Name______________________________ Date_____________________ Period____________

AB Calculus Practice Test: f, f', f''   NO CALCULATOR

Part I: Multiple Choice

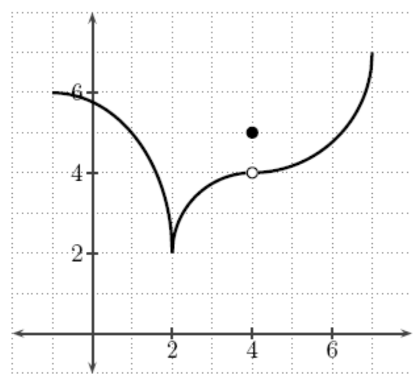
_____ 1. If $f(x) = x + \dfrac{1}{x}$, then the set of values for which *f* increases is

(A) $(-\infty, -1] \cup [1, \infty)$ (B) $[-1, 1]$ (C) $(-\infty, \infty)$ (D) $(0, \infty)$ (E) $(-\infty, 0) \cup (0, \infty)$

_____ 2. An equation of the tangent line to $y = x^3 + 3x^2 + 2$ at its point of inflection is

(A) $y = -6x - 6$ (B) $y = -3x + 1$ (C) $y = 2x + 10$ (D) $y = 3x - 1$ (E) $y = 4x + 1$

_____ 3. If *f* is the function whose graph is given at right



Which of the following properties does *f* NOT have?

(A) $\lim_{x \to 4} f(x) = 4$

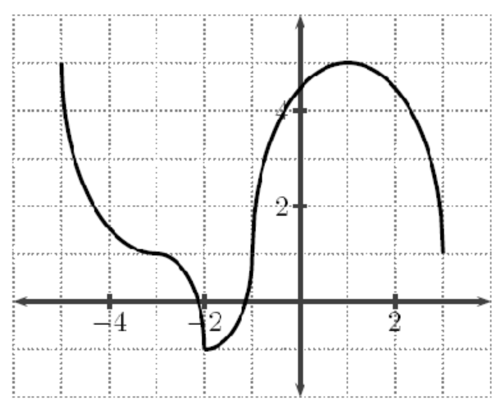
(B) $f'(x) < 0$ on $(-1, 2)$

(C) $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x)$

(D) differentiable at $x = 2$

(E) local maximum at $x = 4$

_____ 4. If *f* is a continuous function on $(-5, 3)$ whose graph is at right, which of the following properties are satisfied?



I. $f''(x) > 0$ on $(-2, 1)$

II. *f* has exactly 2 local extrema

III. *f* has exactly 4 critical points.

(A) all of them   (B) B only   (C) none of them

(D) A and C only   (E) C only   (F) B and C only

(G) A only

_____ 5. For what value of *k* will $x + \dfrac{k}{x}$ have relative maximum at $x = -2$?

(A) -4 (B) -2 (C) 2   (D) 4   (E) None of these

_____ 6. At $x = 0$, which of the following is true of the function *f* defined by $f(x) = x^2 + e^{-2x}$?

(A) *f* is increasing   (B) *f* is decreasing   (C) *f* is discontinuous

(D) *f* has a relative minimum   (E) *f* has a relative maximum

_____ 7. If a function *f* is continuous for all x and if *f* has a relative maximum at $(-1,4)$ and a relative minimum at $(3,-2)$, which of the following statements must be true?

(A) The graph of *f* has a point of inflection somewhere between $x=-1$ and $x=3$

(B) $f'(-1)=0$

(C) The graph of *f* has a horizontal asymptote

(D) The graph of *f* has a horizontal tangent line at $x=3$

(E) The graph of *f* intersects both axes

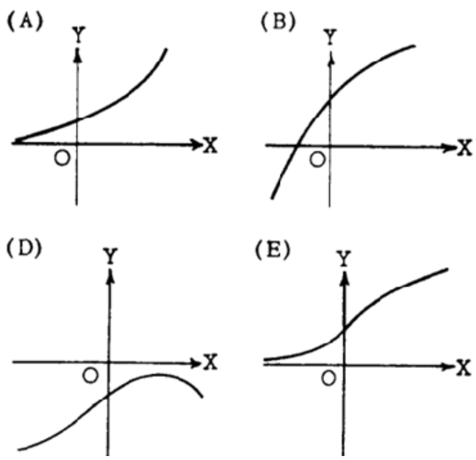
_____ 8. What are the coordinates of the inflection point on the graph of $y=(x+1)\arctan x$?

