$\qquad$ Date $\qquad$ Period $\qquad$
TEST Taylor Polynomials and Taylor Series

## Calculator Permitted

## Multiple Choice

_1. The Taylor series for $\sin x$ about $x=0$ is $x-\frac{x^{3}}{3!}+\frac{x^{5}}{5!}-\cdots$. If $f$ is a function such that $f^{\prime}(x)=\sin \left(x^{2}\right)$, then the coefficient of $x^{7}$ in the Taylor series for $f(x)$ about $x=0$ is
(A) $\frac{1}{7!}$
(B) $\frac{1}{7}$
(C) 0
(D) $-\frac{1}{42}$
(E) $-\frac{1}{7!}$
_2. What are all values of $x$ for which the series $\sum_{n=1}^{\infty} \frac{(x-2)^{n}}{n \cdot 3^{n}}$ converges?
(A) $-3 \leq x \leq 3$
(B) $-3<x<3$
(C) $-1<x \leq 5$
(D) $-1 \leq x \leq 5$
(E) $-1 \leq x<5$
$\qquad$ 3. Let $f$ be the following function given by $f(x)=\ln (3-x)$. The third-degree Taylor polynomial for $f$ about $x=2$ is
(A) $-(x-2)+\frac{(x-2)^{2}}{2}-\frac{(x-2)^{3}}{3}$
(B) $-(x-2)-\frac{(x-2)^{2}}{2}-\frac{(x-2)^{3}}{3}$
(C) $(x-2)+(x-2)^{2}+(x-2)^{3}$
$\begin{array}{ll}\text { (D) }(x-2)+\frac{(x-2)^{2}}{2}+\frac{(x-2)^{3}}{3} & \text { (E) }(x-2)-\frac{(x-2)^{2}}{2}+\frac{(x-2)^{3}}{3}\end{array}$
___ 4. If $f(x)=\sum_{k=1}^{\infty}\left(\sin ^{2} x\right)^{k}$, then $f(1)$ is
(A) 0.369
(B) 0.585
(C) 2.400
(D) 2.426
(E) 3.426
___ 5. A function $f$ has Maclaurin series given by $1+\frac{x^{2}}{2!}+\frac{x^{4}}{4!}+\frac{x^{6}}{6!}+\cdots+\frac{x^{2 n}}{(2 n)!}+\cdots$. Which of the following is an expression for $f(x)$ ?
(A) $\cos x$
(B) $e^{x}-\sin x$
(C) $e^{x}+\sin x$
(D) $\frac{1}{2}\left(e^{x}+e^{-x}\right)$
(E) $e^{x^{2}}$

## Free Response

(2002-BC6) The Maclaurin series for the function $f$ is given by

$$
f(x)=\sum_{n=0}^{\infty} \frac{(2 x)^{n+1}}{n+1}=2 x+\frac{4 x^{2}}{2}+\frac{8 x^{3}}{3}+\frac{16 x^{4}}{4}+\cdots+\frac{(2 x)^{n+1}}{n+1}+\cdots
$$

on its interval of convergence.
(a) Find the interval of convergence of the Maclaurin series for $f$. Justify your answer.
(b) Find the first four terms and the general term for the Maclaurin series for $f^{\prime}(x)$.
(c) Use the Maclaurin series you found in part (b) to find the value of $f^{\prime}\left(-\frac{1}{3}\right)$.

