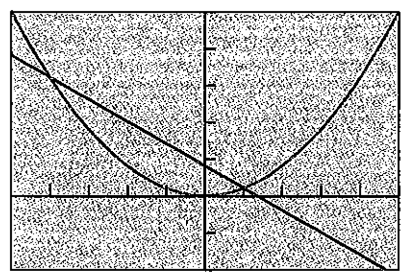
8. $f(x) = x^3$; $g(x) = \sqrt[3]{1-x^3}$

9. $f(x) = x^2$ and $g(x) = 1/x$ are shown below in the viewing window [0, 5] by [0, 5]. Sketch the graph of the sum $(f+g)(x)$ by adding the y-coordinates directly from the graphs. Then graph the sum on your calculator and see how close you came.



[0, 5] by [0, 5]

10. The graphs of $f(x) = x^2$ and $g(x) = 4-3x$ are shown in the viewing window [−5, 5] by [−10, 25]. Sketch the graph of the difference $(f-g)(x)$ by subtracting the y-coordinates directly from the graphs. Then graph the difference on your calculator and see how close you came.



[−5, 5] by [−10, 25]

In Exercises 11–14, find $(f \circ g)(3)$ and $(g \circ f)(-2)$

11. $f(x) = 2x-3$; $g(x) = x+1$

12. $f(x) = x^2-1$; $g(x) = 2x-3$

13. $f(x) = x^2+4$; $g(x) = \sqrt{x+1}$

14. $f(x) = \dfrac{x}{x+1}$; $g(x) = 9-x^2$

In Exercises 15–22, find $f(g(x))$ and $g(f(x))$. State the domain of each.

15. $f(x) = 3x+2$; $g(x) = x-1$

16. $f(x) = x^2-1$; $g(x) = \dfrac{1}{x-1}$

17. $f(x) = x^2-2$; $g(x) = \sqrt{x+1}$

18. $f(x) = \dfrac{1}{x-1}$; $g(x) = \sqrt{x}$

19. $f(x) = x^2$; $g(x) = \sqrt{1-x^2}$

20. $f(x) = x^3$; $g(x) = \sqrt[3]{1-x^3}$

21. $f(x) = \dfrac{1}{2x}$; $g(x) = \dfrac{1}{3x}$

22. $f(x) = \dfrac{1}{x+1}$; $g(x) = \dfrac{1}{x-1}$

In Exercises 23–30, find $f(x)$ and $g(x)$ so that the function can be described as $y = f(g(x))$. (There may be more than one possible decomposition.)

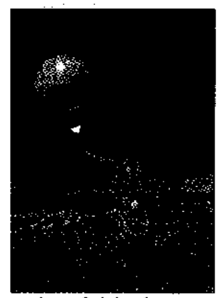
23. $y = \sqrt{x^2 - 5x}$ **24.** $y = (x^3 + 1)^2$

25. $y = |3x - 2|$ **26.** $y = \dfrac{1}{x^3 - 5x + 3}$

27. $y = (x - 3)^5 + 2$ **28.** $y = e^{\sin x}$

29. $y = \cos(\sqrt{x})$ **30.** $y = (\tan x)^2 + 1$

31. Weather Balloons A high-altitude spherical weather balloon expands as it rises due to the drop in atmospheric pressure. Suppose that the radius r increases at the rate of 0.03 inches per second and that $r = 48$ inches at time $t = 0$. Determine an equation that models the volume V of the balloon at time t and find the volume when $t = 300$ seconds.



32. A Snowball's Chance Jake stores a small cache of 4-inch diameter snowballs in the basement freezer, unaware that the freezer's self-defrosting feature will cause each snowball to lose about 1 cubic inch of volume every 40 days. He remembers them a year later (call it 360 days) and goes to retrieve them. What is their diameter then?

33. Satellite Photography A satellite camera takes a rectangular-shaped picture. The smallest region that can be photographed is a 5-km by 7-km rectangle. As the camera zooms out, the length l and width w of the rectangle increase at a rate of 2 km/sec. How long does it take for the area A to be at least 5 times its original size?

34. Computer Imaging New Age Special Effects, Inc., prepares computer software based on specifications prepared by film directors. To simulate an approaching vehicle, they begin with a computer image of a 5-cm by 7-cm by 3-cm box. The program increases each dimension at a rate of 2 cm/sec. How long does it take for the volume V of the box to be at least 5 times its initial size?

35. Which of the ordered pairs (1, 1), (4, −2), and (3, −1) are in the relation given by $3x + 4y = 5$?

36. Which of the ordered pairs (5, 1), (3, 4), and (0, −5) are in the relation given by $x^2 + y^2 = 25$?

In Exercises 37–44, find two functions defined implicitly by the given relation.

37. $x^2 + y^2 = 25$ **38.** $x + y^2 = 25$

39. $x^2 - y^2 = 25$ **40.** $3x^2 - y^2 = 25$

Standardized Test Questions

45. True or False The domain of the quotient function $(f/g)(x)$ consists of all numbers that belong to both the domain of f and the domain of g. Justify your answer.

46. True or False The domain of the product function $(fg)(x)$ consists of all numbers that belong to either the domain of f or the domain of g. Justify your answer.

You may use a graphing calculator when solving Exercises 47–50.

47. Multiple Choice Suppose f and g are functions with domain all real numbers. Which of the following statements is *not* necessarily true?

(A) $(f + g)(x) = (g + f)(x)$ **(B)** $(fg)(x) = (gf)(x)$

(C) $f(g(x)) = g(f(x))$ **(D)** $(f - g)(x) = -(g - f)(x)$

(E) $(f \circ g)(x) = f(g(x))$

48. Multiple Choice If $f(x) = x - 7$ and $g(x) = \sqrt{4 - x}$, what is the domain of the function f/g?

(A) $(-\infty, 4)$ **(B)** $(-\infty, 4]$ **(C)** $(4, \infty)$

(D) $[4, \infty)$ **(E)** $(4, 7)\cup(7, \infty)$

49. Multiple Choice If $f(x) = x^2 + 1$, then $(f \circ f)(x) =$

(A) $2x^2 + 2$ **(B)** $2x^2 + 1$ **(C)** $x^4 + 1$

(D) $x^4 + 2x^2 + 1$ **(E)** $x^4 + 2x^2 + 2$

50. Multiple Choice Which of the following relations defines the function $y = |x|$ implicitly?

(A) $y = x$ **(B)** $y^2 = x^2$ **(C)** $y^3 = x^3$

(D) $x^2 + y^2 = 1$ **(E)** $x = |y|$

Explorations

51. Three on a Match Match each function f with a function g and a domain D so that $(f \circ g)(x) = x^2$ with domain D.

f	g	D
e^x	$\sqrt{2 - x}$	$x \neq 0$
$(x^2 + 2)^2$	$x + 1$	$x \neq 1$
$(x^2 - 2)^2$	$2 \ln x$	$(0, \infty)$
$\dfrac{1}{(x - 1)^2}$	$\dfrac{1}{x - 1}$	$[2, \infty)$
$x^2 - 2x + 1$	$\sqrt{x - 2}$	$(-\infty, 2]$
$\left(\dfrac{x + 1}{[illegible]}\right)^2$	$\dfrac{x + 1}{[illegible]}$	$(-\infty, \infty)$

52. Be a g Whiz Let $f(x) = x^2 + 1$. Find a function g so that

(a) $(fg)(x) = x^4 - 1$

(b) $(f + g)(x) = 3x^2$

(c) $(f/g)(x) = 1$

(d) $f(g(x)) = 9x^4 + 1$

(e) $g(f(x)) = 9x^4 + 1$

Extending the Ideas

53. Identifying Identities An *identity* for a function operation is a function that combines with a given function f to return the same function f. Find the identity functions for the following operations:

(a) Function addition. That is, find a function g such that $(f + g)(x)) = (g + f)(x) = f(x)$.

