$\qquad$ Date $\qquad$ Period $\qquad$
AP Calculus AB/BC
TEST: 3.1 to 3.8, Calculator OK
$\qquad$ 1. How many values of $c$ satisfy the Mean Value Theorem for the equation $f(x)=x \cos (\sqrt{x})$, $0 \leq x \leq 50$ ?
(A) 0
(B) 1
(C) 2
(D) 3
(E) 4
$\qquad$ 2. The function $f$ is twice differentiable with $f(2)=1, f^{\prime}(2)=4$, and $f^{\prime \prime}(2)=3$. What is the value of the approximation of $f(1.9)$ using the line tangent to the graph of $f$ at $x=2$ ?
(A) 0.4
(B) 0.6
(C) 0.7
(D) 1.3
(E) 1.4
_3. A rectangle has one side on the $x$-axis and the upper two vertices on the graph of $y=e^{-2 x^{2}}$. Give a decimal approximation for the maximum possible area for this rectangle.
(A) 1.649
(B) 1
(C) 2.031
(D) 0.545
(E) 0.606
$\qquad$ 4. Let $f$ be the function given by $f(x)=2 x e^{x}$. The graph of $f$ is concave down when
(A) $x<-2$
(B) $x>-2$
(C) $x<-1$
(D) $x>-1$
(E) $x<0$
$\qquad$ 5. The radius of a sphere is decreasing at a rate of 2 centimeters per second. At the instant when the radius of the sphere is 3 centimeters, what is the rate of change, in square centimeters per second, of the surface area of the sphere?
(A) $-108 \pi$
(B) $-72 \pi$
(C) $-48 \pi$
(D) $-24 \pi$
(E) $-16 \pi$
$\qquad$ 6. Let $f$ be the function with derivative given by $f^{\prime}(x)=\sin \left(x^{2}+1\right)$. How many relative extrema does $f$ have on the interval $2<x<4$ ?
(A) One
(B) Two
(C) Three
(D) Four
(E) Five
$\qquad$ 7. The second derivative of a function $f$ is given by $f^{\prime \prime}(x)=x(x-a)(x-b)^{2}$. The graph of $f^{\prime \prime}$ is shown at right. For what values of $x$ does the graph of $f$ have a point of inflection?
(A) 0 and $a$ only
(B) 0 and $m$ only

(C) $j$ and $b$ only
(D) $0, a$, and $b$
(E) $j, b$, and $k$
$\qquad$ 8. If $f(x)=3 x^{2}+x, x=2$, and $d x=0.002$, find $d y$.
(A) 0.02
(B) 0.026
(C) 0.028
(D) 0.014
(E) 0.26

Part II: Free Response. Do all work below the line. Label each part. Notation, Notation, Notation.
10. (1984-AB5) The volume $V$ of a cone is increasing at the rate of $28 \pi$ cubic inches per second. At the instant when the radius $r$ on the cone is 3 inches, its volume is $12 \pi$ cubic inches, and the radius is increasing at $\frac{1}{2}$ inches per second.
(a) At the instant when the radius of the cone is 3 inches, what is the rate of change of the area of the base?
(b) At the instant when the radius of the cone is 3 inches, what is the rate of change of its height $h$ ?
(c) At the instant when the radius of the cone is 3 inches, what is the instantaneous rate of change of the area of its base with respect to its height $h$ ?

## 1973 AB 6

A manufacturer finds it costs him $x^{2}+5 x+7$ dollars to produce $x$ tons of an item. At production levels above 3 tons, he must hire additional workers, and his costs increase by $3(x-3)$ dollars on his total production. If the price he receives is $\$ 13$ per ton regardless of how much he manufactures and if he has a plant capacity of 10 tons, what level of output maximizes his profits?

## 1976 AB 4

a. A point moves on the hyperbola $3 x^{2}-y^{2}=23$ so that its $y$-coordinate is increasing at a constant rate of 4 units per second. How fast is the $x$ coordinate changing when $x=4$ ?
b. For what values of $k$ will the line $2 x+9 y+k=0$ be normal to the hyperbola $3 x^{2}-y^{2}=23 ?$

## 1982 AB 4

A ladder 15 feet long is leaning against a building so that the end X is on level ground and end $Y$ is on the wall. $X$ is moved away from the building at the constant rate of $\frac{1}{2}$ foot per second.
a. Find the rate in feet per second at which the length OY is changing when $X$ is 9 feet from the building.
b. Find the rate of change in square feet per second of the area of the triangle XOY when X is 9 feet from the building.

