$\qquad$ Date $\qquad$
$\qquad$
TEST: 3.1-3.6, NO CALCULATOR
Part I: Multiple Choice: Put the letter in the letter place.
$\qquad$ 1. The graph of the derivative, $g^{\prime}(x)$, of a function $g(x)$ is shown below


Which of the following must be true about the function $g(x)$ on the interval $[-4,4]$ ?
I. $g(x)$ is increasing for $x>2$ only
II. $g(x)$ is not differentiable at four points
III. $g(x)$ is concave down for $-2<x<1$
(A) I, II, and III
(B) I only
(C) I and III only
(D) I and II only
(E) II only
$\qquad$ 2. On what open intervals is $f(x)=\frac{2 x-3}{x^{2}}$ increasing?
(A) $(3, \infty)$
(B) $(0, \infty)$
(C) $(-\infty, 3)$
(D) $(0,3)$
(E) $(-\infty,-3)$
$\qquad$ 3. If $\lim _{h \rightarrow 0} \frac{f(-3+h)-f(-3)}{h}=2.718$, then the graph of $f(x)$ at $x=-3$ is
(A) increasing
(B) concave up
(C) decreasing
(D) stationary
(E) concave down
—_4. On the interval $[0, \pi]$, the graph of $f(x)=\frac{1}{2} x+\sin x$ has a critical value at $x=$
(A) $\pi$
(B) $\frac{2 \pi}{3}$
(C) $\frac{5 \pi}{6}$
(D) 0
(E) $\frac{\pi}{3}$
$\qquad$ 5. The graph of the derivative, $f^{\prime}(x)$, of a function $f(x)$ is shown below


At what value of $x$ does $f(x)$ have a local maximum?
(A) -2
(B) -1
(C) 3
(D) 1
(E) 0
$\qquad$ 6. Selected values for the derivative, $f^{\prime}(x)$, of a differentiable function $f(x)$ are shown in the table below.

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f^{\prime}(x)$ | 8 | 4 | 0 | -4 | -8 | -12 |

If $f^{\prime}(x)$ is strictly decreasing, which of the following statements must be true?
(A) The graph of $f(x)$ is symmetric with respect to the line $x=3$
(B) $f(x)$ is concave up for for all $x$
(C) $f(x)$ changes concavity at $x=3$
(D) $f(x)$ has a relative maximum at $x=3$
(E) $f(x)$ has a relative minimum at $x=3$
7. The function $g$ is defined by the equation $g(x)=6 x^{5}-10 x^{3}$. On what open intervals is the graph of $g(x)$ concave up? HINT: $\frac{\sqrt{2}}{2} \approx 0.707$
(A) $\left(-\infty,-\frac{\sqrt{2}}{2}\right) \cup\left(0, \frac{\sqrt{2}}{2}\right)$
(B) $\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$
(C) $\left(-\frac{\sqrt{2}}{2}, 0\right) \cup\left(\frac{\sqrt{2}}{2}, \infty\right)$
(D) $\left(-\frac{\sqrt{2}}{2}, \infty\right)$
(D) $\left(-\infty, \frac{\sqrt{2}}{2}\right)$
$\qquad$ 8. The shortest distance from the curve $y=\sqrt{x}$ and the point $(4,0)$ is
(A) $\sqrt{15}$
(B) $\frac{\sqrt{14}}{2}$
(C) $\frac{\sqrt{15}}{2}$
(D) $\frac{7}{2}$
(E) $\sqrt{14}$
$\qquad$ 9. The diagram below shows a rectangle inscribed in a semicircle.


If the radius of the semicircle is 2 meters, what is the maximum area, in square meters, of the rectangle?
(A) $4 \sqrt{2}$
(B) $2 \sqrt{2}$
(C) 4
(D) 8
(E) 2
10. The graph of a function is shown below.


On the closed interval $[a, b]$, at how many points is the Mean Value Theorem satisfied?
(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

Part II: Free Response
11. Let $f$ be the function defined by $f(x)=36 x^{1 / 3}-9 x^{4 / 3}$
(a) What is the domain of $f(x)$ ?
(b) Show that $f^{\prime}(x)=\frac{-12(x-1)}{\sqrt[3]{x^{2}}}$. Show the work that leads to your answer.
(c) Find the intervals on which $f$ is decreasing.
(d) At each critical value, determine if $f(x)$ has a local maximum, a local minimum, or neither. Justify.