(b) Function multiplication. That is, find a function g such that $(fg)(x) = (gf)(x) = f(x)$.

(c) Function composition. That is, find a function g such that $(f \circ g)(x) = (g \circ f)(x) = f(x)$.

54. Is Function Composition Associative? You already know that function composition is not commutative; that is, $(f \circ g)(x) \neq (g \circ f)(x)$. But is function composition associative? That is, does $(f \circ (g \circ h))(x) = ((f \circ g) \circ h))(x)$? Explain your answer.

55. Revisiting Example 6 Solve $x^2y + y^2 = 5$ for y using the quadratic formula and graph the pair of implicit functions.

QUICK REVIEW 1.5 (*For help, go to Section P.3 and P.4.*)

In Exercises 1–10, solve the equation for y.

1. $x = 3y - 6$

2. $x = 0.5y + 1$

3. $x = y^2 + 4$

4. $x = y^2 - 6$

5. $x = \dfrac{y-2}{y+3}$

6. $x = \dfrac{3y-1}{y+2}$

7. $x = \dfrac{2y+1}{y-4}$

8. $x = \dfrac{4y+3}{3y-1}$

9. $x = \sqrt{y+3},\ y \ge -3$

10. $x = \sqrt{y-2},\ y \ge 2$

SECTION 1.5 EXERCISES

In Exercises 1–4, find the (x, y) pair for the value of the parameter.

1. $x = 3t$ and $y = t^2 + 5$ for $t = 2$

2. $x = 5t - 7$ and $y = 17 - 3t$ for $t = -2$

3. $x = t^3 - 4t$ and $y = \sqrt{t+1}$ for $t = 3$

4. $x = |t + 3|$ and $y = 1/t$ for $t = -8$

In Exercises 5–8, complete the following. (a) Find the points determined by $t = -3, -2, -1, 0, 1, 2,$ and 3. (b) Find a direct algebraic relationship between x and y and determine whether the parametric equations determine y as a function of x. (c) Graph the relationship in the xy-plane.

5. $x = 2t$ and $y = 3t - 1$

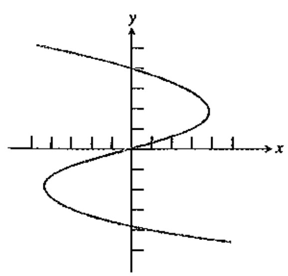
6. $x = t + 1$ and $y = t^2 - 2t$

7. $x = t^2$ and $y = t - 2$

8. $x = \sqrt{t}$ and $y = 2t - 5$

In Exercises 9–12, the graph of a relation is shown. **(a)** Is the relation a function? **(b)** Does the relation have an inverse that is a function?

9.



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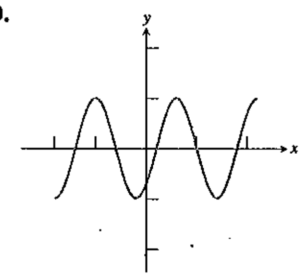

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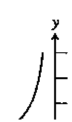

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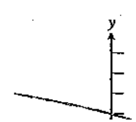


In Exercises 13–22, find a formula for $f^{-1}(x)$. Give the domain of f^{-1}, including any restrictions "inherited" from f.

13. $f(x) = 3x - 6$

14. $f(x) = 2x + 5$

15. $f(x) = \dfrac{2x-3}{x+1}$

16. $f(x) = \dfrac{x+3}{x-2}$

17. $f(x) = \sqrt{x-3}$

18. $f(x) = \sqrt{x+2}$

19. $f(x) = x^3$

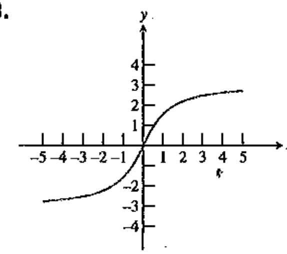
20. $f(x) = x^3 + 5$

21. $f(x) = \sqrt[3]{x+5}$

22. $f(x) = \sqrt[3]{x-2}$

In Exercises 23–26, determine whether the function is one-to-one. If it is one-to-one, sketch the graph of the inverse.

23.



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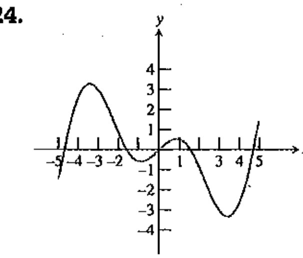

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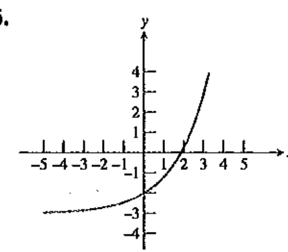

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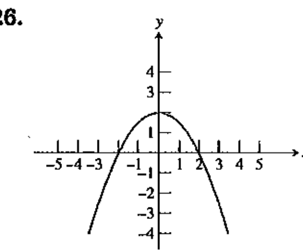

In Exercises 27–32, confirm that f and g are inverses by showing that $f(g(x)) = x$ and $g(f(x)) = x$.

27. $f(x) = 3x - 2$ and $g(x) = \dfrac{x+2}{3}$

28. $f(x) = \dfrac{x+3}{4}$ and $g(x) = 4x - 3$

29. $f(x) = x^3 + 1$ and $g(x) = \sqrt[3]{x-1}$

30. $f(x) = \dfrac{7}{x}$ and $g(x) = \dfrac{7}{x}$

31. $f(x) = \dfrac{x+1}{x}$ and $g(x) = \dfrac{1}{x-1}$

32. $f(x) = \dfrac{x+3}{x-2}$ and $g(x) = \dfrac{2x+3}{x-1}$

33. Currency Conversion In May of 2002 the exchange rate for converting U.S. dollars (x) to euros (y) was $y = 1.08x$.

(a) How many euros could you get for $100 U.S.?

(b) What is the inverse function, and what conversion does it represent?

(c) In the spring of 2002, a tourist had an elegant lunch in Provence, France ordering from a "fixed price" 48-euro menu. How much was that in U.S. dollars?

34. Temperature Conversion The formula for converting Celsius temperature (x) to Kelvin temperature is $k(x) = x + 273.16$. The formula for converting Fahrenheit temperature (x) to Celsius temperature is $c(x) = (5/9)(x - 32)$.

(a) Find a formula for $c^{-1}(x)$. What is this formula used for?

(b) Find $(k \circ c)(x)$. What is this formula used for?

35. Which pairs of basic functions (Section 1.3) are inverses of each other?

36. Which basic functions (Section 1.3) are their own inverses?

37. Which basic function can be defined parametrically as follows?

$$x = t^3 \text{ and } y = \sqrt{t^6} \text{ for } -\infty < t < \infty$$

38. Which basic function can be defined parametrically as follows?

$$x = 8t^3 \text{ and } y = (2t)^3 \text{ for } -\infty < t < \infty$$

Standardized Test Questions

39. True or False If f is a one-to-one function with domain D and range R, then f^{-1} is a one-to-one function with domain R and range D. Justify your answer.