(A) $(-1,0)$ (B) $(0,0)$ (C) $(0,1)$ (D) $\left(1,\frac{\pi}{4}\right)$ (E) $\left(1,\frac{\pi}{2}\right)$

_____ 9. The Mean Value Theorem guarantees the existence of a special point on the graph of $y=\sqrt{x}$ between $(0,0)$ and $(4,2)$. What are the coordinates of this point?

(A) $(2,1)$ (B) $(1,1)$ (C) $(2,\sqrt{2})$ (D) $\left(\frac{1}{2},\frac{1}{\sqrt{2}}\right)$ (E) None of these

_____ 10. If y is a function of x such that $y'>0$ for all x and $y''<0$ for all x, which of the following could be part of the graph of $y=f(x)$?



______ 11. Given the function defined by $f(x)=3x^5-20x^3$, find all values of x for which the graph of *f* is concave up.

(A) $x>0$ (B) $-\sqrt{2}<x<0$ or $x>\sqrt{2}$ (C) $-2<x<0$ (D) $x>\sqrt{2}$ (E) $-2<x<2$

_____ 12. If $f(x)=x+\frac{1}{x}$, then the set of values for which *f* increases is

(A) $(-\infty,-1]\cup[1,\infty)$ (B) $[-1,1]$ (C) $(-\infty,\infty)$ (D) $(0,\infty)$ (E) $(-\infty,0)\cup(0,\infty)$

_____ 13. If $f(x)=\frac{1}{3}x^3-4x^2+12x-5$ and the domain is the set of all x such that $0\le x\le 9$, then the absolute maximum value of the function *f* occurs when x is

(A) 0 (B) 2 (C) 4 (D) 6 (E) 9

_____ 14. Let g be a continuous function on the closed interval $[0,1]$. Let $g(0)=1$ and $g(1)=0$. Which of the following is NOT necessarily true?

(A) $\exists h \in [0,1] \ni g(h) \geq g(x) \forall x \in [0,1]$

(B) $\forall a,b \in [0,1]$, if $a=b$, then $g(a)=g(b)$.

(C) $\exists h \in [0,1] \ni g(h) = \frac{1}{2}$

(D) $\exists h \in [0.1] \ni g(h) = \frac{3}{2}$

(E) $\forall h \in (0,1)$, $\lim_{x \to h} g(x) = g(h)$

_____ 15. If f is a continuous function defined for all real numbers x and if the maximum value of $f(x)$ is 5 and the minimum value of $f(x)$ is -7, then which of the following must be true?

I. The maximum value of $f(|x|)$ is 5.

II. The maximum value of $|f(x)|$ is 7.

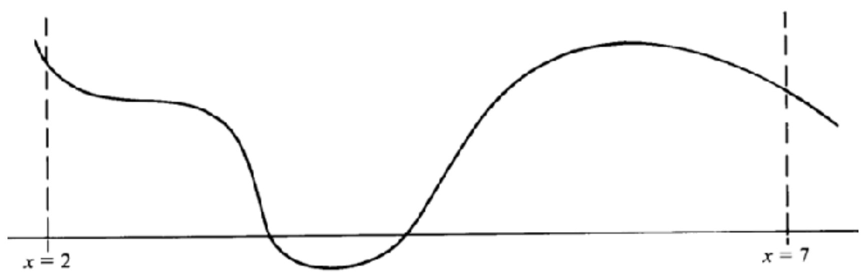
III. The minimum value of $f(|x|)$ is 0.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, and III

_____ 16. Let f be the function given by $f(x)=x^3-3x^2$. What are the values of c that satisfy the conclusion of the Mean Value Theorem of differential calculus on the closed interval $[0,3]$?

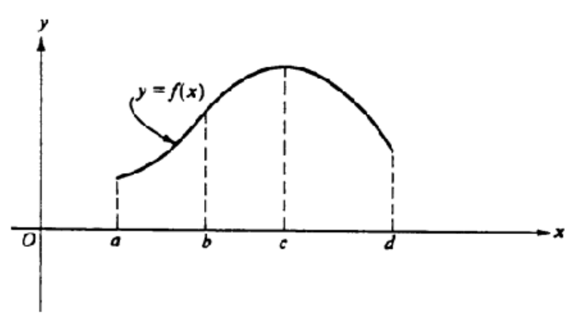
(A) 0 only (B) 2 only (C) 3 only (D) 0 and 3 (E) 2 and 3

_____ 17. The graph of $y=f(x)$ on the closed interval $[2,7]$ is shown below. How many points of inflection does this graph have on this interval?