40. True or False The set of points $(t + 1, 2t + 3)$ for all real numbers t form a line with slope 2. Justify your answer.

41. Multiple Choice Which ordered pair is in the *inverse* of the relation given by $x^2y + 5y = 9$?

(A) (2, 1) **(B)** (−2, 1) **(C)** (−1, 2) **(D)** (2, −1)
(E) (1, −2)

42. Multiple Choice Which ordered pair is not in the *inverse* of the relation given by $xy^2 - 3x = 12$?

(A) (0, −4) **(B)** (4, 1) **(C)** (3, 2) **(D)** (2, 12)
(E) (1, −6)

43. Multiple Choice Which function is the *inverse* of the function $f(x) = 3x - 2$?

(A) $g(x) = \dfrac{x}{3} + 2$

(B) $g(x) = 2 - 3x$

(C) $g(x) = \dfrac{x+2}{3}$

(D) $g(x) = \dfrac{x-3}{2}$

(E) $g(x) = \dfrac{x-2}{3}$

44. Multiple Choice Which function is the *inverse* of the function $f(x) = x^3 + 1$?

(A) $g(x) = \sqrt[3]{x-1}$

(B) $g(x) = \sqrt[3]{x} - 1$

(C) $g(x) = x^3 - 1$

(D) $g(x) = \sqrt[3]{x+1}$

(E) $g(x) = 1 - x^3$

Explorations

45. Function Properties Inherited by Inverses There are some properties of functions that are automatically shared by inverse functions (when they exist) and some that are not. Suppose that f has an inverse function f^{-1}. Give an algebraic or graphical argument (not a rigorous formal proof) to show that each of these properties of f must necessarily be shared by f^{-1}.

(a) f is continuous.

(b) f is one-to-one.

(c) f is odd (graphically, symmetric with respect to the origin).

(d) f is increasing.

46. Function Properties Not Inherited by Inverses There are some properties of functions that are not necessarily shared by inverse functions, even if the inverses exist. Suppose that f has an inverse function f^{-1}. For each of the following properties, give an example to show that f can have the property while f^{-1} does not.

(a) f has a graph with a horizontal asymptote.

(b) f has domain all real numbers.

(c) f has a graph that is bounded above.

(d) f has a removable discontinuity at $x = 5$.

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47. Scaling Algebra Grades A teacher gives a challenging algebra test to her class. The lowest score is 52, which she decides to scale to 70. The highest score is 88, which she decides to scale to 97.

(a) Using the points (52, 70) and (88, 97), find a linear equation that can be used to convert raw scores to scaled grades.

(b) Find the inverse of the function defined by this linear equation. What does the inverse function do?

48. Writing to Learn (Continuation of Exercise 47) Explain why it is important for fairness that the scaling function used by the teacher be an *increasing* function. (Caution: It is *not* because "everyone's grade must go up." What would the scaling function in Exercise 47 do for a student who does enough "extra credit" problems to get a raw score of 136?)

Extending the Ideas

49. Modeling a Fly Ball Parametrically A baseball that leaves the bat at an angle of 60° from horizontal traveling 110 feet per second follows a path that can be modeled by the following pair of parametric equations. (You might enjoy verifying this if you have studied motion in physics.)

$x = 110(t)\cos(60°)$

$y = 110(t)\sin(60°) - 16t^2$

You can simulate the flight of the ball on a grapher. Set your grapher to parametric mode and put the functions above in for X2T and Y2T. Set X1T = 325 and Y1T = 5T to draw a 30-foot fence 325 feet from home plate. Set Tmin = 0, Tmax = 6, Tstep = 0.1, Xmin = 0, Xmax = 350, Xscl = 0, Ymin = 0, Ymax = 300, and Yscl = 0.

(a) Now graph the function. Does the fly ball clear the fence?

(b) Change the angle to 30° and run the simulation again. Does the ball clear the fence?

(c) What angle is optimal for hitting the ball? Does it clear the fence when hit at that angle?

50. The Baylor GPA Scale Revisited *(See Problem 78 in Section 1.2.)* The function used to convert Baylor School percentage grades to GPAs on a 4-point scale is

$$y = \left(\frac{3^{1.7}}{30}(x - 65)\right)^{\frac{1}{1.7}} + 1.$$

The function has domain [65, 100]. Anything below 65 is a failure and automatically converts to a GPA of 0.

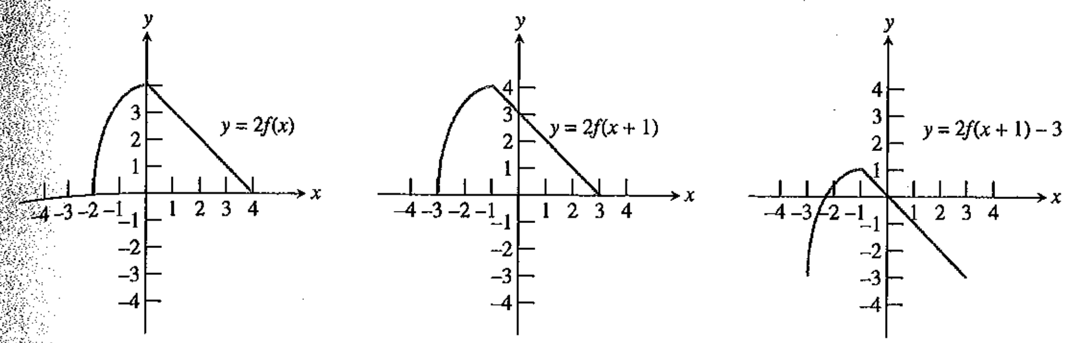
(a) Find the inverse function algebraically. What can the inverse function be used for?

(b) Does the inverse function have any domain restrictions?

(c) Verify with a graphing calculator that the function found in (a) and the given function are really inverses.

51. Group Activity (Continuation of Exercise 50) The number 1.7 that appears in two places in the GPA scaling formula is called the scaling factor (k). The value of k can be changed to alter the curvature of the graph while keeping the points (65, 1) and (95, 4) fixed. It was felt that the lowest D (65) needed to be scaled to 1.0, while the middle A (95) needed to be scaled to 4.0. The faculty's Academic Council considered several values of k before settling on 1.7 as the number that gives the "fairest" GPAs for the other percentage grades.

Try changing k to other values between 1 and 2. What kind of scaling curve do you get when $k = 1$? Do you agree with the Baylor decision that $k = 1.7$ gives the fairest GPAs?



Vertical stretch of factor 2

(a)

Horizontal translation left 1 unit

(b)

Vertical translation down 3 units

(c)

FIGURE 1.79 Transforming the graph of $y = f(x)$ in Figure 1.78 to get the graph of $y = 2f(x + 1) - 3$. (Example 7)

QUICK REVIEW 1.6 *(For help, go to Section A.2.)*

In Exercises 1–6, write the expression as a binomial squared.

1. $x^2 + 2x + 1$ **2.** $x^2 - 6x + 9$

3. $x^2 + 12x + 36$ **4.** $4x^2 + 4x + 1$

5. $x^2 - 5x + \frac{25}{4}$ **6.** $4x^2 - 20x + 25$

In Exercises 7–10, perform the indicated operations and simplify.