(A) one (B) two (C) three (D) four (E) five

_____ 18. The graph of $y=f(x)$ is shown at right. On which of the following intervals are $\frac{dy}{dx} > 0$ and $\frac{d^2y}{dx^2} < 0$?



I. $a < x < b$
II. $b < x < c$
III. $c < x < d$

(A) I only (B) II only (C) III only (D) I and II (E) II and III

Free Response:

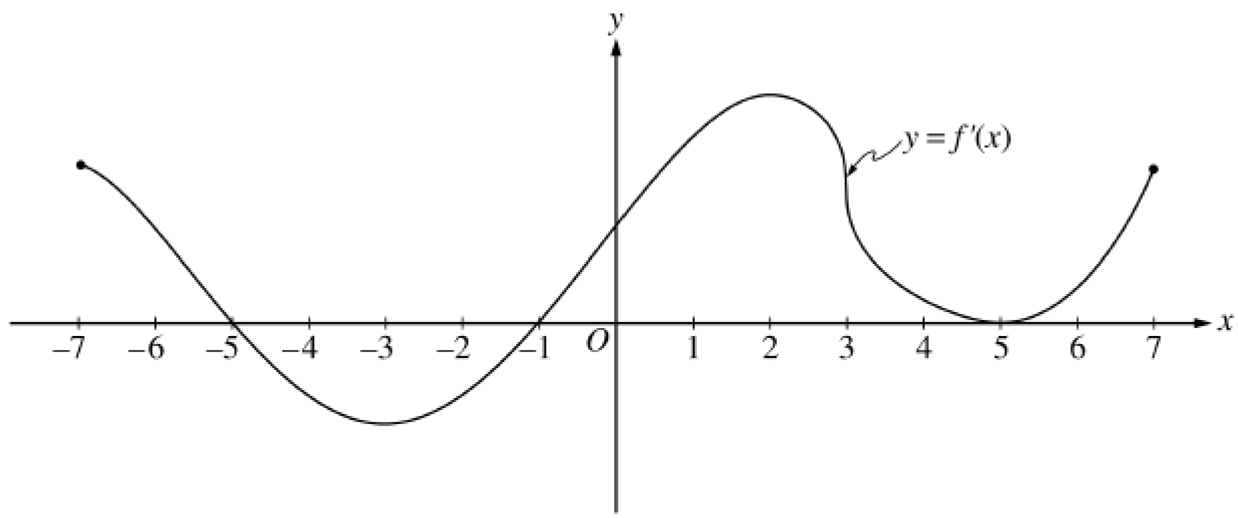
2001 AB4

Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by

$h'(x) = \dfrac{x^2 - 2}{x}$ for all $x \neq 0$.

a) Find all the values of x for which the graph of h has a horizontal tangent, and the determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.
b) On what intervals, if any, is the graph of h concave up? Justify your answer.
c) Write an equation for the tangent line to the graph of h at $x = 4$.
d) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

2000 AB3



The figure above shows the graph of f', the derivative of the function f, for $-7 \leq x \leq 7$. The graph of f' has horizontal tangent lines at $x = -3$, $x = 2$, and $x = 5$, and a vertical tangent line at $x = 3$.

(a) Find all values of x, for $-7 < x < 7$, at which f attains a relative minimum. Justify your answer.

(b) Find all values of x, for $-7 < x < 7$, at which f attains a relative maximum. Justify your answer.

(c) Find all values of x, for $-7 < x < 7$, at which $f''(x) < 0$.

(d) At what value of x, for $-7 \leq x \leq 7$, does f attain its absolute maximum? Justify your answer.

1999 AB4

Suppose that the function f has a continuous second derivative for all x, and that $f(0) = 2$, $f'(0) = -3$, and $f''(0) = 0$. Let g be a function whose derivative is given by $g'(x) = e^{-2x}\left(3f(x) + 2f'(x)\right)$ for all x.

a) Write an equation of the line tangent to the graph of f at the point where $x = 0$
b) Is there sufficient information to determine whether or not the graph of f has a point of inflection when $x = 0$? Explain your answer.
c) Given that $g(0) = 4$, write an equation of the line tangent to the graph of g at the point where $x = 0$.
d) Show that $g''(x) = e^{-2x}\left(-6f(x) - f'(x) + 2f''(x)\right)$. Does g have a local maximum at $x = 0$? Justify your answer.