7. $(x - 2)^2 + 3(x - 2) + 4$

8. $2(x + 3)^2 - 5(x + 3) - 2$

9. $(x - 1)^3 + 3(x - 1)^2 - 3(x - 1)$

10. $2(x + 1)^3 - 6(x + 1)^2 + 6(x + 1) - 2$

SECTION 1.6 EXERCISES

In Exercises 1–8, describe how the graph of $y = x^2$ can be transformed to the graph of the given equation.

1. $y = x^2 - 3$ **2.** $y = x^2 + 5.2$

3. $y = (x + 4)^2$ **4.** $y = (x - 3)^2$

5. $y = (100 - x)^2$ **6.** $y = x^2 - 100$

7. $y = (x - 1)^2 + 3$ **8.** $y = (x + 50)^2 - 279$

In Exercises 9–12, describe how the graph of $y = \sqrt{x}$ can be transformed to the graph of the given equation.

9. $y = -\sqrt{x}$ **10.** $y = \sqrt{x - 5}$

11. $y = \sqrt{-x}$ **12.** $y = \sqrt{3 - x}$

In Exercises 17–20, describe how to transform the graph of f into the graph of g.

17. $f(x) = \sqrt{x + 2}$ and $g(x) = \sqrt{x - 4}$

18. $f(x) = (x - 1)^2$ and $g(x) = -(x + 3)^2$

19. $f(x) = (x - 2)^3$ and $g(x) = -(x + 2)^3$

20. $f(x) = |2x|$ and $g(x) = 4|x|$

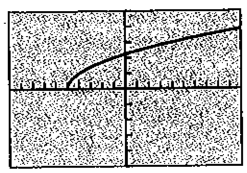
In Exercises 21–24, sketch the graphs of f, g, and h by hand. Support your answers with a grapher.

21. $f(x) = (x + 2)^2$
$g(x) = 3x^2 - 2$

22. $f(x) = x^3 - 2$
$g(x) = (x + 4)^3 - 1$

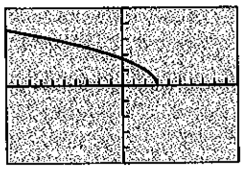
In Exercises 25–28, the graph is that of a function $y = f(x)$ that can be obtained by transforming the graph of $y = \sqrt{x}$. Write a formula for the function f.

25.



$[-10, 10]$ by $[-5, 5]$

26.



$[-10, 10]$ by $[-5, 5]$

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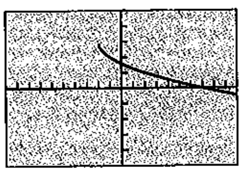

$[-10, 10]$ by $[-5, 5]$

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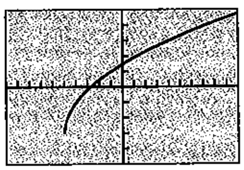


$[-10, 10]$ by $[-5, 5]$
Vertical stretch = 2

In Exercises 29–32, find the equation of the reflection of f across **(a)** the x-axis and **(b)** the y-axis.

29. $f(x) = x^3 - 5x^2 - 3x + 2$ **30.** $f(x) = 2\sqrt{x+3} - 4$

31. $f(x) = \sqrt[3]{8x}$ **32.** $f(x) = 3|x + 5|$

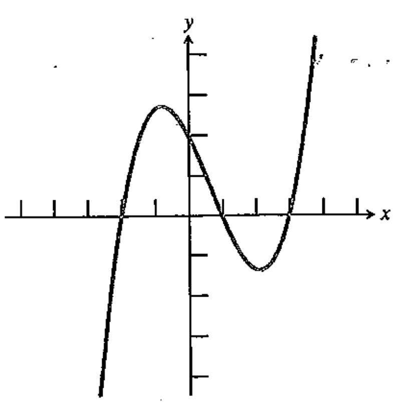
33. Reflecting Odd Functions Prove that the graph of an odd function is the same when reflected across the x-axis as it is when reflected across the y-axis.

34. Reflecting Odd Functions Prove that if an odd function is reflected about the y-axis and then reflected again about the x-axis, the result is the original function.

Exercises 35–38 refer to the graph of $y = f(x)$ shown below. In each case, sketch a graph of the new function.

35. $y = |f(x)|$ **36.** $y = f(|x|)$

37. $y = -f(|x|)$ **38.** $y = |f(|x|)|$



In Exercises 39–42, transform the given function by **(a)** a vertical stretch by a factor of 2, and **(b)** a horizontal shrink by a factor of 1/3.

39. $f(x) = x^3 - 4x$ **40.** $f(x) = |x + 2|$

41. $f(x) = x^2 + x - 2$ **42.** $f(x) = \dfrac{1}{x+2}$

In Exercises 43–46, describe a basic graph and a sequence of transformations that can be used to produce a graph of the given function.

43. $y = 2(x - 3)^2 - 4$ **44.** $y = -3\sqrt{x+1}$

45. $y = (3x)^2 - 4$ **46.** $y = -2|x + 4| + 1$

In Exercises 47–50, a graph G is obtained from a graph of y by the sequence of transformations indicated. Write an equation whose graph is G.

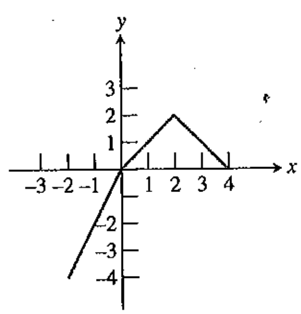
47. $y = x^2$: a vertical stretch by a factor of 3, then a shift right 4 units.

48. $y = x^2$: a shift right 4 units, then a vertical stretch by a factor of 3.

49. $y = |x|$: a shift left 2 units, then a vertical stretch by a factor of 2, and finally a shift down 4 units.

50. $y = |x|$: a shift left 2 units, then a horizontal shrink by a factor of 1/2, and finally a shift down 4 units.

Exercises 51–54 refer to the function f whose graph is shown below.



51. Sketch the graph of $y = 2 + 3f(x + 1)$.

52. Sketch the graph of $y = -f(x + 1) + 1$.

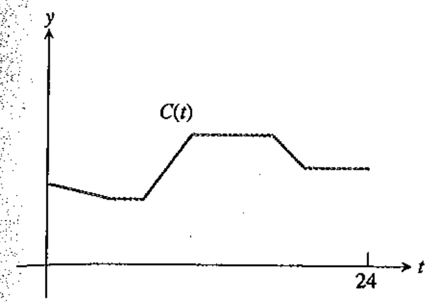
53. Sketch the graph of $y = f(2x)$.

54. Sketch the graph of $y = 2f(x - 1) + 2$.

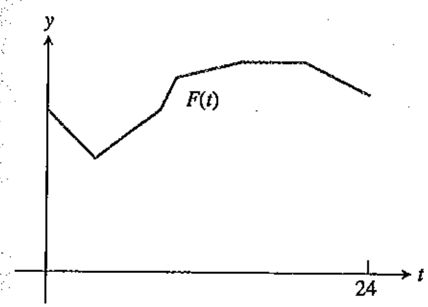
55. Writing to Learn Graph some examples to convince yourself that a reflection and a translation can have a different effect when combined in one order than when combined in the opposite order. Then explain in your own words why this can happen.

56. Writing to Learn Graph some examples to convince yourself that vertical stretches and shrinks do not affect a graph's x-intercepts. Then explain in your own words why this is so.

57. Celsius vs. Fahrenheit The graph shows the temperature in degrees Celsius in Windsor, Ontario, for one 24-hour period. Describe the transformations that convert this graph to one showing degrees Fahrenheit. [*Hint:* $F(t) = (9/5)C(t) + 32$.]



58. Fahrenheit vs. Celsius The graph shows the temperature in degrees Fahrenheit in Mt. Clemens, Michigan, for one 24-hour period. Describe the transformations that convert this graph to one showing degrees Celsius.
[*Hint:* $F(t) = (9/5)C(t) + 32$.]



Standardized Test Questions

59. True or False The function $y = f(x + 3)$ represents a translation to the right by 3 units of the graph of $y = f(x)$. Justify your answer.

60. True or False The function $y = f(x) - 4$ represents a translation down 4 units of the graph of $y = f(x)$. Justify your answer.

In Exercises 61–64, you may use a graphing calculator to answer the question.

61. Multiple Choice Given a function f, which of the following represents a vertical stretch by a factor of 3?

(A) $y = f(3x)$ **(B)** $y = f(x/3)$

(C) $y = 3f(x)$ **(D)** $y = f(x)/3$

(E) $y = f(x) + 3$

63. Multiple Choice Given a function f, which of the following represents a vertical translation of 2 units upward, followed by a reflection across the y-axis?

(A) $y = f(-x) + 2$ **(B)** $y = 2 - f(x)$

(C) $y = f(2 - x)$ **(D)** $y = -f(x - 2)$

(E) $y = f(x) - 2$

64. Multiple Choice Given a function f, which of the following represents reflection across the x-axis, followed by a horizontal shrink by a factor of 1/2?

(A) $y = -2f(x)$ **(B)** $y = -f(x)/2$

(C) $y = f(-2x)$ **(D)** $y = -f(x/2)$

(E) $y = -f(2x)$

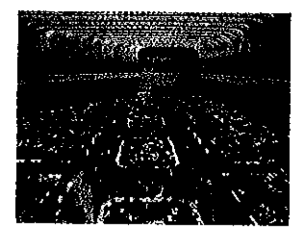
Explorations

65. International Finance Table 1.11 shows the price of a share of stock in Dell Computer for the first eight months of 2004:

Table 1.11 Dell Computer

Month	Price ($)
1	33.44
2	32.65
3	33.62
4	34.78
5	35.24
6	35.82
7	35.47
8	34.84

Source: Yahoo! Finance



(a) Graph price (y) as a function of month (x) as a line graph, connecting the points to make a continuous graph.

(b) Explain what transformation you would apply to this graph to produce a graph showing the price of the stock in Japanese yen.

66. Group Activity Get with a friend and graph the function $y = x^2$ on both your graphers. Apply a horizontal or vertical stretch or shrink to the function on one of the graphers. Then change the *window* of that grapher to make the two graphs look the same.

Extending the Ideas

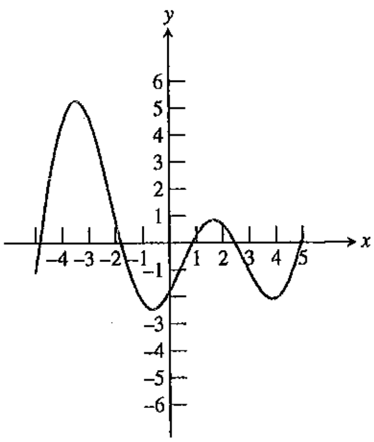
67. The Absolute Value Transformation Graph the function $f(x) = x^4 - 5x^3 + 4x^2 + 3x + 2$ in the viewing window $[-5, 5]$ by $[-10, 10]$. (Put the equation in Y1.)

(a) Study the graph and try to predict what the graph of $y = |f(x)|$ will look like. Then turn Y1 off and graph Y2 = abs (Y1). Did you predict correctly?

(b) Study the original graph again and try to predict what the graph of $y = f(|x|)$ will look like. Then turn Y1 off and graph Y2 = Y1(abs (X)). Did you predict correctly?

(c) Given the graph of $y = g(x)$ shown below, sketch a graph of $y = |g(x)|$.

(d) Given the graph of $y = g(x)$ shown below, sketch a graph of $y = g(|x|)$.



68. Parametric Circles and Ellipses Set your grapher to parametric and radian mode and your window as follows:

Tmin = 0, Tmax = 7, Tstep = 0.1

Xmin = −4.7, Xmax = 4.7, Xscl = 1

Ymin = −3.1, Ymax = 3.1, Yscl = 1

(a) Graph the parametric equations $x = \cos t$ and $y = \sin t$. You should get a circle of radius 1.

(b) Use a transformation of the parametric function of x to produce the graph of an ellipse that is 4 units wide and 2 units tall.

(c) Use a transformation of both parametric functions to produce a circle of radius 3.

(d) Use a transformation of both functions to produce an ellipse that is 8 units wide and 4 units tall.

(You will learn more about ellipses in Chapter 8.)

QUICK REVIEW 1.7 *(For help, go to Section P.3 and P.4.)*

In Exercises 1–10, solve the given formula for the given variable.

1. **Area of a Triangle** Solve for h: $A = \frac{1}{2}bh$
2. **Area of a Trapezoid** Solve for h: $A = \frac{1}{2}(b_1 + b_2)h$
3. **Volume of a Right Circular Cylinder** Solve for h: $V = \pi r^2 h$
4. **Volume of a Right Circular Cone** Solve for h: $V = \frac{1}{3}\pi r^2 h$
5. **Volume of a Sphere** Solve for r: $V = \frac{4}{3}\pi r^3$
6. **Surface Area of a Sphere** Solve for r: $A = 4\pi r^2$
7. **Surface Area of a Right Circular Cylinder** Solve for h: $A = 2\pi rh + 2\pi r^2$
8. **Simple Interest** Solve for t: $I = Prt$
9. **Compound Interest** Solve for P: $A = P\left(1 + \frac{r}{n}\right)^{nt}$
10. **Free-Fall from Height H** Solve for t: $s = H - \frac{1}{2}gt^2$

SECTION 1.7 EXERCISES

In Exercises 1–10, write a mathematical expression for the quantity described verbally:

1. Five more than three times a number x.
2. A number x increased by 5 and then tripled.
3. Seventeen percent of a number x.
4. Four more than 5% of a number x.
5. **Area of a Rectangle** The area of a rectangle whose length is 12 more than its width x.
6. **Area of a Triangle** The area of a triangle whose altitude is 2 more than its base length x.
7. **Salary Increase** A salary after a 4.5% increase, if the original salary is x dollars.
8. **Income Loss** Income after a 3% drop in the current income of x dollars.
9. **Sale Price** Sale price of an item marked x dollars, if 40% is discounted from the marked price.
10. **Including Tax** Actual cost of an item selling for x dollars if the sales tax rate is 8.75%.

In Exercises 11–14, choose a variable and write a mathematical expression for the quantity described verbally.

11. **Total Cost** The total cost is $34,500 plus $5.75 for each item produced.
12. **Total Cost** The total cost is $28,000 increased by 9% plus $19.85 for each item produced.
13. **Revenue** The revenue when each item sells for $3.75.
14. **Profit** The profit consists of a franchise fee of $200,000 plus 12% of all sales.

In Exercises 15–20, write the specified quantity as a function of the specified variable. It will help in each case to draw a picture.

15. The height of a right circular cylinder equals its diameter. Write the volume of the cylinder as a function of its radius.
16. One leg of a right triangle is twice as long as the other. Write the length of the hypotenuse as a function of the length of the shorter leg.
17. The base of an isosceles triangle is half as long as the two equal sides. Write the area of the triangle as a function of the length of the base.
18. A square is inscribed in a circle. Write the area of the square as a function of the radius of the circle.
19. A sphere is contained in a cube, tangent to all six faces. Find the surface area of the cube as a function of the radius of the sphere.
20. An isosceles triangle has its base along the x-axis with one base vertex at the origin and its vertex in the first quadrant on the graph of $y = 6 - x^2$. Write the area of the triangle as a function of the length of the base.

In Exercises 21–36, write an equation for the problem and solve the problem.

21. One positive number is 4 times another positive number. The sum of the two numbers is 620. Find the two numbers.
22. When a number is added to its double and its triple, the sum is 714. Find the three numbers.
23. **Salary Increase** Mark received a 3.5% salary increase. His salary after the raise was $36,432. What was his salary before the raise?

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24. Consumer Price Index The consumer price index for food and beverages in 2003 was 184.0 after a hefty 2.3% increase from the previous year. What was the consumer price index for food and beverages in 2002? (Source: *U.S. Bureau of Labor Statistics*)

25. Travel Time A traveler averaged 52 miles per hour on a 182-mile trip. How many hours were spent on the trip?

26. Travel Time On their 560-mile trip, the Bruins basketball team spent two more hours on the interstate highway than they did on local highways. They averaged 45 mph on local highways and 55 mph on the interstate highways. How many hours did they spend driving on local highways?

27. Sale Prices At a shirt sale, Jackson sees two shirts that he likes equally well. Which is the better bargain, and why?

28. Job Offers Ruth is weighing two job offers from the sales departments of two competing companies. One offers a base salary of $25,000 plus 5% of gross sales; the other offers a base salary of $20,000 plus 7% of gross sales. What would Ruth's gross sales total need to be to make the second job offer more attractive than the first?

29. Personal Computers From 1996 to 1997, the worldwide shipments of personal computers grew from 71,065,000 to 82,400,000. What was the percentage increase in worldwide personal computer shipments? (Source: *Dataquest*)

30. Personal Computers From 1996 to 1997, the U.S. shipments of personal computers grew from 26,650,000 to 30,989,000. What was the percentage increase in U.S. personal computer shipments? (Source: *Dataquest*)

31. Mixing Solutions How much 10% solution and how much 45% solution should be mixed together to make 100 gallons of 25% solution?

(a) Write an equation that models this problem.

(b) Solve the equation graphically.

32. Mixing Solutions The chemistry lab at the University of Hardwoods keeps two acid solutions on hand. One is 20% acid and the other is 35% acid. How much 20% acid solution and how much 35% acid solution should be used to prepare 25 liters of a 26% acid solution?

33. Maximum Value Problem A square of side x inches is cut out of each corner of a 10 in. by 18 in. piece of cardboard and the sides are folded up to form an open-topped box.

(a) Write the volume V of the box as a function of x.

(b) Find the domain of your function, taking into account the restrictions that the model imposes in x.

(c) Use your graphing calculator to determine the dimensions of the cut-out squares that will produce the box of maximum volume.

34. Residential Construction DDL Construction is building a rectangular house that is 16 feet longer than it is wide. A rain gutter is to be installed in four sections around the 136-foot perimeter of the house. What lengths should be cut for the four sections?

35. Protecting an Antenna In Example 3, suppose the parabolic dish has a 32 in. diameter and is 8 in. deep, and the radius of the cardboard cylinder is 8 in. Now how tall must the cylinder be to fit in the middle of the dish and be flush with the top of the dish?

36. Interior Design Renée's Decorating Service recommends putting a border around the top of the four walls in a dining room that is 3 feet longer than it is wide. Find the dimensions of the room if the total length of the border is 54 feet.

37. Finding the Model and Solving Water is stored in a conical tank with a faucet at the bottom. The tank has depth 24 inches and radius 9 in., and it is filled to the brim. If the faucet is opened to allow the water to flow at a rate of 5 cubic inches per second, what will the depth of the water be after 2 minutes?

38. Investment Returns Reggie invests $12,000, part at 7% annual interest and part at 8.5% annual interest. How much is invested at each rate if Reggie's total annual interest is $900?

39. Unit Conversion A tire of a moving bicycle has radius 16 in. If the tire is making 2 rotations per second, find the bicycle's speed in miles per hour.

40. Investment Returns Jackie invests $25,000, part at 5.5%

Standardized Test Questions

41. True or False A correlation coefficient gives an indication of how closely a regression line or curve fits a set of data. Justify your answer.

42. True or False Linear regression is useful for modeling the position of an object in free fall. Justify your answer.

In Exercises 43–46, tell which type of regression is likely to give the most accurate model for the scatter plot shown without using a calculator.

(A) Linear regression

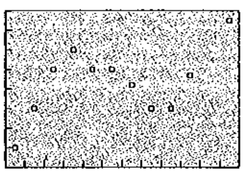
(B) Quadratic regression

(C) Cubic regression

(D) Exponential regression

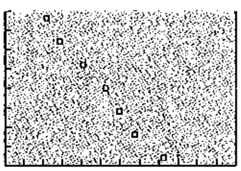
(E) Sinusoidal regression

43. Multiple Choice



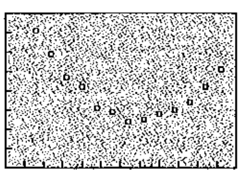
[0, 12] by [0, 8]

44. Multiple Choice



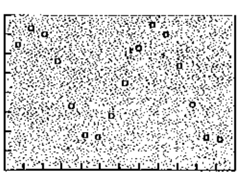
[0, 12] by [0, 8]

45. Multiple Choice



[0, 12] by [0, 8]

46. Multiple Choice



[0, 12] by [0, 8]

Exploration

47. Manufacturing The Buster Green Shoe Company determines that the annual cost C of making x pairs of one type of shoe is \$30 per pair plus \$100,000 in fixed overhead costs. Each pair of shoes that is manufactured is sold wholesale for \$50.

(a) Find an equation that models the cost of producing x pairs of shoes.

(b) Find an equation that models the revenue produced from selling x pairs of shoes.

(c) Find how many pairs of shoes must be made and sold in order to break even.

(d) Graph the equations in (a) and (b). What is the graphical interpretation of the answer in (c)?

48. Employee Benefits John's company issues employees a contract that identifies salary and the company's contributions to pension, health insurance premiums, and disability insurance. The company uses the following formulas to calculate these values.

Salary	x (dollars)
Pension	12% of salary
Health Insurance	3% of salary
Disability Insurance	0.4% of salary

If John's total contract with benefits is worth \$48,814.20, what is his salary?

49. Manufacturing Queen, Inc., a tennis racket manufacturer, determines that the annual cost C of making x rackets is \$23 per racket plus \$125,000 in fixed overhead costs. It costs the company \$8 to string a racket.

(a) Find a function $y_1 = u(x)$ that models the cost of producing x unstrung rackets.

(b) Find a function $y_2 = s(x)$ that models the cost of producing x strung rackets.

(c) Find a function $y_3 = R_u(x)$ that models the revenue generated by selling x unstrung rackets.

(d) Find a function $y_4 = R_s(x)$ that models the revenue generated by selling x rackets.

(e) Graph y_1, y_2, y_3, and y_4 simultaneously in the window [0, 10,000] by [0, 500,000].

(f) Writing to Learn Write a report to the company recommending how they should manufacture their rackets, strung or unstrung. Assume that you can include the viewing window in (e) as a graph in the report, and use it to support your recommendation.

50. Hourly Earnings of U.S. Production Workers The average hourly earnings of U.S. production workers for 1990–2003 are shown in Table 1.13.

Table 1.13 Average Hourly Earnings

Year	Average Hourly Earnings
1990	10.19
1991	10.50
1992	10.76
1993	11.03
1994	11.32
1995	11.64
1996	12.03
1997	12.49
1998	13.00
1999	13.47
2000	14.00
2001	14.53
2002	14.95
2003	15.35

Source: Bureau of Labor Statistics, U.S. Dept. of Labor, as reported in The World Almanac and Book of Facts 2005.

(a) Produce a scatter plot of the hourly earnings (y) as a function of years since 1990 (x).

(b) Find the linear regression equation. Round the coefficients to the nearest 0.001.

(c) Does the value of r suggest that the linear model is appropriate?

(d) Find the quadratic regression equation. (Round the coefficients to the nearest 0.01.)

(e) Does the value of R^2 suggest that a quadratic model is appropriate?

(f) Use both curves to predict the housing CPI for the year 2010. How different are the estimates?

(g) Writing to Learn Use the results of parts (a)–(f) to explain why it is risky to predict y-values for x-values that are not very close to the data points, even when the regression plots fit the data points very well.

Extending the Ideas

51. Newton's Law of Cooling A 190° cup of coffee is placed on a desk in a 72° room. According to Newton's Law of Cooling, the temperature T of the coffee after t minutes will be $T = (190 - 72)b^t + 72$, where b is a constant that depends on

Table 1.14 Cooling a Cup of Coffee

Time	Temp	Time	Temp
1	184.3	11	140.0
2	178.5	12	136.1
3	173.5	13	133.5
4	168.6	14	130.5
5	164.0	15	127.9
6	159.2	16	125.0
7	155.1	17	122.8
8	151.8	18	119.9
9	147.0	19	117.2
10	143.7	20	115.2

(a) Make a scatter plot of the data, with the times in list L1 and the temperatures in list L2.

(b) Store L2 − 72 in list L3. The values in L3 should now be an exponential function ($y = a \times b^x$) of the values in L1.

(c) Find the exponential regression equation for L3 as a function of L1. How well does it fit the data?

52. Group Activity Newton's Law of Cooling If you have access to laboratory equipment (such as a CBL or CBR unit for your grapher), gather experimental data such as in Exercise 51 from a cooling cup of coffee. Proceed as follows:

(a) First, use the temperature probe to record the temperature of the room. It is a good idea to turn off fans and air conditioners that might affect the temperature of the room during the experiment. It should be a constant.

(b) Heat the coffee. It need not be boiling, but it should be at least 160°. (It also need not be coffee.)

(c) Make a new list consisting of the temperature values minus the room temperature. Make a scatter plot of this list (y) against the time values (x). It should appear to approach the x-axis as an asymptote.

(d) Find the equation of the exponential regression curve. How well does it fit the data?

(e) What is the equation predicted by Newton's Law of Cooling? (Substitute your initial coffee temperature and the temperature of your room for the 190 and 72 in the equation in Exercise 51.)

(f) Group Discussion What sort of factors would affect the value of b in Newton's Law of Cooling? Discuss your ideas with the group.

CHAPTER 1 Review Exercises

The collection of exercises marked in red could be used as a chapter test.

In Exercises 1–10, match the graph with the corresponding function (a)–(j) from the list below. Use your knowledge of function behavior, *not* your grapher.

(a) $f(x) = x^2 - 1$ **(b)** $f(x) = x^2 + 1$

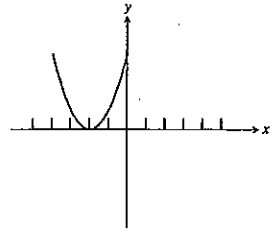
(c) $f(x) = (x - 2)^2$ **(d)** $f(x) = (x + 2)^2$

(e) $f(x) = \dfrac{x - 1}{2}$ **(f)** $f(x) = |x - 2|$

(g) $f(x) = |x + 2|$ **(h)** $f(x) = -\sin x$

(i) $f(x) = e^x - 1$ **(j)** $f(x) = 1 + \cos x$

1.



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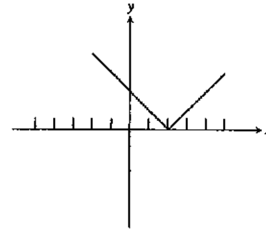

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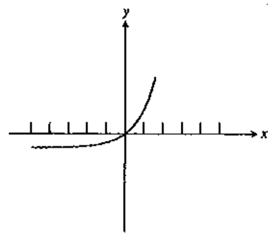

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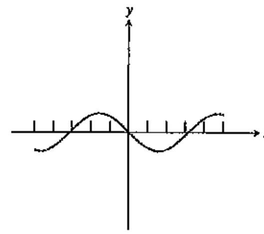

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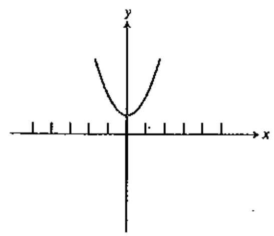

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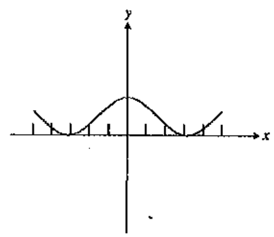

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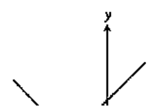

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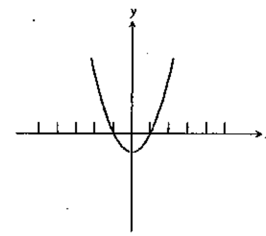

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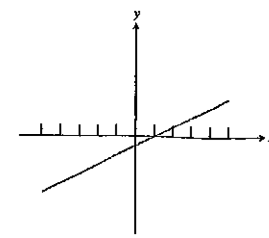


In Exercises 11–18, find **(a)** the domain and **(b)** the range of the function.

11. $g(x) = x^3$

12. $f(x) = 35x - 602$

13. $g(x) = x^2 + 2x + 1$

14. $h(x) = (x - 2)^2 + 5$

15. $g(x) = 3|x| + 8$

16. $k(x) = \sqrt{4 - x^2} - 2$

17. $f(x) = \dfrac{x}{x^2 - 2x}$

18. $k(x) = \dfrac{1}{\sqrt{9 - x^2}}$

In Exercises 19 and 20, graph the function, and state whether the function is continuous at $x = 0$. If it is discontinuous, state whether the discontinuity is removable or nonremovable.

19. $f(x) = \dfrac{x^2 - 3}{x + 2}$

20. $k(x) = \begin{cases} 2x + 3 & \text{if } x > 0 \\ 3 - x^2 & \text{if } x \le 0 \end{cases}$

In Exercises 21–24, find all **(a)** vertical asymptotes and **(b)** horizontal asymptotes of the graph of the function. Be sure to state your answers as equations of lines.

21. $y = \dfrac{5}{x^2 - 5x}$

22. $y = \dfrac{3x}{x - 4}$

23. $y = \dfrac{7x}{\sqrt{x^2 + 10}}$

24. $y = \dfrac{|x|}{x + 1}$

In Exercises 25–28, graph the function and state the intervals on which the function is *increasing*.

25. $y = \dfrac{x^3}{6}$

26. $y = 2 + |x - 1|$

27. $y = \dfrac{x}{1 - x^2}$

28. $y = \dfrac{x^2 - 1}{x^2 - 4}$

In Exercises 29–32, graph the function and tell whether the function is bounded above, bounded below, or bounded.

29. $f(x) = x + \sin x$

30. $g(x) = \dfrac{6x}{x^2 + 1}$

31. $h(x) = 5 - e^x$

32. $k(x) = 1000 + \dfrac{x}{1000}$

In Exercises 33–36, use a grapher to find all **(a)** relative maximum

In Exercises 37–40, graph the function and state whether the function is odd, even, or neither.

37. $y = 3x^2 - 4|x|$

38. $y = \sin x - x^3$

39. $y = \frac{x}{e^x}$

40. $y = x \cos (x)$

In Exercises 41–44, find a formula for $f^{-1}(x)$.

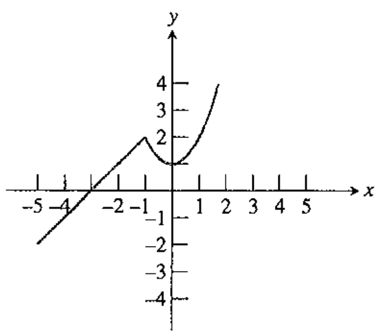
41. $f(x) = 2x + 3$

42. $f(x) = \sqrt[3]{x - 8}$

43. $f(x) = \frac{2}{x}$

44. $f(x) = \frac{6}{x + 4}$

Exercises 45–52 refer to the function $y = f(x)$ whose graph is given below.



45. Sketch the graph of $y = f(x) - 1$.

46. Sketch the graph of $y = f(x - 1)$.

47. Sketch the graph of $y = f(-x)$.

48. Sketch the graph of $y = -f(x)$.

49. Sketch a graph of the inverse relation.

50. Does the inverse relation define y as a function of x?

51. Sketch a graph of $y = f(|x|)$.

52. Define f algebraically as a piecewise function. [*Hint*: the pieces are translations of two of our "basic" functions.]

In Exercises 53–58, let $f(x) = \sqrt{x}$ and let $g(x) = x^2 - 4$.

53. Find an expression for $(f \circ g)(x)$ and give its domain.

54. Find an expression for $(g \circ f)(x)$ and give its domain.

55. Find an expression for $(fg)(x)$ and give its domain.

56. Find an expression for $\left(\frac{f}{g}\right)(x)$ and give its domain.

57. Describe the end behavior of the graph of $y = f(x)$.

58. Describe the end behavior of the graph of $y = f(g(x))$.

In Exercises 59–64, write the specified quantity as a function of the specified variable. Remember that drawing a picture will help.

59. Square Inscribed in a Circle A square of side s is inscribed in a circle. Write the area of the circle as a function of s.

60. Circle Inscribed in a Square A circle is inscribed in a square of side s. Write the area of the circle as a function of s.

61. Volume of a Cylindrical Tank A cylindrical tank with diameter 20 feet is partially filled with oil to a depth of h feet. Write the volume of oil in the tank as a function of h.

62. Draining a Cylindrical Tank A cylindrical tank with diameter 20 feet is filled with oil to a depth of 40 feet. The oil begins draining at a constant rate of 2 cubic feet per second. Write the volume of the oil remaining in the tank t seconds later as a function of t.

63. Draining a Cylindrical Tank A cylindrical tank with diameter 20 feet is filled with oil to a depth of 40 feet. The oil begins draining at a constant rate of 2 cubic feet per second. Write the depth of the oil remaining in the tank t seconds later as a function of t.

64. Draining a Cylindrical Tank A cylindrical tank with diameter 20 feet is filled with oil to a depth of 40 feet. The oil begins draining so that the depth of oil in the tank decreases at a constant rate of 2 feet per hour. Write the volume of oil remaining in the tank t hours later as a function of t.

65. U.S. Crude Oil Imports The imports of crude oil to the U.S. from Canada in the years 1995–2004 (in thousands of barrels per day) are given in Table 1.15.

Table 1.15 Crude Oil Imports from Canada

Year	Barrels/day × 1000
1995	1,040
1996	1,075
1997	1,198
1998	1,266
1999	1,178
2000	1,348
2001	1,356
2002	1,445
2003	1,549
2004	1,606

Source: Energy Information Administration, Petroleum Supply Monthly, as reported in The World Almanac and Book of Facts 2005.

(a) Sketch a scatter plot of import numbers in the right-hand column (y) as a function of years since 1990 (x).

(b) Find the equation of the regression line and superimpose it on the scatter plot.

(c) Based on the regression line, approximately how many thousands of barrels of oil would the U.S. import from Canada in 2010?

66. The winning times in the women's 100-meter freestyle event at the Summer Olympic Games since 1952 are shown in Table 1.16:

Table 1.16 Women's 100-Meter Freestyle

Year	Time	Year	Time
1952	66.8	1980	54.79
1956	62.0	1984	55.92
1960	61.2	1988	54.93
1964	59.5	1992	54.64
1968	60.0	1996	54.50
1972	58.59	2000	53.83
1976	55.65	2004	53.84

Source: The World Almanac and Book of Facts 2005.

(a) Sketch a scatter plot of the times (y) as a function of the years (x) beyond 1900. (The values of x will run from 52 to 104.)

(b) Explain why a linear model cannot be appropriate for these times over the long term.

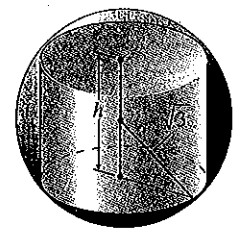
(c) The points appear to be approaching a horizontal asymptote of $y = 52$. What would this mean about the times in this Olympic event?

(d) Subtract 52 from all the times so that they will approach an asymptote of $y = 0$. Redo the scatter plot with the new y-values. Now find the *exponential* regression curve and superimpose its graph on the vertically-shifted scatter plot.

(e) According to the regression curve, what will be the winning time in the women's 100-meter freestyle event at the 2008 Olympics?

67. **Inscribing a Cylinder Inside a Sphere** A right circular cylinder of radius r and height h is inscribed inside a sphere of radius $\sqrt{3}$ inches.

(a) Use the Pythagorean Theorem to write h as a function of r.



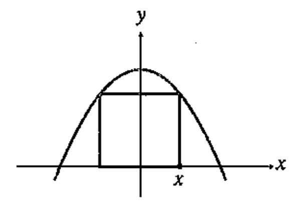
(b) Write the volume V of the cylinder as a function of r.

(c) What values of r are in the domain of V?

(d) Sketch a graph of $V(r)$ over the domain $[0, \sqrt{3}]$.

(e) Use your grapher to find the maximum volume that such a cylinder can have.

68. **Inscribing a Rectangle Under a Parabola** A rectangle is inscribed between the x-axis and the parabola $y = 36 - x^2$ with one side along the x-axis, as shown in the figure below.



(a) Let x denote the x-coordinate of the point highlighted in the figure. Write the area A of the rectangle as a function of x.

(b) What values of x are in the domain of A?

(c) Sketch a graph of $A(x)$ over the domain.

(d) Use your grapher to find the maximum area that such a rectangle can have.